Online supplement document

Suppose we are to decompose the difference in total number of deaths between two populations (j=1, 2). The two populations could be defined by time periods, geographic places, or both. Each population has p age groups (i = 1, 2, …, p). Let d_{ij}, n_{ij}, and m_{ij} denote the number of deaths, population size, ASMR for the ij^{th} subgroup, respectively; and s_{ij} represents the proportion of population size of the i^{th} group to total population size for the j^{th} population, respectively, (i = 1, 2, …, p, j=1, 2) (**Table 1**).

Table 1. Meaning of mathematical symbols in decomposition formula

Age group	Population 1 (j=1)				Population 2 (j=2)			
	d_{i1}	n_{i1}	m_{i1}	s_{i1}	d_{i2}	n_{i2}	m_{i2}	s_{i2}
1	d_{11}	n_{11}	m_{11}	s_{11}	d_{12}	n_{12}	m_{12}	s_{12}
2	d_{21}	n_{21}	m_{21}	s_{21}	d_{22}	n_{22}	m_{22}	s_{22}
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
p	d_{p1}	n_{p1}	m_{p1}	s_{p1}	d_{p2}	n_{p2}	m_{p2}	s_{p2}
Total	D_1	N_1	M_1	S_1=1	D_2	N_2	M_2	S_2=1

Note: d_{ij}, n_{ij}, m_{ij}, and s_{ij} are the number of deaths, population size, age-specific mortality rate and proportion of group population to total population of the ij^{th} subgroup. D_1 and D_2, N_1 and N_2, M_1 and M_2 represent total number of deaths, population size and crude mortality rate of populations 1 and 2, respectively.

$$D_1 = \sum_{i=1}^{p} d_{i1}$$

$$D_2 = \sum_{i=1}^{p} d_{i2}$$

$$N_1 = \sum_{i=1}^{p} n_{i1}$$

$$N_2 = \sum_{i=1}^{p} n_{i2}$$

$$M_1 = \sum_{i=1}^{p} m_{i1} s_{i1}$$

$$M_2 = \sum_{i=1}^{p} m_{i2} s_{i2}$$

$$m_{ij} = \frac{d_{ij}}{n_{ij}}$$

$s_{ij} = \frac{n_{ij}}{N_j}$

1. Formula derivation of method I

Step 1: calculate the expected deaths in population 1 (D_{1e}) and population 2 (D_{2e}) by adjusting the population size of two populations to 100,000 persons:

$D_{1e} = \sum_{i=1}^{p} s_{i1} m_{i1} 10^5$

$D_{2e} = \sum_{i=1}^{p} s_{i2} m_{i2} 10^5$

Step 2: calculate the expected deaths ($D_{2e}^{ASMR_1}$) by applying age-specific mortality rates of population 1 to the simulated population of 100,000 persons for population 2:

$D_{2e}^{ASMR_1} = \sum_{i=1}^{p} s_{i2} m_{i1} 10^5$

Step 3: calculate the number of deaths attributed to age structure for population 1:

$\left(D_{2e}^{ASMR_1} - D_{1e}\right)/D_{1e} \times D_1$

$= \left(\sum_{i=1}^{p} s_{i2} m_{i1} 10^5 - \sum_{i=1}^{p} s_{i1} m_{i1} 10^5\right)/\sum_{i=1}^{p} s_{i1} m_{i1} 10^5 \times \sum_{i=1}^{p} d_{i1}$

$= \frac{\left(\sum_{i=1}^{p} s_{i2} m_{i1} - \sum_{i=1}^{p} s_{i1} m_{i1}\right)}{\sum_{i=1}^{p} s_{i1} m_{i1}} \times \sum_{i=1}^{p} d_{i1}$

$= \frac{\left(\sum_{i=1}^{p} s_{i2} m_{i1} - \sum_{i=1}^{p} s_{i1} m_{i1}\right)}{\sum_{i=1}^{p} \frac{n_{i1}}{N_1} \times \frac{d_{i1}}{n_{i1}}} \times \sum_{i=1}^{p} d_{i1}$

$= \frac{\left(\sum_{i=1}^{p} s_{i2} m_{i1} - \sum_{i=1}^{p} s_{i1} m_{i1}\right)}{\sum_{i=1}^{p} \frac{d_{i1}}{N_1}} \times \sum_{i=1}^{p} d_{i1}$

$= \frac{\left(\sum_{i=1}^{p} s_{i2} m_{i1} - \sum_{i=1}^{p} s_{i1} m_{i1}\right)}{\frac{\sum_{i=1}^{p} d_{i1}}{N_1}} \times \sum_{i=1}^{p} d_{i1}$

$= \left(\sum_{i=1}^{p} s_{i2} m_{i1} - \sum_{i=1}^{p} s_{i1} m_{i1}\right) N_1$

$= \sum_{i=1}^{p} N_1 (s_{i2} - s_{i1})\, m_{i1}$

$= M_s$

According to the factorial experiment design of three factors, M_s denotes the main effect of age structure.

Step 4: calculate the number of deaths attributed to ASMR for population 1:

$$\left(D_{2e} - D_{2e}^{ASMR_1}\right)/D_{1e} \times D_1$$

$$= \left(\sum_{i=1}^{p} s_{i2} m_{i2} 10^5 - \sum_{i=1}^{p} s_{i2} m_{i1} 10^5\right)/\sum_{i=1}^{p} s_{i1} m_{i1} 10^5 \times \sum_{i=1}^{p} d_{i1}$$

$$= \frac{\left(\sum_{i=1}^{p} s_{i2} m_{i2} - \sum_{i=1}^{p} s_{i2} m_{i1}\right)}{\sum_{i=1}^{p} s_{i1} m_{i1}} \times \sum_{i=1}^{p} d_{i1}$$

$$= \frac{\left(\sum_{i=1}^{p} s_{i2} m_{i2} - \sum_{i=1}^{p} s_{i2} m_{i1}\right)}{\sum_{i=1}^{p} \frac{n_{i1}}{N_1} \times \frac{d_{i1}}{n_{i1}}} \times \sum_{i=1}^{p} d_{i1}$$

$$= \frac{\left(\sum_{i=1}^{p} s_{i2} m_{i2} - \sum_{i=1}^{p} s_{i2} m_{i1}\right)}{\sum_{i=1}^{p} \frac{d_{i1}}{N_1}} \times \sum_{i=1}^{p} d_{i1}$$

$$= \frac{\left(\sum_{i=1}^{p} s_{i2} m_{i2} - \sum_{i=1}^{p} s_{i2} m_{i1}\right)}{\frac{\sum_{i=1}^{p} d_{i1}}{N_1}} \times \sum_{i=1}^{p} d_{i1}$$

$$= \left(\sum_{i=1}^{p} s_{i2} m_{i2} - \sum_{i=1}^{p} s_{i2} m_{i1}\right) N_1$$

$$= \sum_{i=1}^{p} N_1 s_{i2} (m_{i2} - m_{i1})$$

$$= \sum_{i=1}^{p} N_1 s_{i1} (m_{i2} - m_{i1}) + \sum_{i=1}^{p} N_1 (s_{i2} - s_{i1})(m_{i2} - m_{i1})$$

$$= M_m + I_{sm}$$

According to the factorial experiment design of the three factors, the first part is the main effect of ASMR (M_m), and the second part is the two-way interaction of ASMR and age structure (I_{sm}).

Step 5: calculate number of deaths attributed to population size:

$$(D_2 - D_1) - \left(D_{2e}^{ASMR_1} - D_{1e}\right)/D_{1e} \times D_1 - \left(D_{2e} - D_{2e}^{ASMR_1}\right)/D_{1e} \times D_1$$

$$= \sum_{i=1}^{p} N_2 s_{i2} m_{i2} - \sum_{i=1}^{p} N_1 s_{i1} m_{i1} - \sum_{i=1}^{p} N_1 (s_{i2} - s_{i1}) m_{i1} - \sum_{i=1}^{p} N_1 s_{i1} (m_{i2} - m_{i1}) - \sum_{i=1}^{p} N_1 (s_{i2} - s_{i1})(m_{i2} - m_{i1})$$

$$= \sum_{i=1}^{p} (N_2 - N_1) s_{i1} m_{i1} + \sum_{i=1}^{p} (N_2 - N_1) s_{i1} (m_{i2} - m_{i1}) + \sum_{i=1}^{p} (N_2 - N_1)(s_{i2} - s_{i1}) m_{i1} + \sum_{i=1}^{p} (N_2 - N_1)(s_{i2} - s_{i1})(m_{i2} - m_{i1})$$

$$= S_p + I_{ps} + I_{pm} + I_{psm}$$

According to the factorial experiment design of the three factors, the first part is the main effect of population size (M_p), and the remaining three parts are two-way interactions of population size and ASMR (I_{pm}), population size and age structure (I_{ps}), and three-way interaction of three factors (I_{psm}), respectively.

2. Formula derivation of method II

Step 1: calculate expected deaths ($D_{2e}^{(AS_1,\ ASMR_1)}$) of population 2 by applying both age-specific mortality rates and population structure of population 1 to population 2:

$$D_{2e}^{(AS_1,\ ASMR_1)} = \sum_{i=1}^{p} N_2 s_{i1} m_{i1}$$

Step 2: calculate expected deaths ($D_{2e}^{ASMR_1}$) of population 2 by applying only age-specific mortality rates of population 1 to population 2:

$$D_{2e}^{ASMR_1} = \sum_{i=1}^{p} N_2 s_{i2} m_{i1}$$

Step 3: calculate number of deaths attributed to population size:

$$D_{2e}^{(AS_1,\ ASMR_1)} - D_1$$

$$= \sum_{i=1}^{p} N_2 s_{i1} m_{i1} - \sum_{i=1}^{p} N_1 s_{i1} m_{i1}$$

$$= \sum_{i=1}^{p} (N_2 - N_1) s_{i1} m_{i1}$$

$$= M_p$$

According to the factorial experiment design of the three factors, M_p represents the main effect of population size.

Step 4: calculate number of deaths attributed to age structure:

$$\left(D_{2e}^{ASMR_1} - D_1\right) - \left(D_{2e}^{(AS_1,\ ASMR_1)} - D_1\right)$$

$$= \left(D_{2e}^{ASMR_1} - D_{2e}^{(AS_1,\ ASMR_1)}\right)$$

$$= \sum_{i=1}^{p} N_2 s_{i2} m_{i1} - \sum_{i=1}^{p} N_2 s_{i1} m_{i1}$$

$= \sum_{i=1}^{p} N_2(s_{i2} - s_{i1})m_{i1}$

$= \sum_{i=1}^{p} N_1(s_{i2} - s_{i1})m_{i1} + \sum_{i=1}^{p}(N_2 - N_1)(s_{i2} - s_{i1})m_{i1}$

$= M_s + I_{ps}$

According to the factorial experiment design of the three factors, M_s denotes the main effect of age structure and I_{ps} represents the two-way interaction of population size and age structure.

Step 5: calculate number of deaths attributed to ASMR:

$(D_2 - D_1) - \left(D_{2e}^{ASMR_1} - D_{2e}^{(AS_1,\, ASMR_1)}\right) - \left(D_{2e}^{(AS_1,\, ASMR_1)} - D_1\right)$

$= \left(D_2 - D_{2e}^{ASMR_1}\right)$

$= \sum_{i=1}^{p} N_2 s_{i2} m_{i2} - \sum_{i=1}^{p} N_2 s_{i2} m_{i1}$

$= \sum_{i=1}^{p} N_2 s_{i2}(m_{i2} - m_{i1})$

$= \sum_{i=1}^{p} N_1 s_{i1}(m_{i2} - m_{i1}) + \sum_{i=1}^{p} N_1(s_{i2} - s_{i1})(m_{i2} - m_{i1}) + \sum_{i=1}^{p}(N_2 - N_1)s_{i1}(m_{i2} - m_{i1}) + \sum_{i=1}^{p}(N_2 - N_1)(s_{i2} - s_{i1})(m_{i2} - m_{i1})$

$= S_m + I_{sm} + I_{pm} + I_{psm}$

According to the factorial experiment design of the three factors, S_m is the simple effect of ASMR, and the remaining three parts are two-way interactions of ASMR and age structure (I_{sm}), population size and ASMR (I_{pm}), and the three-way interaction of the three factors (I_{psm}), respectively.

3. Assessing the performance of methods I and II

3.1. Consistency between methods I and II

Using AS_I (AS_{II}), $ASMR_I$ ($ASMR_{II}$) and PS_I (PS_{II}) to represent the number of deaths attributed to age structure, ASMR and population size defined by method I (II), we calculate differences in the contribution of the three factors between the two methods:

$$AS_I - AS_{II} = M_s - (M_s + I_{ps}) = -I_{ps}$$

$$ASMR_I - ASMR_{II} = M_m + I_{sm} - (M_m + I_{sm} + I_{pm} + I_{psm}) = -I_{pm} - I_{psm}$$

$$PS_I - PS_{II} = M_p + I_{ps} + I_{pm} + I_{psm} - M_p = I_{ps} + I_{pm} + I_{psm}$$

The results show that the decomposition results from methods I and II are different unless there are no interactions between two and three factors, a situation that would be extremely rare in practice.

3.2. Stability of method I

Using AS_I', $ASMR_I'$ and PS_I' to represent the number of deaths attributed to age structure, ASMR and population size when selecting population 2 as the reference, , respectively, we calculate as follows:

$$AS_I' = (D_{1e}^{ASMR_2} - D_{2e})/D_{2e} \times D_2$$

$$= (\sum_{i=1}^{p} s_{i1}m_{i2}10^5 - \sum_{i=1}^{p} s_{i2}m_{i2}10^5)/\sum_{i=1}^{p} s_{i2}m_{i2}10^5 \times \sum_{i=1}^{p} d_{i2}$$

$$= \frac{(\sum_{i=1}^{p} s_{i1}m_{i2} - \sum_{i=1}^{p} s_{i2}m_{i2})}{\sum_{i=1}^{p} \frac{n_{i2}}{N_2} \times \frac{d_{i2}}{n_{i2}}} \times \sum_{i=1}^{p} d_{i2}$$

$$= (\sum_{i=1}^{p} s_{i1}m_{i2} - \sum_{i=1}^{p} s_{i2}m_{i2})N_2$$

$$= \sum_{i=1}^{p} N_2(s_{i1} - s_{i2})\, m_{i2}$$

$$ASMR_I' = (D_{1e} - D_{1e}^{ASMR_2})/D_{2e} \times D_2$$

$$= (\sum_{i=1}^{p} s_{i1}m_{i1}10^5 - \sum_{i=1}^{p} s_{i1}m_{i2}10^5)/\sum_{i=1}^{p} s_{i2}m_{i2}10^5 \times \sum_{i=1}^{p} d_{i2}$$

$$= \frac{(\sum_{i=1}^{p} s_{i1}m_{i1} - \sum_{i=1}^{p} s_{i1}m_{i2})}{\sum_{i=1}^{p} \frac{n_{i2}}{N_2} \times \frac{d_{i2}}{n_{i2}}} \times \sum_{i=1}^{p} d_{i2}$$

$$= (\sum_{i=1}^{p} s_{i1}m_{i1} - \sum_{i=1}^{p} s_{i1}m_{i2})N_2$$

$= \sum_{i=1}^{p} N_2 s_{i1}(m_{i1} - m_{i2})$

$PS'_I = (D_1 - D_2) - \left(D_{1e}^{ASMR_2} - D_{2e}\right)/D_{2e} \times D_2 - \left(D_{1e} - D_{1e}^{ASMR_2}\right)/D_{2e} \times D_2$

$= \sum_{i=1}^{p} N_1 s_{i1} m_{i1} - \sum_{i=1}^{p} N_2 s_{i2} m_{i2} - \sum_{i=1}^{p} N_2 (s_{i1} - s_{i2})\, m_{i2} -$

$\sum_{i=1}^{p} N_2 s_{i1}(m_{i1} - m_{i2})$

$= \sum_{i=1}^{p}(N_1 - N_2) s_{i1} m_{i1}$

Calculating differences in decomposition results of the three factors by changing the reference population from population 1 to 2, we derive the following formulas:

$AS'_I - AS_I = \sum_{i=1}^{p}(s_{i1} - s_{i2})(N_2 m_{i2} - N_1 m_{i1})$

$ASMR'_I - ASMR_I = \sum_{i=1}^{p}(m_{i1} - m_{i2})(N_2 s_{i1} - N_1 s_{i2})$

$PS'_I - PS_I = \sum_{i=1}^{p}(N_1 - N_2)(s_{i1} m_{i1} - s_{i2} m_{i2})$

The decomposition results are different after changing the reference population unless there are no interaction between three factors, a situation that is extremely rare in practice.

3.3. Stability of method II

Using AS'_{II}, $ASMR'_{II}$ and PS'_{II} to represent the number of deaths attributed to age structure, ASMR and population size when selecting population 2 as the reference, respectively, we calculate as follows:

$PS'_{II} = D_{1e}^{(AS_2,\, ASMR_2)} - D_2$

$= \sum_{i=1}^{p} N_1 s_{i2} m_{i2} - \sum_{i=1}^{p} N_2 s_{i2} m_{i2}$

$= \sum_{i=1}^{p}(N_1 - N_2) s_{i2} m_{i2}$

$AS'_{II} = \left(D_{1e}^{ASMR_2} - D_2\right) - \left(D_{1e}^{(AS_2,\, ASMR_2)} - D_2\right)$

$= \left(D_{1e}^{ASMR_2} - D_{1e}^{(AS_2,\, ASMR_2)}\right)$

$$= \sum_{i=1}^{p} N_1 s_{i1} m_{i2} - \sum_{i=1}^{p} N_1 s_{i2} m_{i2}$$

$$= \sum_{i=1}^{p} N_1 (s_{i1} - s_{i2}) m_{i2}$$

$$ASMR'_{II} = (D_1 - D_2) - \left(D_{1e}^{ASMR_2} - D_{1e}^{(AS_2,\, ASMR_2)}\right) - \left(D_{1e}^{(AS_2,\, ASMR_2)} - D_2\right)$$

$$= \left(D_1 - D_{1e}^{ASMR_2}\right)$$

$$= \sum_{i=1}^{p} N_1 s_{i1} m_{i1} - \sum_{i=1}^{p} N_1 s_{i1} m_{i2}$$

$$= \sum_{i=1}^{p} N_1 s_{i1} (m_{i1} - m_{i2})$$

Compared with the original results of selecting population 1 as the reference, we obtain:

$$AS'_{II} - AS_{II} = \sum_{i=1}^{p} (s_{i1} - s_{i2})(N_1 m_{i2} - N_2 m_{i1})$$

$$ASMR'_{II} - ASMR_{II} = \sum_{i=1}^{p} (m_{i1} - m_{i2})(N_1 s_{i1} - N_2 s_{i2})$$

$$PS'_{II} - PS_{II} = \sum_{i=1}^{p} (N_1 - N_2)(s_{i2} m_{i2} - s_{i1} m_{i1})$$

The decomposition results are different after changing the reference population unless there are no interactions between the three factors.

4. Formula derivation of method III

To overcome the deficiencies of methods I and II, we developed an innovative approach, method III, based on the principle that a reliable decomposition method should generate stable and consistent results regardless of the change of reference population.

Based on the factorial experiment design of three factors and methods I and II, we first calculate the main effect, two-way interactions, and three-way interactions between population size, age structure and ASMR on the difference in total deaths between the two populations. We use population 1 as the reference first, and then we conduct decomposition analysis using population 2 as the reference.

Using population 1 as the reference, main effects (M_p, M_s and M_m) and two-way and three-way interactions (I_{ps}, I_{pm}, I_{sm} and I_{psm}) of the three factors are calculated as follows:

$M_p = \sum_{i=1}^{p}(N_2 - N_1)s_{i1}m_{i1}$

$M_s = \sum_{i=1}^{p} N_1(s_{i2} - s_{i1})m_{i1}$

$M_m = \sum_{i=1}^{p} N_1 s_{i1}(m_{i2} - m_{i1})$

$I_{ps} = \sum_{i=1}^{p}(N_2 - N_1)(s_{i2} - s_{i1})m_{i1}$

$I_{pm} = \sum_{i=1}^{p}(N_2 - N_1)s_{i1}(m_{i2} - m_{i1})$

$I_{sm} = \sum_{i=1}^{p} N_1(s_{i2} - s_{i1})(m_{i2} - m_{i1})$

$I_{psm} = \sum_{i=1}^{p}(N_2 - N_1)(s_{i2} - s_{i1})(m_{i2} - m_{i1})$

Using population 2 as the reference, the formulas are calculated as follows:

$M'_p = \sum_{i=1}^{p}(N_1 - N_2)s_{i2}m_{i2}$

$M'_s = \sum_{i=1}^{p} N_2(s_{i1} - s_{i2})m_{i2}$

$M'_m = \sum_{i=1}^{p} N_2 s_{i2}(m_{i1} - m_{i2})$

$I'_{ps} = \sum_{i=1}^{p}(N_1 - N_2)(s_{i1} - s_{i2})m_{i2}$

$I'_{pm} = \sum_{i=1}^{p}(N_1 - N_2)s_{i2}(m_{i1} - m_{i2})$

$I'_{sm} = \sum_{i=1}^{p} N_2(s_{i1} - s_{i2})(m_{i1} - m_{i2})$

$I'_{psm} = \sum_{i=1}^{p}(N_1 - N_2)(s_{i1} - s_{i2})(m_{i1} - m_{i2})$

The contribution of each factor should include its main effect and partly of related interactions.

(1) Suppose *a*%, *b*% and *c*% of the two-way interaction between population size and age structure, population size and ASMR, and age structure and ASMR are

allocated to the first factor, respectively. Accordingly, (100-*a*)%, (100-*b*)% and (100-*c*)% of the three two-way interactions are allocated to the second factor.

And (2) suppose d_1%, d_2% and (100- d_1-d_2)% of the three-way interaction are allocated to population size, age structure, and ASMR, respectively.

Using AS_{III} (AS'_{III}), $ASMR_{III}$ ($ASMR'_{III}$) and PS_{III} (PS'_{III}) to represent the number of deaths attributed to age structure, ASMR and population size defined by method III when using population 1 (2) as reference, the contributions of the three factors can be calculated as follows:

$$PS_{III} = M_p + a\%I_{ps} + b\%I_{pm} + d_1\%I_{psm}$$

$$AS_{III} = M_s + (100 - a)\%I_{ps} + \mathrm{c}\%I_{sm} + d_2\%I_{psm}$$

$$ASMR_{III} = M_m + (100 - b)\%I_{pm} + (100 - \mathrm{c})\%I_{sm} + (100 - d_1 - d_2)\%I_{psm}$$

$$PS'_{III} = M'_p + a\%I'_{ps} + b\%I'_{pm} + d_1\%I'_{psm}$$

$$AS'_{III} = M'_s + (100 - a)\%I'_{ps} + \mathrm{c}\%I'_{sm} + d_2\%I'_{psm}$$

$$ASMR'_{III} = M'_m + (100 - b)\%I'_{pm} + (100 - \mathrm{c})\%I'_{sm} + (100 - d_1 - d_2)\%I'_{psm}$$

According to the principle above, the decomposition results should remain unchanged in absolute value when the reference population changes, so we have a group of three equations:

$$\begin{cases} PS_{III} \equiv -PS'_{III} \\ AS_{III} \equiv -AS'_{III} \\ ASMR_{III} \equiv -ASMR'_{III} \end{cases}$$

Through formula derivation, we have three simplified equations:

$$\begin{cases} \sum_{i=1}^{p}(N_2 - N_1)[(s_{i1}m_{i1} - s_{i2}m_{i2})(100 - a - b)\% + (s_{i2}m_{i1} - s_{i1}m_{i2})(a - b)\%] \equiv 0 \\ \sum_{i=1}^{p}(s_{i2} - s_{i1})[(N_2m_{i1} - N_1m_{i2})(100 - a - c)\% + (N_1m_{i1} - N_2m_{i2})(a - c)\%] \equiv 0 \\ \sum_{i=1}^{p}(m_{i2} - m_{i1})[(N_2s_{i2} - N_1s_{i1})(100 - b - c)\% + (N_1s_{i2} - N_2s_{i1})(b - c)\%] \equiv 0 \end{cases}$$

These three equations cannot be true all the time unless *a*, *b*, and *c* all equal 50.

The three equations have no requirements for d_1 and d_2. Given there is no theoretical guidance to allocate the three-way interaction of three factors, we divide it equally, d_1=d_2=1/3×100.

Last, we have the attribution formulas of the three factors as follows when using population 1 as reference:

$$PS_{III} = M_p + 1/2I_{pm} + 1/2I_{ps} + 1/3I_{psm}$$

$$AS_{III} = M_s + 1/2I_{sm} + 1/2I_{ps} + 1/3I_{psm}$$

$$ASMR_{III} = M_m + 1/2I_{pm} + 1/2I_{sm} + 1/3I_{psm}$$