Exercise sheet n° 1

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Exercise 1.

1. The AMPL solution for Rosenbrock is found in "exercise1/solution.out".
2. Let $a = 100$, we have

$$\begin{aligned}\frac{\delta g}{\delta x_1} &= \frac{\mathrm{d}}{\mathrm{d}x_1}\left[(1-x_1)^2 + a(x_2 - x_1^2)^2\right] \\ &= -2(1-x_1) - 2ax_1(x_2 - x_1^2);\end{aligned}$$

$$\begin{aligned}\frac{\delta g}{\delta x_k} &= \frac{\mathrm{d}}{\mathrm{d}x_k}\left[a(x_k - x_{k-1}^2)^2 + (1-x_k)^2 + a(x_{k+1} - x_k^2)^2\right] \\ &= 2a(x_k - x_{k-1}^2) - 2(1-x_k) - 2ax_k(x_{k+1} - x_k^2), \text{ for } 2 \leq k \leq n-1;\end{aligned}$$

$$\begin{aligned}\frac{\delta g}{\delta x_n} &= \frac{\mathrm{d}}{\mathrm{d}x_n}\left[a(x_n - x_{n-1}^2)^2\right] \\ &= 2a(x_n - x_{n-1}^2);\end{aligned}$$

The MATLAB script is found in "exercise1/exercise1.m".

Exercise 2.

1. The solution is found in "exercise2/solution.out".

2. We have

$$\begin{aligned}\frac{\delta f}{\delta x_1} &= 2(1-x_2^3)(x_1(1-x_2^3) - 2.625) - 2(1-x_2^2)(2.25 - x_1(1-x_2)^2) - 2(1-x_2)(1.5 - x_1(1-x_2)), \\ \frac{\delta f}{\delta x_2} &= -6x_1x_2^2(x_1(1-x_2^3) - 2.625) + 4x_1x_2(2.25 - x_1(1-x_2)^2) + 2x_1x_2(1.5 - x_1(1-x_2)).\end{aligned}$$

Exercise 3.

1. The script is found in "exercise2/exercise2.m". The matrix of $(1, t_i, \cdots, t_i^p)$ becomes to singular when p is 4 or 5, leading to imprecise results. Meanwhile, using the weights returned by the backslash operator, we can calculate that the residual does not reduce when p get larger. We conclude that the polynomial model is not a good one.

2.

a. We have $\lim_{t\to\infty} f_x(t) = x_2$, so we can approximate $x_2 \approx \operatorname{argmax} f_x(t) = 300$.

b. Using $x_2 = 300$ and the average of $f_x(0)$ given by the data, we have $f_x(0) = \dfrac{300}{1+300x_3} = 19.5$. Therefore, $x_3 \approx 0.04795$.

c. We have

$$f_x'(t) = \frac{-x_2(-x_1 x_2 x_3 e^{-x_1 t})}{(1 + x_2 x_3 e^{-x_1 t})^2} = x_1 x_3 f_x(t)^2 e^{-x_1 t}.$$

Hence

$$\begin{aligned} f_x''(t) &= x_1 x_3 \left[2 f_x(t) f_x'(t) e^{-x_1 t} - x_1 e^{-x_1 t} f_x(t)^2\right] \\ &= x_1 x_3 \left[2 x_1 x_3 f_x(t)^3 e^{-2x_1 t} - x_1 f_x(t)^2 e^{-x_1 t}\right] \\ &= x_1^2 x_3 f_x(t)^2 e^{-x_1 t} \left[2 x_3 f_x(t) e^{-x_1 t} - 1\right]. \end{aligned}$$

Therefore, to determine whether $f_x''(t) = 0$, we only have to determine whether $2x_3 f_x(t) e^{-x_1 t} = 1$, that is

$$\frac{2 x_2 x_3 e^{-x_1 t}}{1 + x_2 x_3 e^{-x_1 t}} = \frac{2 x_2 x_3}{e^{x_1 t} + x_2 x_3} = 1.$$

The estimation of f_x'' in the MATLAB script is not a good one, we can approximate $x_1 = \dfrac{\ln(x_2 x_3)}{20} \approx 0.1333$.

The plot appears after running "exercise2/plot_user.m"