Master 2 Mathematics and Computer Science Symbolic Dynamics

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- Course format: 8 sessions, each 3 hours long
- Course assessment: a written exam (two hours).

Course description

- Lecture 1: Shift spaces. Shifts of finite type. Sofic shifts. Irreducible sofic shifts. Minimal deterministic presentation. Decidability of finite type.
- Lecture 2: Conjugacy and state-splitting. Perron-Frobenius theorem. Conjugacy invariants. Entropy, zeta function.
- Lecture 3: One-sided shift spaces, Williams' theory.
- Lecture 4: Invited lectures: Valérie Berthé and Olivier Carton, Novembre 20, 2025.
- Lecture 5: Substitutions, substitution shifts. Primitive substitutions, linear complexity, linear recurrence, minimality, and uniform recurrence, return words.
- Lecture 6: Recognizability in substitution shifts
- Lecture 7: (if time) Synchronization problems: Road Coloring Theorem

Books on symbolic dynamics and non-negative matrices

- Douglas Lind and Brian Marcus. *An Introduction to Symbolic Dynamics and Coding*. Cambridge University Press, Cambridge, 1995. 2nd edition, 2021.

- `https://www.cambridge.org/core/books/`
  `an-introduction-to-symbolic-dynamics-and-coding/`
  `55BF49690F146403335C79ABD84C4C25`

- B. P. Kitchens, *Symbolic Dynamics: One-sided, two-sided and countable state Markov shifts*, Universitext, Springer-Verlag, 1998.

- E. Senata, *Non-negative matrices and Markov Chains*, Springer Series in Statistics, second edition 2006.

Books on combinatorics on words, symbolic dynamics, and substitutions

- Marie-Pierre Béal, Fabien Durand, and Dominique Perrin, *Symbolic Dynamics and Substitutions*. preprint.

- Jean Berstel, Dominique Perrin, and. Christophe Reutenauer, *Codes and Automata*. Cambridge University Press, Cambridge, 2013.

- Combinatorics, Automata and Number Theory, V. Berthé, M. Rigo (eds.). 2010, Encyclopedia Math. Appl., vol. 135, Cambridge University Press, 2013.

Books on combinatorics on words, symbolic dynamics, and substitutions

- Fabien Durand and Dominique Perrin, *Dimension Groups and Dynamical Systems*. Cambridge University Press, 2022, `https://arxiv.org/abs/2007.15721`.

- Martine Queffélec. *Substitution dynamical systems—spectral analysis*, volume 1294 of Lecture Notes in Mathematics. Springer-Verlag, Berlin, 1987. Second edition: 2010.

- N. Pytheas Fogg, *Substitutions in Dynamics, Arithmetics and Combinatorics*, V. Berthé and S. Ferenczi and C. Mauduit and A. Siegel (eds), Lecture Notes in Mathematics, vol. 1794, Springer-Verlag.