Evaluations of Korean Mortality Forecasting Models

STATISTICS KOREA

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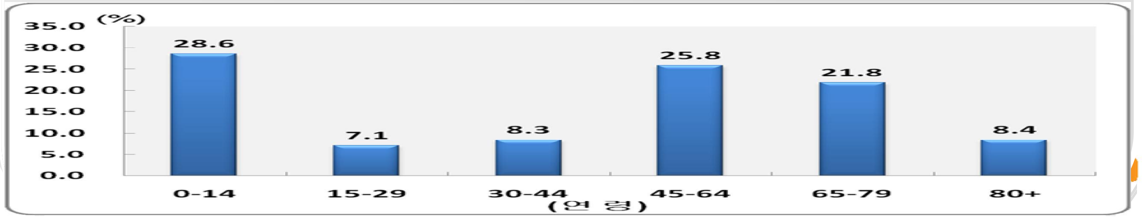
Introduction

Life expectancy at birth for Korea has increased dramatically by 19 years over the past 40 years

Years	1970	2010	Diff.
Males	58.7	77.2	18.5
Females	65.6	84.1	18.5

This excessive gain has resulted mainly from the improved survival of children. However, mortality decline is decelerating at younger ages and accelerating at older ages



Introduction (cont.)

Mortality forecasting, including detailed old age mortality, is of growing interest in the context of population projection, as well as for economic, social, and health planning

- The shortage of historical mortality rates for the elderly needs to be overcome before forecasting mortality in Korea

The growing uncertainty is requesting stochastic models in forecasting

Introduction (cont.)

The purposes of this study are to find the best models to extend mortality for the elderly and to forecast Korean mortality rates stochastically for the period from 2010 to 2060

First, it examines the way to estimate mortality rates for the elderly

- It compares two models to extend the elderly mortality rates and presents the results of an evaluation of the models
- 2-parameter logistic model and the Brass-Logit model

Introduction (cont.)

Second, it evaluates four kinds of Lee-Carter models which are known well for their simplicity and robustness to forecast mortality rates.

Lee-Carter Model, adjusted Lee-Carter Model, Lee-Miller Model, and Li-Lee Model(Coherent Lee-Carter)

Finally, this study presents the results of forecasted life expectancies from 2011 to 2060 from the four kinds of models.

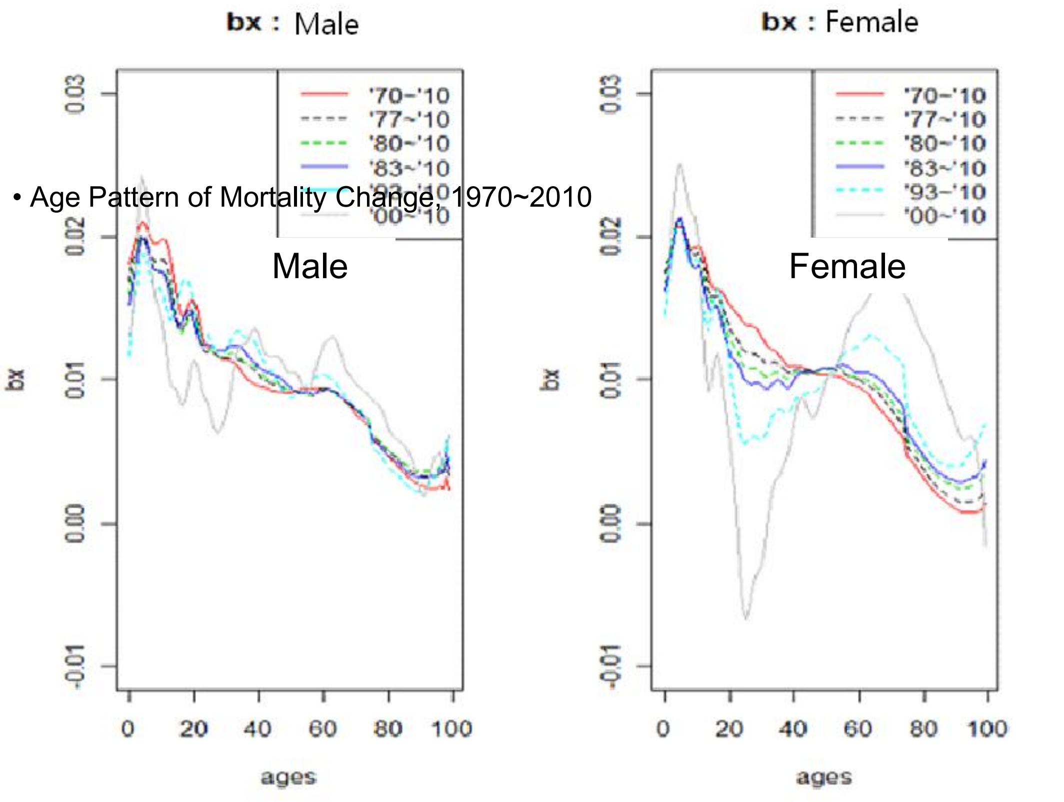
Data

Data
Age Specific Death Rates(ASDR) from Vital Statistics for Korea, 1970-2010

- The Available Data

. 1970s : 0~75+
. 1980s : 0~85+
. 1990s : 0~95+
. 2000s : 0~100+

• Age Pattern of Mortality Change, 1970~2010



Extending Mortality Rates for the Elderly

Estimating Methods of Mortality Rates for Advanced Ages

2-parameter logistic model

$$m_x = \frac{Be^{\beta x}}{1 + Be^{\beta x}}$$

The Brass-Logit model based on 2010 Korean life table

$$Logit(m_{x,t}) = \alpha + \beta \cdot Logit(m_{x,s})$$

$$Logit(m_{x,t}) = 0.5 \cdot \ln\left(\frac{m_{x,t}}{1 - m_{x,t}}\right)$$

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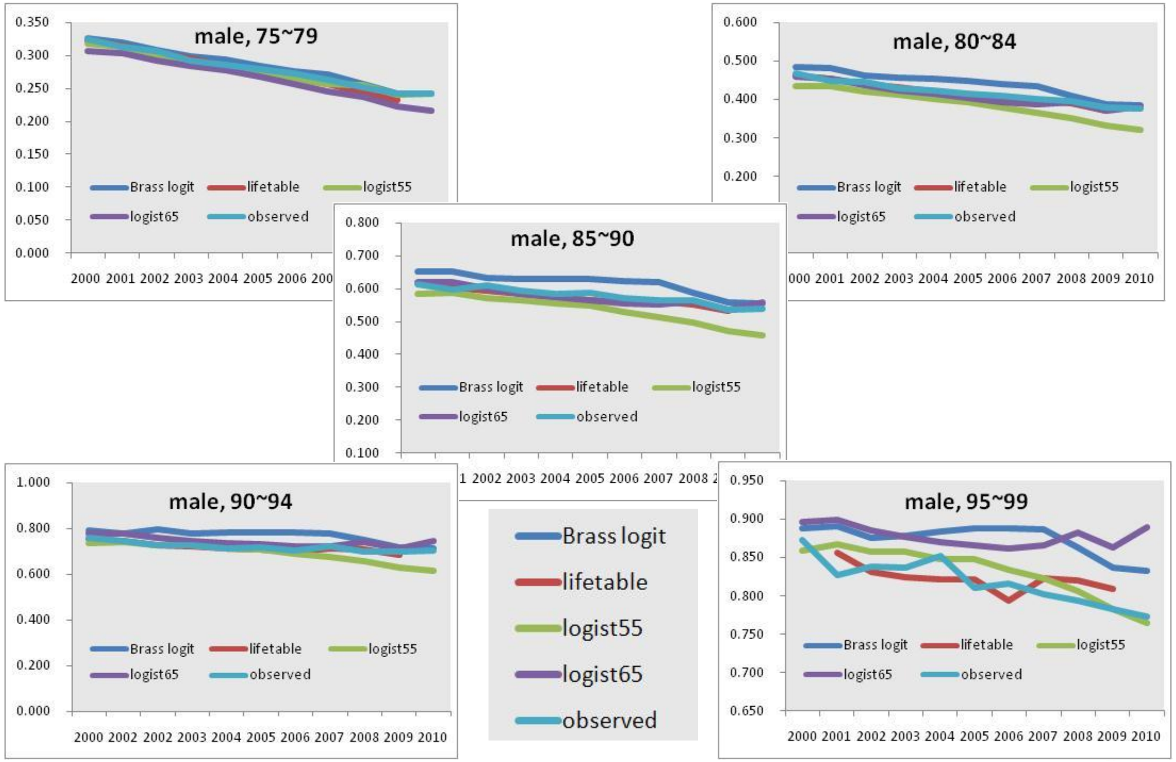
Extending Mortality Rates for the Elderly

The two methods were

1) fitted with death probabilities of age 65 to 75 and age 55 to 75
2) used to estimate death probabilities from age 75 to 100 between 2000 and 2010
3) evaluated by comparing forecasting figures with actual probabilities during that period.

Estimated Mortality Rates for Males Above Age75 using Different Methods



Mean Absolute Percent Error between Estimated and Observed Probabilities of Death for Ages Over 75

Sex	Male			Female		
Method	2-parameter logistic		Brass-Logit	2-parameter logistic		Brass-Logit
	Fitting ages 55 to 75	Fitting ages 65 to 75		Fitting ages 55 to 75	Fitting ages 65 to 75	
MAPE	0.16124	<u>0.09812</u>	0.10006	0.28468	0.14249	<u>0.12844</u>
RMSE	0.02933	<u>0.02325</u>	0.03078	0.04124	<u>0.02727</u>	0.03357

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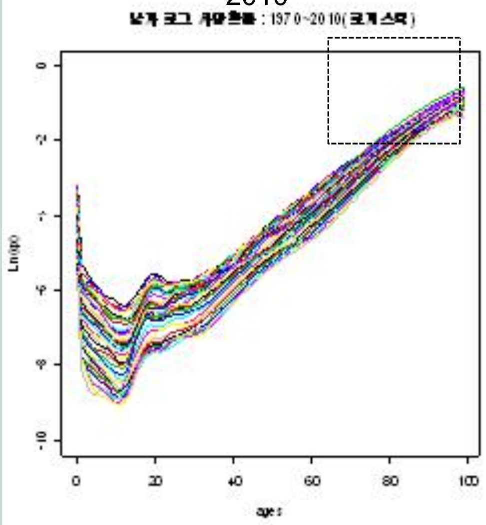
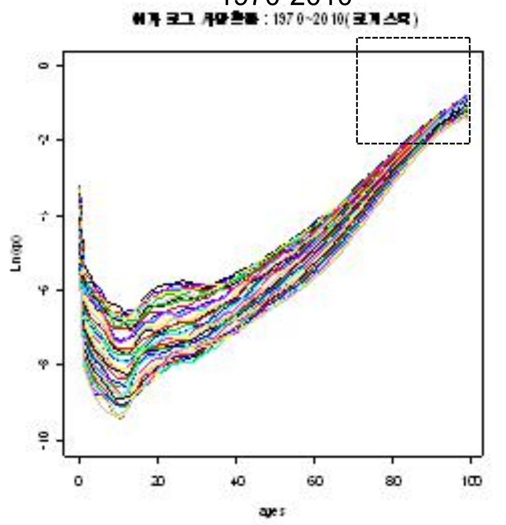
The Extended Age Specific Death Rates for the Period of 1970 ~2010

	Male	Female
2-parameter logistic	Log death rates for male, 1970-2010 	Log death rates for female, 1970-2010 

Evaluations of Mortality Forecasting Models for Korea

We compared four kinds of Lee-Carter Models to find the most appropriate model to forecast Korean mortality

- The Original Lee-Carter model
- The Lee-Carter model with adjusted jump off bias
- The Lee-Miller Model
- The Li-Lee model (The Coherent Lee-Carter model)

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Evaluations of Mortality Forecasting Models for Korea

The evaluation procedure involves fitting the different models with data up to 2010

Forecasting death probabilities for the period of 1970 – 2010

Comparing the forecasts with actual probabilities in that period

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Mortality Forecasting Models

Lee-Carter Model

$$\ln[m(x,t)] = a_x + b_x \cdot k_t + \epsilon_{x,t} \;,$$

Kt : Random Walk with Drift

Lee- Carter model with adjustment jump off bias

$$\ln m(x,t) = \ln m(x,T) + b_x [k_t - k_T], t = 1,2,\cdots,T$$

$$k_t = k_{t-1} + d + e_t \text{ (Random walk with drift model)}$$

Lee-Miller model

$$\ln m(x,t) = \ln m(x,T) + b_x [k_t - k_T], t = 1,2,\cdots,T$$

- adjustment of K_t involves $e(0)$ in year t

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Forecasting Methods of Mortality

Li-Lee model(The Coherent LC model with adjustment jump off bias)

$$\ln m(x,t,i) = \ln m(x,T,i) + B_x[K_t - K_T] + b_{x,i}[k_{t,i} - k_{T,i}] \ , \ t = 1,2,\cdots,T$$

$B_x K_t$: pattern of combined Mortality

$b_{x,i} k_{t,i}$: pattern of separate Mortality

$K_t = K_{t-1} + d + e_t$ (Random walk with drift model)

$k_t = k_{t-1} + e_t$ (Random walk without drift model)

Examination of Goodness-of-fit for the Models

The MAPE of ASDR for Male, 1970~2010

	Male			
	Lee-Carter	LC with adj.	Lee-Miller	Li-Lee (Coh. LC)
Overall	0.0889	0.1850	0.1401	0.0607
1970	0.0783	0.1386	0.0855	0.0739
1980	0.0527	0.2023	0.0613	0.0719
1990	0.0967	0.2622	0.1303	0.0381
2000	0.0628	0.2205	0.1243	0.0566
2010	0.1821	0.0006	0.3050	0.0980

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Examination of Goodness-of-fit for the Models

< Table 2 > The MAPE of ASDR for Female, 1970~2010

	Female			
	Lee-Carter	LC with adj.	Lee-Miller	Li-Lee (Coh. LC)
Overall	0.0655	0.1047	0.0755	0.0548
1970	0.1007	0.0673	0.1082	0.0497
1980	0.1030	0.1692	0.1071	0.1312
1990	0.0503	0.1121	0.0512	0.0296
2000	0.0437	0.1164	0.0462	0.0369
2010	0.0795	0.0016	0.0881	0.0613

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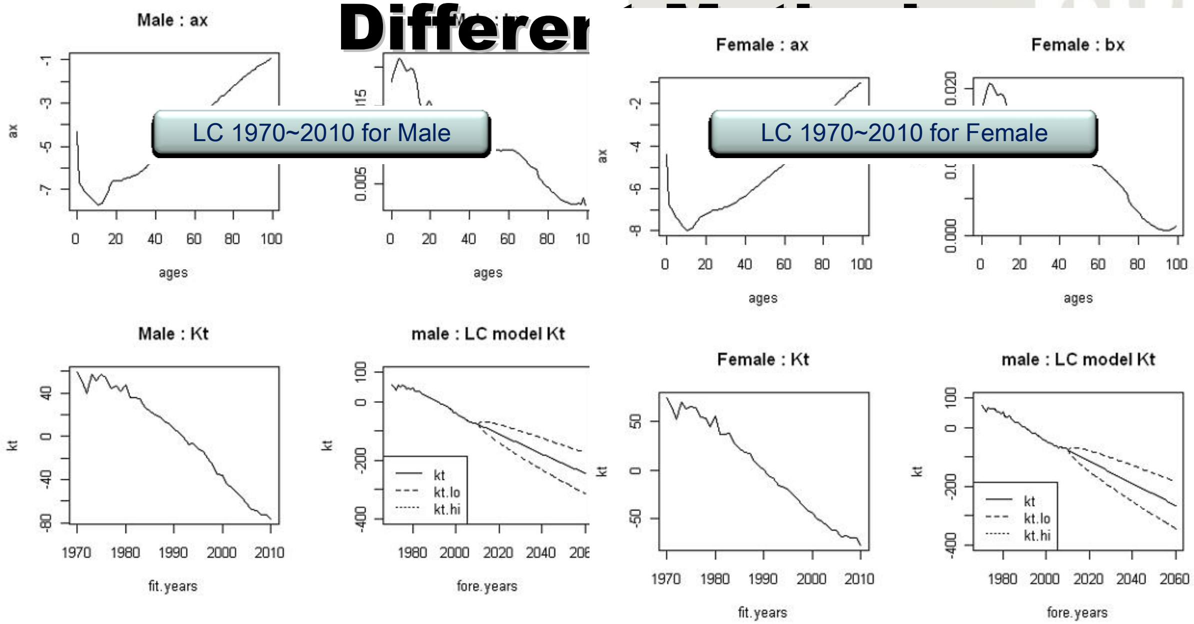
Estimates of a_x , b_x , and K_t by Differen



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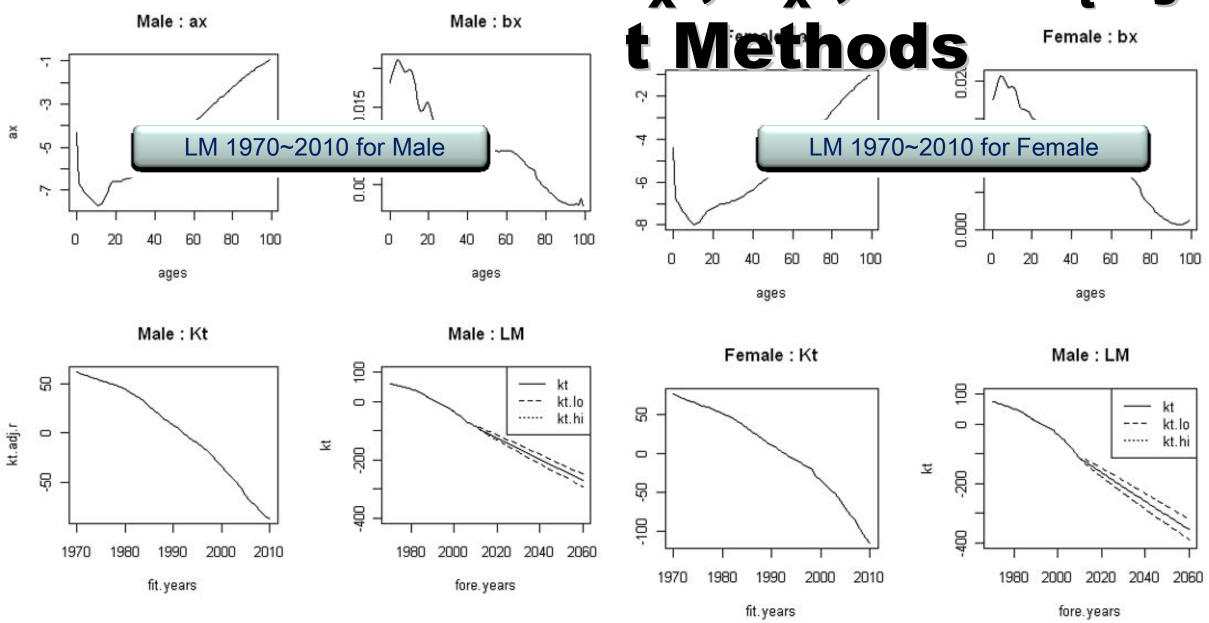
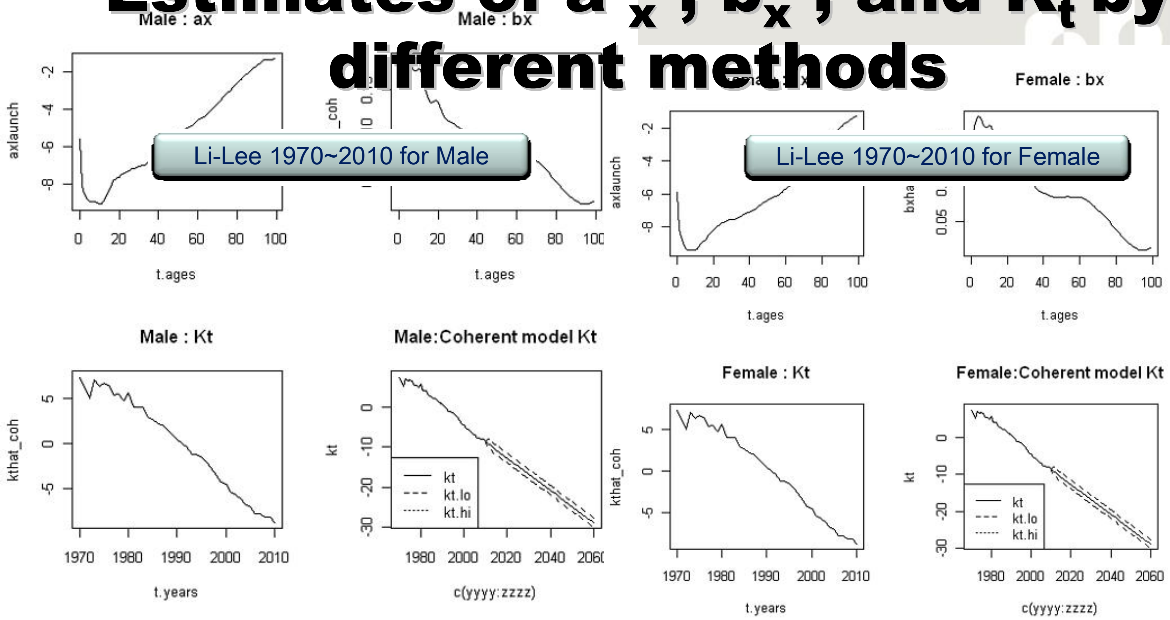
Estimates of a_x , b_x , and K_t by
t Methods



Estimates of a_x , b_x , and K_t by different methods



Forecasted Life Expectancy at Birth by Four Models, 2010~2060

Sex	Male				Female			
Method	Lee-Carter	LC with adj.	Lee-Miller	Li-Lee (Coh. LC)	Lee-Carter	LC with adj.	Lee-Miller	Li-Lee (Coh. LC)
2010	76.34	77.19	77.18	76.91	82.24	84.1	84.06	83.78
2020	78.90	79.76	79.79	79.53	83.95	85.75	85.62	85.56
2030	81.16	81.98	82.12	81.75	85.29	87.07	86.93	87.06
2040	83.14	83.91	84.14	83.65	86.4	88.14	87.99	88.35
2050	84.89	85.62	85.92	85.3	87.34	89.04	88.88	89.48
2060	86.45	87.14	87.5	86.73	88.14	89.81	89.65	90.48

Sex Differentials of Life Expectancy at Birth by Four Models, 2010~2060

Method	Sex Differential			
	Lee-Carter	LC with adj.	Lee-Miller	Li-Lee (Coh. LC)
2010	5.90	6.91	6.88	6.87
2020	5.05	5.99	5.83	6.04
2030	4.14	5.09	4.81	5.31
2040	3.27	4.23	3.85	4.70
2050	2.45	3.42	2.96	4.18
2060	1.69	2.67	2.15	3.74

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Concluding Remarks

The 2-parameter logistic model shows better goodness of fit to extend Korean mortality rates at older ages

According to the evaluation of four forecasting models, the Li-Lee model(Coh. LC) consistently shows more accurate results than the other models in estimation of Korean death probability

Concluding Remarks

The Li-Lee model(Coh. LC) yields the highest life expectancy at birth in 2060 among the compared models.

- The Li-Lee model(Coh. LC) provides more moderate sex differential in life expectancy at birth at 3.7 years in 2060 compared to the smaller ones in the LC(1.69), the LM(2.15), and the adj. LC(2.67)

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Thank You !