Exercise sheet n° 1

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Exercise 1.

1. The AMPL solution for Rosenbrock is found in "exercise1/solution.out".
2. Let $a = 100$, we have

$$\begin{aligned}\frac{\delta g}{\delta x_1} &= \frac{\mathrm{d}}{\mathrm{d}x_1}\left[(1-x_1)^2 + a(x_2 - x_1^2)^2\right]\\ &= -2(1-x_1) - 2ax_1(x_2 - x_1^2);\end{aligned}$$

$$\begin{aligned}\frac{\delta g}{\delta x_k} &= \frac{\mathrm{d}}{\mathrm{d}x_k}\left[a(x_k - x_{k-1}^2)^2 + (1-x_k)^2 + a(x_{k+1} - x_k^2)^2\right]\\ &= 2a(x_k - x_{k-1}^2) - 2(1-x_k) - 2ax_k(x_{k+1} - x_k^2), \text{ for } 2 \le k \le n-1;\end{aligned}$$

$$\begin{aligned}\frac{\delta g}{\delta x_n} &= \frac{\mathrm{d}}{\mathrm{d}x_n}\left[a(x_n - x_{n-1}^2)^2\right]\\ &= 2a(x_n - x_{n-1}^2);\end{aligned}$$

The MATLAB script is found in "exercise1/stationary.m".

Exercise 2. f(x) =

1. The solution is found in "exercise2/solution.out".

2. We have

$$\begin{aligned}\frac{\delta f}{\delta x_1} &= 2(1-x_2^3)(x_1(1-x_2^3) - 2.625) - 2(1-x_2^2)(2.25 - x_1(1-x_2)^2) - 2(1-x_2)(1.5 - x_1(1-x_2)),\\ \frac{\delta f}{\delta x_2} &= -6x_1x_2^2(x_1(1-x_2^3) - 2.625) + 4x_1x_2(2.25 - x_1(1-x_2)^2) + 2x_1x_2(1.5 - x_1(1-x_2)).\end{aligned}$$

Exercise 3.

1.

2.

a. We have $\lim\limits_{t\to\infty} f_x(t) = x_2$, so we can approximate $x_2 \approx \operatorname{argmax} f_x(t) = 300$.

b. Using $x_2 = 300$ and the average of $f_x(0)$ given by the data, we have $f_x(0) = \dfrac{300}{1+300x_3} = 19.5$. Therefore, $x_3 \approx 0.04795$.

c. Using the data, we approximate $f_x(t)$ as followings

t	f_x	t	f_x
0	19.5	21	
1	20		
2	23.5		

We have

$$f_x'(t) = \frac{-x_2(-x_1x_2x_3e^{-x_1t})}{(1+x_2x_3e^{-x_1t})^2} = x_1x_3f_x(t)^2e^{-x_1t}.$$

Hence

$$\begin{aligned}
f_x''(t) &= x_1x_3\left[2f_x(t)f_x'(t)e^{-x_1t} - x_1e^{-x_1t}f_x(t)^2\right] \\
&= x_1x_3\left[2x_1x_3f_x(t)^3e^{-2x_1t} - x_1f_x(t)^2e^{-x_1t}\right] \\
&= x_1^2x_3f_x(t)^2e^{-x_1t}\left[2x_3f_x(t)e^{-x_1t} - 1\right].
\end{aligned}$$

Therefore, to determine whether $f_x''(t) = 0$, we only have to determine whether $2x_3f_x(t)e^{-x_1t} = 1$, that is

$$\frac{2x_2x_3e^{-x_1t}}{1+x_2x_3e^{-x_1t}} = \frac{2x_2x_3}{e^{x_1t}+x_2x_3} = 1.$$

Thus, we can approximate $x_1 =$