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4) No. Rock-paper-scissors has no PNE to converge.

EXERCISE 3

1) NEs returned by nashpy are

$$\left[\begin{pmatrix}1\\0\end{pmatrix},\begin{pmatrix}1\\0\end{pmatrix}\right] \quad \left[\begin{pmatrix}0\\1\end{pmatrix},\begin{pmatrix}0\\1\end{pmatrix}\right] \quad \left[\begin{pmatrix}0,5\\0,5\end{pmatrix},\begin{pmatrix}0,75\\0,25\end{pmatrix}\right].$$

2) Payoff vectors:

$$v_1 = \cancel{4x_{2C}+2x_{2D}} \begin{pmatrix}4x_{2C}\\ 2x_{2C}+6x_{2D}\end{pmatrix} = \begin{pmatrix}4x_{2C}\\ 6-4x_{2C}\end{pmatrix}$$

$$v_2 = \begin{pmatrix}6x_{1A}\\ 2x_{1A}+4x_{1B}\end{pmatrix} = \begin{pmatrix}6-6x_{1B}\\ 2+2x_{1B}\end{pmatrix}.$$

$$\dot{x}_{1B} = x_{1B}\left[v_1(B) - u_1\left(B, \overset{x}{\begin{pmatrix}x_{2C}\\ x_{2D}\end{pmatrix}}\right)\right]$$

$$= x_{1B}\left[6-4x_{2C} - 4x_{1A}x_{2C} - x_{1B}(6-4x_{2C})\right]$$

$$= x_{1B}\left[6-4x_{2C} - 4(1-x_{1B})x_{2C} - x_{1B}(6-4x_{2C})\right]$$

$$= x_{1B}\left[6-4x_{2C} - 4x_{2C} + 4x_{1B}x_{2C} - 6x_{1B} + 4x_{1B}x_{2C}\right]$$

$$= x_{1B}\left[6 - 6x_{1B} - 8x_{2C} + 8x_{1B}x_{2C}\right]$$

$$= x_{1B}(1-x_{1B})(6-8x_{2C}).$$

$$\dot{x}_{2C} = x_{2C}\left[v_2(C) - u_2(x)\right]$$

$$= x_{2C}\left[6-6x_{1B} - x_{2C}(6-6x_{1B}) - x_{2D}(2+2x_{1B})\right]$$

$$= x_{2C}\left[6-6x_{1B} - x_{2C}(6-6x_{1B}) - (1-x_{2C})(2+2x_{1B})\right]$$

$$= x_{2C}\left[6-6x_{1B} - 6x_{2C} + 6x_{1B}x_{2C} - 2 - 2x_{1B} + 2x_{2C} + 2x_{1B}x_{2C}\right]$$

$$= x_{2C}\left[4 - 8x_{1B} - 4x_{2C} + 8x_{1B}x_{2C}\right]$$

$$= 4x_{2C}(1-2x_{1B})(\cancel{4}-\cancel{4}x_{2C})$$

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Faculté des Sciences et Techniques

NOTE : 19

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Date : 24/10/2024 Session :
Diplôme :
UE :
Nombre de feuilles intercalaires :

Exercise 1

1) Denote $x_1 = (x_{1A}, x_{1B}, x_{1C})^T$ $\quad x = \begin{pmatrix}x_1\\x_2\end{pmatrix}$

$x_2 = (x_{2X}, x_{2Y}, x_{2Z})^T$

We have $v_1(x) = \begin{pmatrix}6x_{2X}+6x_{2Y}+4x_{2Z}\\ 7x_{2X}+4x_{2Y}\\ 5x_{2Z}\end{pmatrix}$

$v_2(x) = \begin{pmatrix}5x_{1A}+10x_{1B}+x_{1C}\\ 4x_{1A}+6x_{1B}\\ 3x_{1C}\end{pmatrix}$

2) Suppose that there is a function $\phi : A_1 \times A_2 \to \mathbb{R}$ ~~such that~~ satisfying conditions of a potential function, i.e.

$\phi(A,X) - \phi(B,X) = u_1(A,X) - u_1(B,X) = 6-7 = -1$
$\phi(B,X) - \phi(C,X) = 7$
$\phi(A,Y) - \phi(B,Y) = 2$
$\phi(B,Y) - \phi(C,Y) = 4$
$\phi(A,Z) - \phi(B,Z) = 4$
$\phi(B,Z) - \phi(C,Z) = -5$

Let $\phi(A,X) = c \Rightarrow \phi(B,X) = c+1$
Hence $\phi(C,X) = c-6$.
From $\phi(A,X) - \phi(A,Y) = u_2(A,X) - u_2(A,Y) = 1$
We have $\phi(A,Y) = c-1$
Hence $\phi(B,Y) = c-3$ and $\phi(C,Y) = c-7$.
From $\phi(A,X) - \phi(A,Z) = u_2(A,X) - u_2(A,Z) = 5$
We have $\phi(A,Z) = c-5$ ~~and~~, $\phi(B,Z) = c-9$
$\phi(C,Z) = c-4$

In summary, ϕ is represented in a table as

	X	Y	Z
A	c	c-1	c-5
B	c+1	c-3	c-9
C	c-6	c-7	c-4

We can check that ϕ satisfies other differences of payoffs.

Thus, the game is potential, a valid potential function is given already

3) The maximizer of ϕ is a Nash equilibrium of Γ (NE).
Hence (B, X) is an NE.

EXERCISE 2.

1) Denote $X > Y$ if Y is dominated by X.

Step 1: $D > E$, ~~$B > A$~~. $A > B$.

Remaining

	A	C	D
X	(50, −2)	(50, −2)	(−4, 40)
Y	(0, 0)	(2, −26)	(−4, −10)
Z	(30, 2)	(4, 4)	(24, 0)
V	(40, 2)	(30, −2)	(8, −2)
W	(2, −66)	(−6, 86)	(5, −12)

Step 2. $X > Y$ $Z > W$

	A	C	D

Remaining

Step 1: $B > A$

Remaining

	B	C	D	E
X	(−4, 0)	(50, −2)	(−4, 40)	(−6, 39)
Y	(−2, 30)	(2, −26)	(−4, −10)	(−6, −40)
Z	(4, 10)	(4, 4)	(24, 0)	(4, 6)
V	(0, 4)	(30, −2)	(8, −2)	(0, 8)
W	(−1, 10)	(−6, 86)	(5, −12)	(−2, −24).

Step 2: $Z > Y$, $Z > W$

~~Remaing~~ Remaining

	B	C	D	E
X	(−4, 0)	(50, −2)	(−4, 40)	(−6, 39)
Z	(4, 10)	(4, 4)	(24, 0)	(4, 6)
V	(0, 4)	(30, −2)	(8, −2)	(0, 8)

Step 3: $B > C$

Remaining

	B	D	E
X	(−4, 0)	(−4, 40)	(−6, 39)
Z	(4, 10)	(24, 0)	(4, 6)
V	(0, 4)	(8, −2)	(0, 8)

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Step 4. $Z > X$

Remaining.

	B	D	E
Z	(4, 10)	(24, 0)	(4, 6)
V	(0, 4)	(8, −2)	(0, 8)

Step 5. $B > D$.

Remaining.

	B	E
Z	~~(24, 0)~~ (4, 10)	(4, 6)
V	~~(8, −2)~~ (0, 4)	(0, 8)

Step 6. $Z > V$

Remaining

	B	E
Z	~~(24, 0)~~ (4, 10)	(4, 6).

Step 7. $B > E$

Remaining (Z, B).

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2) The game is dominance solvable, since the remaining after elimination is a singleton $\{(Z, B)\}$

3).

$$(W, E) \xrightarrow[C = \arg\max_b u_2(W, b)]{Z = \arg\max_a u_1(a, E)} (Z, C).$$

at each step, pick "one" player i to choose his best action while fixing the action of the other player.

$$\xrightarrow[B = \arg\max_b u_2(Z, C)]{X = \arg\max_a u_1(a, C)} (X, B).$$

0

(Similarly) $\rightarrow (Z, D) \rightarrow (V, B)$
$\rightarrow (Z, E) \rightarrow \underbrace{(Z, B) \rightarrow (Z, B)}_{\text{converges.}}$

The convergent point is (Z, B).

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3) RD approaches $\left[\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right]$

4) $u_1\left(\begin{pmatrix}0\\1\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right) = \cancel{\text{argma}} > u_1\left[\begin{pmatrix}1\\0\end{pmatrix}, \begin{pmatrix}0\\1\end{pmatrix}\right]$.

$\Rightarrow$ ~~x_1~~ X_1 is not a NE, hence not stable.

Similarly for X_4.

From Nashpy, X_2, X_3, X_5 are NEs.

~~$v_1\left(\begin{pmatrix}1\\0\end{pmatrix}\right) = 4x_{1A} + 2x_{1B} =$~~

$$u_1\left(\begin{pmatrix}x_{1A}\\x_{2B}\end{pmatrix}, \begin{pmatrix}1\\0\end{pmatrix}\right) = 4x_{1A} + 2x_{2B}$$

$$= 4 - 2x_{2B} \leq 4$$

Equality holds iff $x_{2B} = 0$

Similarly for u_2.

Hence X_2 is a strict NE, hence asymtotically stable.

Similarly for X_3.

From ODEs, all points are stationary.

$$u_1\left(\begin{pmatrix}x_{1A}\\x_{1B}\end{pmatrix}, \begin{pmatrix}0,75\\0,25\end{pmatrix}\right) = 3(x_{1A} + x_{2B}) = 3, \forall x_{1A} \in [0, 1$$

Hence X_5 is not a strict NE

~~Rerun RD for starting point~~

Since RD for $x(0) = \begin{pmatrix}0,6\\0,5\end{pmatrix}$ converges to $\begin{pmatrix}1\\0\end{pmatrix}$

There exists ~~a neighborhood~~ $\varepsilon > 0$

such that $\left\| x(t) - \begin{pmatrix}0,5\\0,75\end{pmatrix}\right\| > \varepsilon$

Hence X_5 is not Lyapunov stable.

Summary	Stationary :	X_1	X_2	X_3	X_4	X_5
	NE		X_2	X_3		X_5
	Strict NE		X_2	X_3		
	Lyapunov stable		X_2	X_3		
	Asymtotically Stabe		X_2	X_3.		