Homework 8

See due date on Canvas

COE 347

There is absolutely no tolerance for academic misconduct. All assigned material is to be prepared individually. Late assignments will not be accepted, unless under exceptional circumstances at the instructor's full discretion.

Submit your homework electronically as a PDF via Canvas by 11:59 pm on the due date.

If you are submitting a scanned copy of your handwritten notes, rather than a typeset document, please take the time to reduce the file to a manageable size by adjusting the resolution.

Objectives

The objective of this homework is to explore the integration of non-linear conservative laws with conservative methods.

Instructions

- **Problem 1 (30 pts) - MacCormack method in conservative form.**

  Consider the MacCormack method, which was discussed in class and in the book. The first step provides a *provisional state* U_i^* via a forward difference of the flux

  $$U_i^* = U_i^n - \frac{k}{h}[f(U_{i+1}^n) - f(U_i^n)], \tag{1}$$

  and the second step uses a backward difference to *correct* the provisional value

  $$U_i^{n+1} = \frac{U_i^* + U_i^n}{2} - \frac{k}{2h}[f(U_i^*) - f(U_{i-1}^*)]. \tag{2}$$

  Manipulate the two equations above to find expressions for the left and right numerical fluxes F_i^L and F_i^R. Once you have found the two expressions, argue that the method is a conservative method.

  [Grading rubric: (15 pts) One equation is obtained for U_i^{n+1} and it conforms to the conservative method template. (15 pts) The right and left fluxes are identified and their expressions are correct.]

- **Problem 2 (70 pts) - Solution of the inviscid Burgers' equation.**

  Consider two IVPs discussed in class: *(a)* the right travelling step (or discontinuity), and *(b)* the right travelling hump leading to wave breaking. For both IVPs, you are to solve the problems numerically in a finite domain $x = [-2, 2]$, with N interior cells, from time $t = 0$ to time $t = T$ with the initial conditions discussed below.

(a) Right travelling step.

$$u^0(x) = \begin{cases} 2, & x < 0 \\ 1, & x > 0 \end{cases} \tag{3}$$

(b) Wave breaking.

$$u^0(x) = \begin{cases} [\cos(2\pi x) + 1]/2 + 1, & -1/2 \le x \le 1/2 \\ 1, & \text{otherwise} \end{cases} \tag{4}$$

– **2a (30 pts) -** Implement the Lax-Wendroff (LW), Richtmyer (RM), and MacCormack (MCK) methods inside the function `fluxevaluate.m` and turn in the updated version of the source file.

  [Grading rubric: (10 pts) for each of the three methods' implementation.]

– **2b (20 pts) -** Obtain numerical solutions for IVP (a) at $T = 2/3$ with each of four methods, first-order upwind (FOU), LW, RM, and MCK, with $\sigma = 0.75$ and $N = \{50, 200, 800\}$. Recall that the analytical solution consists of the same step shifted to the right by $Ts = (2/3) \times \times (1+2)/2 = 1$ where $s = (u_L + u_R)/2 = 3/2$ is the step speed.

  Produce 3 figures, one for each level of grid refinement. On each figure, plot the exact solution with a solid black line and the four numerical solutions with lines and symbols. Make sure to label all data, axes, and figures clearly.

  Comment on the similarities and differences among the numerical solutions obtained with each method. Adjust the view in the figures above to best illustrate the conclusions you make.

  [Grading rubric: 5 points for each figure. 5 points for commentary.]

– **2c (20 pts) -** Obtain numerical solutions for IVP (b) at $T = 1/\pi - 10^{-5}$, i.e. just before wave breaking, with each of four methods, first-order upwind (FOU), LW, RM, and MCK, with $\sigma = 0.75$ and $N = \{50, 200, 800\}$. The analytical solution is available to you in `hump_analytical.dat`, included as part of the homework material.

  Produce 3 figures, one for each level of grid refinement. On each figure, plot the exact solution with a solid black line and the four numerical solutions with lines and symbols. Make sure to label all data, axes, and figures clearly.

  Comment on the similarities and differences among the numerical solutions obtained with each method. Adjust the view in the figures above to best illustrate the conclusions you make.

  [Grading rubric: 5 points for each figure. 5 points for commentary.]