GRADIENT DESCENT ON MANIFOLDS

Data Science Project

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Outline

Introduction

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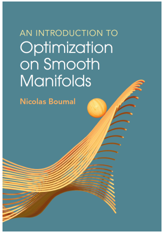
- Many optimization problems involve constraints that can be naturally modeled as manifolds.
- Gradient descent on manifolds extends traditional gradient descent methods to handle these constraints effectively.
- Presentation goals: build up the concept of gradient on manifolds and obtain basic convergence results.
- Main reference: An Introduction to Optimization on Smooth Manifolds by Nicolas Boumal.



Embedded Submanifolds of a Linear Space

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A subset $\mathcal{M}$ of $\mathbb{R}^d$ is an embedded manifold of dimension n if for each point $x \in \mathcal{M}$, there exists a neighborhood in $\mathcal{M}$ of x (i.e. $\mathcal{M} \cap U$ for some open set $U \subset \mathbb{R}^d$ containing x) that is approximate to an open subset of $\mathbb{R}^n$.

We consider smooth submanifolds: for each $x \in \mathcal{M}$, there exists an open set $U \subset \mathbb{R}^d$ containing x and a smooth map $h : U \to \mathbb{R}^{d-n}$ such that $M \cap U = h^{-1}(\{0\})$.

By being approximate to $\mathbb{R}^n$, we mean that for any direction $v \in \mathbb{R}^d$ that is a tangent vector to $\mathcal{M}$ at x, we have

$$h(x + tv) = o(t).$$

Definition
Tangent space

Now we can use Taylor expansion to write

$$h(x + tv) = h(x) + t\mathrm{D}h(x)[v] + o(t) = t\mathrm{D}h(x)[v] + o(t).$$

where $\mathrm{D}h(x)$ is the differential of h at x.

Thank you for listening !