**Reviewer #1:**

*Comment 1:*

*While the analyses provided in this paper is intriguing, the end results are not very exciting as the proposed method is essentially a brute force method and has the same time complexity as integer programming.*

Answer 1:

We consider that a discrete algorithm like integer programming cannot be applied to the NCPP problem on continuous surface studied in the paper as it cannot guarantee to find all optimal solutions. We believe this is the first method to prove such solution to the NCPP problem. We argue that the proposed transformation of the problem from continuous domain to discrete domain by disregarding equivalent cellular decompositions, the central contribution of the paper, is fundamental in effectively making the problem solvable and guaranteeing the existence of a solution. ~~As $n$ is usually less than 10, this method is applicable in practice.~~

~~If the reviewer means that a better algorithm can be utilized for decomposed NCPP, we agree that it may exist. The proposed algorithm is guaranteed to find all optimal solutions.~~

We agree with the reviewer hinting at the fact there may indeed exist an algorithm to decompose the NCPP problem differently, more adept to finding an optimal solution with lower time complexity – e.g. via integer programming as suggested. Yet the proposed scheme guarantees a decomposition whereby discovering all solutions can be assured. Moreover, we show that an exhaustive search is necessary if all optimal solutions are requested, and no algorithm seeking all optimal solutions can perform faster than exponential complexity.

*Comment 2:*

*The cutting paths connecting the "holes" to the outer boundary do not seem to play any important role in these examples. The holes seem to be simply removed because they are covered by other sets.*

Answer 2:

This is not the case. The "hole" that the reviewer mentions refers only to one such case, a "simple hole" in that it has only one edge (its circular boundary), as such no other cells lie in the inner part. However, the relevance of the cutting paths for inner-part “holes” relates to the definition of "topological edges" (6). This is clearly visible in the examples shown in Fig. 5(b), Fig. 6 and Fig. 7-8 (for the illustration of 2-edge, 5-edge and 3-edge inner-parts respectively). In these cases, cutting paths are an essential and intrinsic component of the proposed algorithm to enforce cell connectivity and division equivalences.

Comment 3:

*The experiments provide no comparison and do not report any statistical information about the proposed method. Decompositions from these examples are not demonstrated.*

*Answer 3:*

The optimality of the algorithm is theoretically guaranteed as proven. We agree a comparative analysis with other methods, even when suboptimal, would be beneficial and will be added. Limited space left after the comprehensive proof of the theory poses a challenge; as such extensive demonstrations of the decompositions concepts proposed were given with three experiments in a separate video (footnote 1, Section I). If paper is accepted we will add comparatives with related methods in the video too.

*Comment 4:*

*All images are very small and the tiny text are not really legible. Many figures, such as Fig.3, Fig.6, Fig.7 are not well explained.*

Answer 4:

This will be addressed as advised. Please refer to Answer 5.1 also.

**Reviewer #4:**

*Comment 1:*

*I hope that the authors did some work to provide more insight and intuition before delving into mathematical rigor. It was not an easy read, and requires a few repeats.*

Answer 1:

We agree the proposed construe is mathematically non-trivial, hence the work’s contribution. We are glad that after a few reads this stands out and becomes more apparent. Intuitively, the idea of inductively solving the problem originates from three observations:

(1) Areas in a surface can be classified by the number of “holes” within them.

(2) The problem is defined on a continuous surface, so the problem is finitely solvable only after

eliminating equivalent cellular decompositions thus rendering finite unique cellular decompositions.

(3) For each category, we can find an optimal case, thus calling for an enumeration process to find all optimal solutions.

*Comment 2:*

*One of the reasons why I had a difficulty was that even though provided illustrations are useful, but do not provide clear legends.*

Answer 2:

This will be addressed as advised. Please refer to Answer 5.1 also.

**Reviewer #5:**

*~~Comment 1:~~*

*~~Sec VI focuses on two methods: (1) treating the inner and outer parts as independent problems which can be solved for because each part is a simply-connected cell or (2) transforming the genus-one cell into a simply-connected cell (thus removing the hole) and then further dividing the cell.~~*

~~Answer 1:~~

~~We do not present two methods, but show two situations: (1) transforming the cell into a simply-connected one or (2) regarding the inner and outer parts as two independent problems.~~

*Comment 1:*

*(1) Fig.3 is a large complex figure and it is not immediately apparent in what direction to read it. While the text directs us to see that examples of the different topological divisions described by Eq (15) and (16) are visualized in Fig.3 the separation is not clear. The reader may need a more detailed caption or further annotations to fully appreciate Fig.3.*

*(2) While there are pairs of figures that share similar style, i.e., Fig.5b with Fig.7 and Fig.6 with Fig.8, the reader may need more details (in caption or in text) to interpret them properly. Many theoretical ideas were illustrated via the figures and I think more illustrative description could further solidify the understanding.*

*(3) Given the large number of symbols and terminology, it may be helpful to introduce a table of symbols to aid the reader in keep track of everything. Likewise, a flowchart of the algorithm may help summarize the algorithmic contribution and help guide the reader.*

Answer 1:

(1)(2) Figures will be further annotated and cross-referenced with further textual descriptions to increase clarity as advised.

(3) Since many symbols only appear one time in formulae, it may not be a practical choice to list them all in a table given space restrictions. However, a compromise may be to consolidate in a table those used repetitively for added clarity in referencing. A work-flow figure will also be added to increase guidance to the reader as advised.

*Comment 2:*

*One of the assumptions detailed in Sec. III is that all points on surface are within the robot's dexterous workspace, in that we do not need to consider singularities. This assumption was only made clear to me when watching the supplementary video. How limiting is this assumption?*

Answer 2:

This might be a misinterpretation: singularities are considered in the scope of the problem described in Section III, they sit at the intersection of different configurations and as such contribute to defining the cell sets for each configuration. As such, not all points on the surface are assumed within the robot’s dexterous space, singular points and their neighbours are not. This will be made more clear in the final manuscript if accepted to avoid misinterpretation.

Note: singularities represent poses with null manipulability, thus they are, and ought to be, identified and removed as a manipulator cannot safely operate in a singular configuration (even more so in the case of a contact-oriented task). Neighbouring points where instabilities may also occur are also removed, that’s indeed an arbitrary postulate to increase safety. The effect of this assumption in the final optimal solution has not been quantified but can be expected to be minimal in pursuit of intrinsically safe and stable configurations for optimal NCPP.

*Comment 3:*

*Sec V states that phi and psi, from Eq.13, will be defined later. However, in Sec VI.D, the text refers phi as defined in Eq.13. It is not clear where these quantities are defined.*

Answer 3:

Psi and phi are both defined in (13), the sentence aiming to indicate they are defined then (in Section V) for consistency for when they are used later in the paper (in Section VI.D), as both relate to K - the number of edges. Not that they are defined later. This will be clarified further to avoid the mislead. Moreover, on the back of the reviewer’s comment, we have realised we can easily overcome this misunderstanding further in the final manuscript by removing the definition of Psi altogether, in hindsight is not strictly necessary.

*Comment 4:*

*Given that Eq.13 establishes a bound on phi, should the second equals sign in Eq.25 also be a bound?*

Answer 4:

Yes, all related equations (24-26, 30-36, 38-41, 43-46) represent upper bounds. This is apparent as picked up by the reviewer, but will be made explicit to avoid misinterpretation.

*Comment 5:*

*Sec III, mentions a metric in joint space d(.,.) - is this the Euclidean distance?*

Answer 5:

Yes. Other measures would also be valid, e.g. allocating different weights to different joints. ~~A metric satisfying the triangle inequality is proved to be valid for building NCPP problem.~~

*Comment 6:*

*Is there a way to provide insight into how to interpret the various manipulability thresholds?*

Answer 6:

Generally, as the manipulability threshold is increased, the number of valid configurations decreases (as depicted in Fig. 12 and 13), thus making it more difficult to cover the reachable surface (as opposed to when manipulability may be disregarded altogether: so long as it is non-singular, it is valid). So the manipulability threshold should be chosen low to extend coverage. In contrast, finding a good relative pose between manipulator and object is less straightforward, and an analysis such as that proposed in this work for various relative arrangements can elucidate the most promising one for fuller coverage.