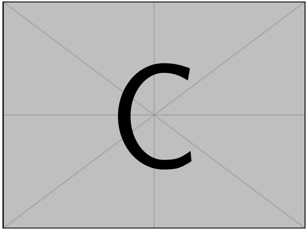
# Algebra



# Groups

**Theorem 1.1 (Lagrange's Theorem. )**

*Let $G$ be a finite group, and let $H$ be a subgroup of $G$. Then the order of $H$ divides the order of $G$.*

**Proof.**

Since $G$ is finite, there exist finitely many left cosets of $H$ in $G$. Let $m$ be the largest integer such that the cosets $g_1H, g_2H, \ldots, g_mH$ are distinct.
*Claim.* $\forall i \in \{1, \ldots, m\}, \mathrm{card}(g_iH) = \mathrm{card}(H)$.
*Claim.* $\forall i, j \in \{1, \ldots, m\}$ and $i \neq j$, $g_iH \cap g_jH = \varnothing$.
*Claim.* $G = \bigcup_{i=1}^{m} g_iH$. $\square$