Business Analytics for Flexible Resource Allocation Under Random Emergencies

Journal: Management Science (2014)

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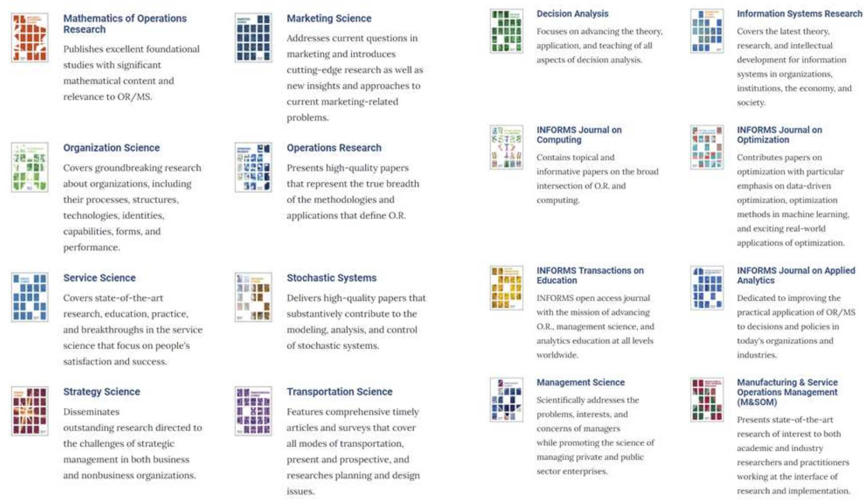
About this Journal

- *Management Science* is a scholarly journal established in 1954
- One of the top Journals in the world (UTD 24 List)
- Articles are primarily based on the foundational disciplines of economics, mathematics, psychology, sociology, and statistics
- Topics covered in Management Science include:
  - Business Strategy
  - Decision Analysis
  - Entrepreneurship
  - Operations
  - Optimization and Modeling
  - Product Development
  - Simulation
  - Social Networks
  - Stochastic Models
  - Supply Chain Management

The Institute for Operations Research and the Management Sciences (Informs)



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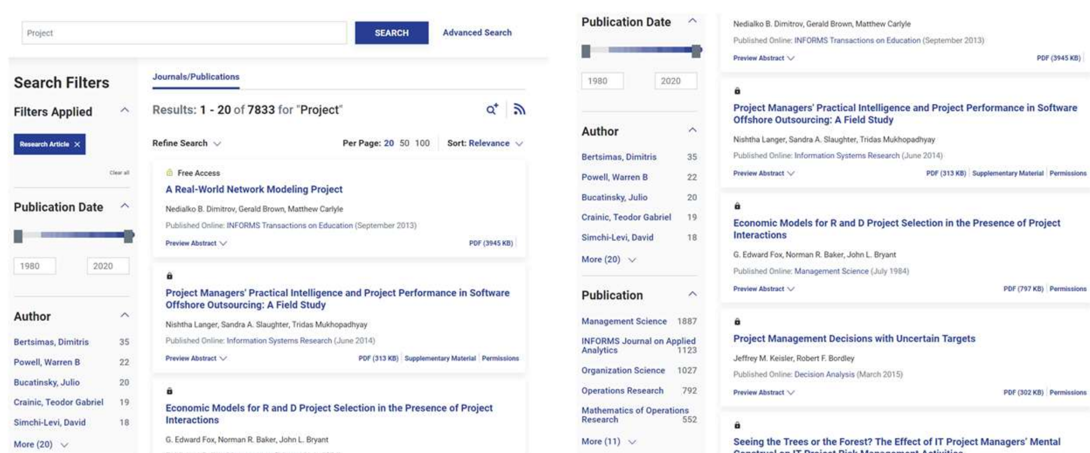


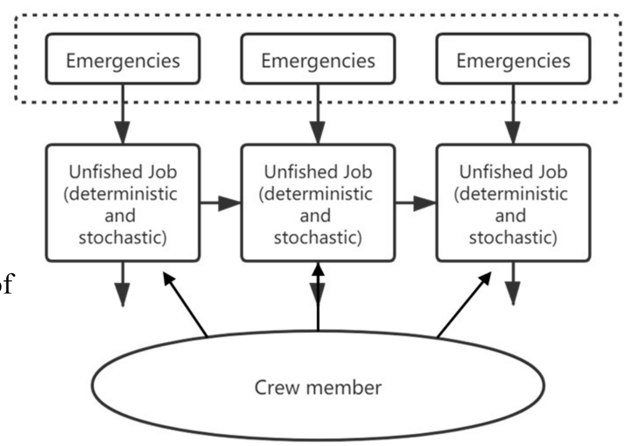
The reason for choosing this article

Keywords: resource allocation; stochastic emergencies; scheduling; gas pipeline maintenance; utility; optimization

- The main topic is to allocate the workload(deterministic and stochastic) to crew members while minimizing the cost
- The author proposed an innovative decomposition method, built two phase models, so as to solve the problems quickly while maintaining high level accuracy
- Solid proof and analysis of the algorithm
- Business analytical function of the tools developed in the paper

Background: Gas Utility Maintenance Operation

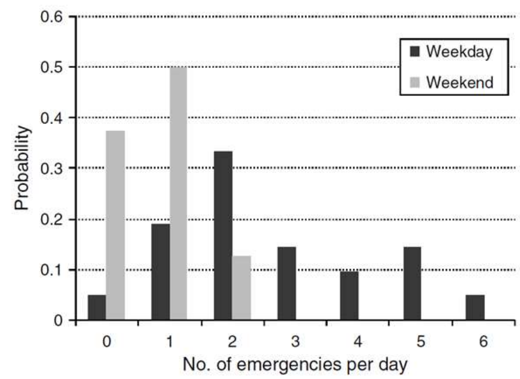
- Gas Utility: Pipeline network
- Deterministic work: maintenance, replace the old pipes, new utility constriction, clear deadline
- Emergency: gas leak, be handled immediately
- *Resource Management Department* set the monthly target and deadlines of every standard job for yards
- *Resource planner* decides which jobs should be done by the yard that day and which crew execute these jobs
- An independent department monitor the emergencies and report to yards



Background: Gas Utility Maintenance Operation

- Once an emergency is reported, any idle crew is immediately dispatched
- Once started, a standard job will not be paused even when an emergency arrives because of the significant startup effort for the job
- Resource planners make crew assignment decisions(standard job) at the beginning of the day, and monitor the arrival of emergencies.
- Yards rarely postpone standard jobs in the case of many emergencies because of the special procedure.
- The assignment is rarely changed

Figure 1 **Historical Distribution of the Number of Emergencies in a Given Yard for April 2011**

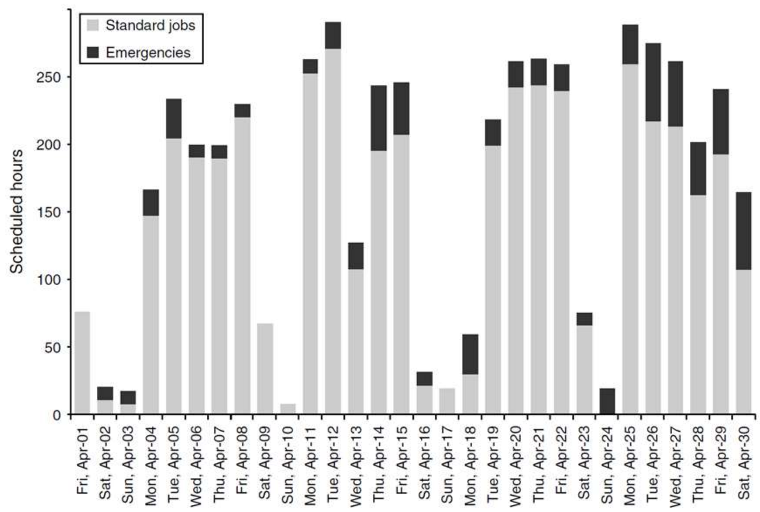


Note. Since most emergencies are found by monitoring, there are often more emergencies discovered during weekdays when more monitoring crews are working.

Problem identification: work overtime and cost

- On average, 25% to 40% crew member work overtime
- The overtime wage is 1.5 to 2 times as much as the regular wage
- The main reason for working overtime:
  - Hours spent working on standard jobs are uneven among the workdays
    - Variation in the number of crew
    - Standard jobs taking longer to finish than anticipated duo to some difficulties
    - Resource planners unable to anticipate variation in standard job duration
  - The stochastic number of emergencies. Resource planner do not account for emergencies when deciding on crew assignment
  - It is preferable for maintenance crews to work overtime to complete standard job assignments rather than postpone them and risk incurring fines.

Problem identification: work overtime and cost



Assumptions

- *Assumption 1.* The number of crews available per day is deterministic, although this number can vary daily.
- *Assumption 2*. There is no preemption of standard jobs.
- *Assumption 3*. Standard jobs have deterministic durations(Regression).
- *Assumption 4*. The number of emergencies per day is stochastic. Emergencies have equal durations.
- *Assumption 5*. Crew assignment does not consider distances
- *Assumption 6*. The specific time that an emergency arrives is ignored

Key elements/notation in the model

Model framework	
T	Length of planning horizon
K_t	Number of crews available for work on day t, where $t = 1, \ldots, T$
Standard jobs	
n	Total number of known jobs
d_i	Duration of job i, where $i = 1, \ldots, n$
τ_i	Deadline of job i, with $\tau_i \leq T$, where $i = 1, \ldots, n$
Emergency jobs	
d_L	Duration of each emergency
$L(\omega)$	Number of emergencies under outcome ω
Ω_t	(finite) set of all outcomes in day t, where $t = 1, \ldots, T$
$P_t(.)$	Probability distribution of events on day t, P_t: $\Omega_t \rightarrow [0,1]$

Decision variables

- At the start of the planning horizon, the job schedule has to be decided
- At the start of the each day, the crew assignments need to be decided before the emergencies is known
- Binary variable X_{it} take a value of 1 if and only if the job i is scheduled to be done at day t
- Binary variable Υ_{itk} take a value of 1 if and only if the job i is done on day t by crew k.
- $Z_{tk}(\omega)$ is the second-stage decision variable denoting the number of emergencies assigned to crew k.

Theoretically joint model

For each day t, given the crew assignment Υ_t, and the outcome of the number of emergencies $L(\omega)$, the objective is to minimize the maximum number of hours.

$$F_t(\Upsilon_t, L(\omega)) = \underset{Z_{tk}(\omega)}{\text{mininize}} \max_{k=1,\dots,K_t} \{d_L Z_{tk}(\omega) + \sum_{i=1}^{n} d_i \Upsilon_{itk}\}$$

Subject to $\sum_{k=1}^{K_t} Z_{tk}(\omega) = L(\omega)$

$$Z_{tk}(\omega) \in \mathbb{Z}^+, k = 1, \dots, K_t$$

The objective of the first-stage is to minimize the maximum expected resource.

$$\underset{X,\Upsilon}{\text{mininize}} \max_{t=1,\dots,t} E_t[F_t(\Upsilon_t, L(\omega))]$$

Subject to $\sum_{t=1}^{\tau_i} X_{it} = 1, i = 1, \dots, n$

$$\sum_{t=1}^{\tau_i} \Upsilon_{itk} = X_{it}, i = 1, \dots, n, t = 1, \dots, T$$

$$X_{it} \in \{0,1\}, i = 1, \dots, n, t = 1, \dots, T$$

$$\Upsilon_{itk} \in \{0,1\}, i = 1, \dots, n, t = 1, \dots, T,$$

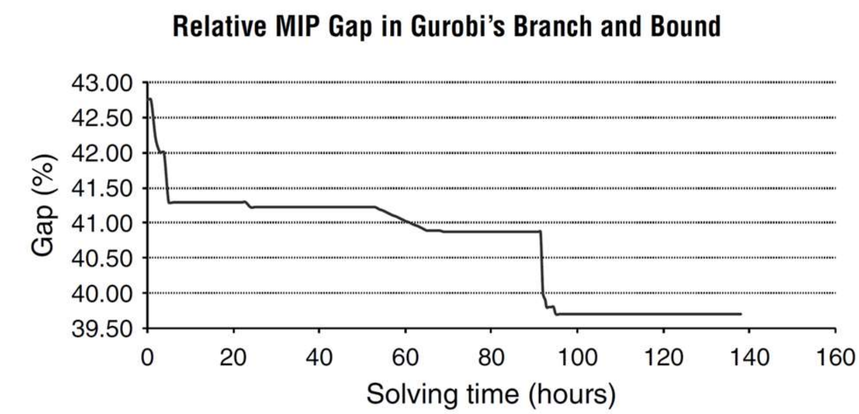
$$k = 1, \dots, K_t$$

Min-Max objective function

- The previous version is to minimize total expected overtime since straight hours are a sunk cost
- Min-Max objective led to better acceptance among all stakeholders
- Management view high overtime labor cost as a *symptom* not the root. Underlying problem is an uneven distribution of workload
- The issue of “fairness”. It sometimes assigns one crew significantly more work hours compared to others.

Practical limitations

- The problem is known to be NP-hard even without emergency.



Decomposition of the joint problem

- Decompose the joint problem into two sequential decisions (job scheduling and crew assignment)
- In job scheduling phase, schedule the jobs on the planning horizon assuming only an average number of emergencies on each day. The goal is to meet all deadlines while evenly distributing work
- In crew assignment phase, the standard jobs are assigned to crews assuming a stochastic number of emergencies. The goal is to minimize the expected maximum hours worked by any crew

Phase I: Job Scheduling (MIP)

- A deterministic mixed integer program and LP relaxation algorithm

$$\underset{X}{\text{minimize}} \max_{t=1,\ldots,T}\{\frac{1}{K_t}(d_L E_t[L(\omega)] + \sum_{i=1}^{n} d_i X_{it})\}$$

Subject to $\sum_{t=1}^{\tau_i} X_{it} = 1, i = 1, \ldots, n,$

$X_{it} \in \{0,1\}, i = 1, \ldots, n, t = 1, \ldots, T$

$\Downarrow$

$\underset{C,X}{\text{minimize}}\ C$

Subject to $d_L E_t[L(\omega)] + \sum_{i=1}^{n} d_i X_{it} \leq K_t C, t = 1, \ldots, T$

$\sum_{t=1}^{\tau_i} X_{it} = 1, i = 1, \ldots, n,$

$X_{it} \in \{0,1\}, i = 1, \ldots, n, t = 1, \ldots, T$

Phase I: Job Scheduling (LP relaxation)

- Solve the LP relaxation and rounding to a feasible schedule
- The constraints $X_{it} \in \{0,1\}$ are replace by $X_{it} \geq 0$
- Fix the jobs that have fractional solutions while re-solving the problem to find schedules for the jobs that have faction solutions
- A job i with a fractional solution can now only be scheduled on a data t when the corresponding LP solution is strictly positive
- The rounding step is still a MIP; however, it only has O(n+T) binary variables, while the original problem has O(nT). (Lenstra et al. 1990)

Phase I: Job Scheduling (LP relaxation)

x_{it}	1	2	3	4	5	6
1	0	0	1	0	0	0
2	0	0.2	0.3	0.3	0.1	0.1

- *Theorem 1*. C^{OPT} is the optimal objective cost of the problem, C^{LP} is the optimal cost of its LP relaxation. X^H is the schedule produced by LP-schedule, has an objective cost C^H, where

$$C^H \leq C^{OPT} \times (1 + \frac{1}{C^{LP}} (\min_{t=1,\ldots,T} K_t)^{-1} \sqrt{\frac{1}{2} (\sum_{i=1}^{n} d_i^2)(1 + ln\delta)})$$

$$\delta = \max_{t=1,\ldots,T} \delta_t \text{ and } \delta_t \triangleq |\{r = 1, \ldots, T: X_{ir}^{LP} > 0\}|$$

Proof of *Theorem 1*

x_{it}	1	2	3	4	5	6
1	0	0.2	0.3	0.3	0.1	0.1

- Introduce $\tilde{X}$ as the schedule derived from X^{LP} as probabilities
- All outcomes of $\tilde{X}$ are all the possible rounding of X^{LP}
- Prove that there exists a positive probability that $\tilde{X}$ has a bound
- Prove that there exists a positive probability that none of the bad events will not occur
- Use *Lovász's Local Lemma* to prove such positive probability and *McDiarmid's Inequality* provides the necessary prerequisite

Some insights so far

$$C^H \leq C^{OPT} \times (1 + \frac{1}{C^{LP}} (\min_{t=1,\dots,T} K_t)^{-1} \sqrt{\frac{1}{2} (\sum_{i=1}^n d_i^2)(1 + ln\delta)})$$

$$\delta = \max_{t=1,\dots,T} \delta_t \text{ and } \delta_t \triangleq |\{r = 1, \dots, T: X_{ir}^{LP} > 0\}|$$

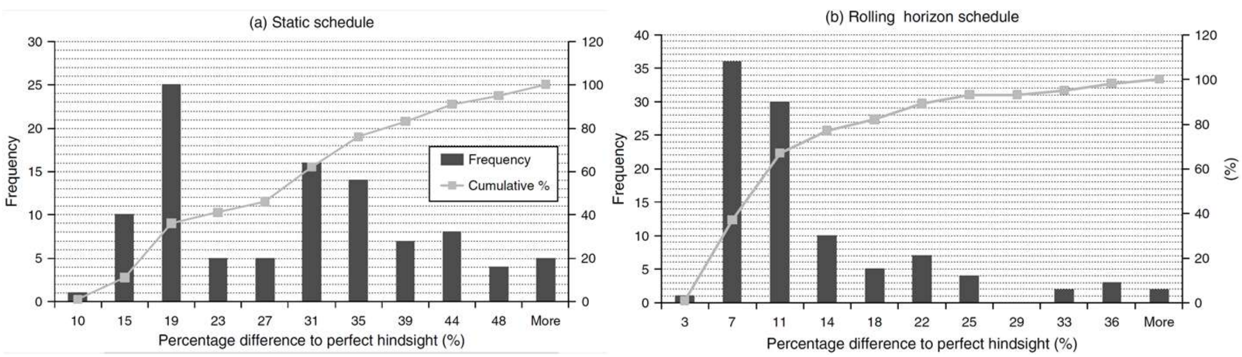
- The algorithm has a cost closer to optimal if the job deadlines are more restrictive or if the job duration are less variant
- C^{LP} takes its smallest value $(\sum_{i=1}^n d_i)/(KT)$ if all jobs are due on the last day, and largest $(\sum_{i=1}^n d_i)/K$ if all jobs are due on the first day
- When $C^{LP} = (\sum_{i=1}^n d_i)/(\alpha K)$, the bound simplifies to $1 + \alpha(||d||_2/||d||_1)\sqrt{1/2\ (1 + ln\delta)}$

Compared to Sensible Resource Planner

- In SRP, the standard jobs are sorted with increasing deadlines so that the job with the earliest deadline comes first in the list.
- We generate 100 problem. In each problem, there are 7 days, 3 crews, 70 standard jobs.
- Algorithm LP-schedule has a sample mean for the difference equal to 3.6%, 95% confidence interval [2.9%, 4.2%]
- SRP has a sample mean 9.7%, 95 confidence interval [9.1%, 10.2%]

Dynamic Job Scheduling

- To implement dynamic scheduling, the LP-schedule is reapplied every day, with the horizon starting from the current day to the end.



Phase II: Crew Assignment(two-stage stochastic MIP)

- At the beginning of each day, assigns all standard jobs in set I to available crews. After the number of emergencies is known, all emergencies must be assigned to the crews

$$\underset{\Upsilon}{\text{mininize}}\, E[F(\Upsilon, L(\omega))]$$

$$\text{Subject to} \sum_{k=1}^{K} \Upsilon_{ik} = 1, i \in I,$$

$$\Upsilon_{ik} \in \{0,1\}, i = I, k = 1, \dots, K,$$

Where $F(\Upsilon, L(\omega))$ is defined as

$$F(\Upsilon, L(\omega)) \triangleq \underset{Z}{\text{mininize}} \max_{k=1,\dots,K} \{d_L Z_k + \sum_{i \in I} d_i \Upsilon_{ik}\}$$

$$\text{Subject to } \sum_{k=1}^{K} Z_k = L(\omega),$$

$$Z_k \in \mathbb{Z}^+, k = 1, \dots, K$$

Phase II: Crew Assignment Heuristic

- An observation of the optimal solution is that if a crew is assigned to work on an emergency in a given emergency outcome, that crew should also be assigned to work on an emergency under outcomes with more emergencies
- *Proposition 2.* There exists an optimal solution(Υ^*, $Z^*(\omega)$, $\omega \in \Omega$) to the stochastic assignment problem where each crew's emergency assignment is monotonic in the number of emergencies. That is, if $L(\omega_1) < L(\omega_2)$ for some $\omega_1, \omega_2 \in \Omega$, then $Z_k^*(\omega_1) \leq Z_k^*(\omega_2)$ for all $k = 1, \ldots, K$

Phase II: Heuristic Algorithm Stoch-LPT

- LPT sorts the standard jobs in decreasing duration. Starting with the longest duration job, each iteration of LPT assigns the current standard job to crew with smallest current load and update the load. (Greedy)
- Stock-LPT first making assignments of emergencies in each outcome
- Then, like LPT, the algorithm sorts the standard jobs in decreasing order of duration. Starting with the longest one, each iteration of Stoch-LPT assign the standard job to a crew considering the expected maximum load. (Greedy)

Some tables for illustration

Firstly, assign emergencies in each outcome

Outcome	1	2	3	4	5	6
1	3	3	3	6	6	6
2	0	3	3	3	6	6
3	0	0	3	3	3	6
Maximum	3	3	3	6	6	6

Then consider the standard job, with the duration of 2 hours

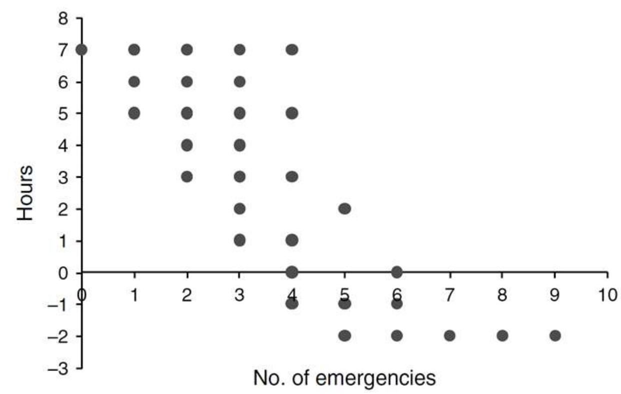
Outcome	1	2	3	4	5	6
1	5	5	5	8	8	8
2	3	5	5	6	8	8
3	3	3	5	6	6	8

Compared to Other algorithm

	Expected maximum hours				% difference to OPT		
	OPT	AVG	1-LPT	Stoch-LPT	AVG	1-LPT	Stoch-LPT
Leak distribution 1	10.66	10.66	11.50	11.50	0.00	7.96	7.96
Leak distribution 2	11.41	11.42	12.13	12.06	0.06	6.28	5.72
Leak distribution 3	11.78	12.18	12.75	12.75	3.39	8.24	8.23
Leak distribution 4	11.79	13.70	14.00	12.67	16.23	18.73	7.50
Leak distribution 5	12.18	12.18	12.77	12.19	0.03	4.80	0.11
Leak distribution 6	12.50	12.95	13.39	13.44	3.59	7.13	7.52
Leak distribution 7	12.85	12.95	13.40	13.34	0.75	4.27	3.79

Dynamic Crew Reassignment

- We assume that the standard jobs can be reassigned every hour
- We assume that the arrival of emergencies follow Poisson process
- The static assignment is conservative. However, with many emergencies, dynamic often results in more work time
- The utility choose to implement a dynamic model, but only once midday



Note. Each data point corresponds to a different sequence of emergency arrivals.

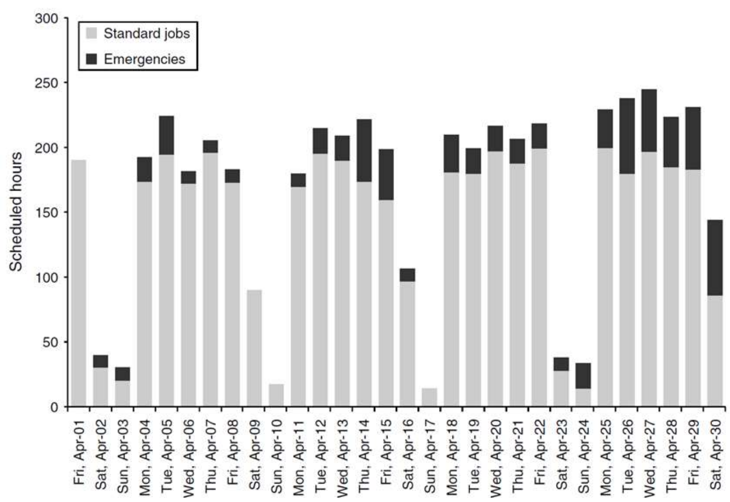
Business Analytics Overview

- Firstly, we created a tool---the Resource Allocation and Planning Tool(RAPT)---to efficiently schedule jobs and assign crews
- Create and improve process to ensure new jobs be added to database
- Analyze the impact of key process and management drivers on operating costs and the ability to meet deadlines using optimization
- Determine the potential impact of the RAPT tool to company

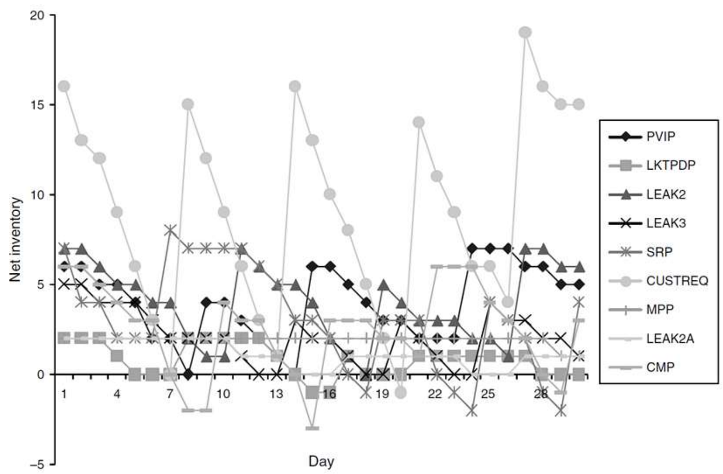
Recommend Process Improvement

- Optimal Work Queue level
- Using crew Productivity Data
- Increasing Supervision over Crews
- Projected Financial Impact from Changes

Recommend Process Improvement



Recommend Process Improvement



Note. Each line is the net inventory of workable jobs for a job type in Table EC.17 of the electronic companion.

Conclusion & Future Direction

- Use stochastic optimization to model the problem faced by gas utility
- Decompose the problem into job scheduling and crew assignment
- LP-based heuristic to solve MIP
- Motivated by the structure of the optimal solution and proposed heuristic to solve two-stage stochastic crew assignment problem
- Consider geography in making decision
- Have emergencies with random duration
- Establish a guarantee for the general case

Merits and Help

- Impression to this paper: Solid, Full and Rigorous
  - Theoretical model, decomposition, proof, algorithm analysis
- MIP and Heuristic: Unpretentious but Practical, Problem-oriented
- Notation and Model description: Precise and Easy to understand

$$\Upsilon_t \triangleq (\Upsilon_{itk})_{ik}$$

Thanks for listening

Q&A