The Field problems

Coding the Matrix, 2015

For auto-graded problems, edit the file The_Field_problems.py to include your solution.

Problem 1: `my_filter(L, num)`
input: list of numbers and a number.
output: list of numbers not containing a multiple of `num`.
example: given `list = [1,2,4,5,7]` and `num = 2`, return `[1,5,7]`.

Problem 2: `my_lists(L)`
input: list `L` of non-negative integers.
output: a list of lists: for every element x in `L` create a list containing $1, 2, \ldots, x$.
example: given `[1,2,4]` return `[[1],[1,2],[1,2,3,4]]`.
example: given `[0]` return `[[]]`.

Problem 3: `my_function_composition(f,g)`
input: two functions f and g, represented as dictionaries, such that $g \circ f$ exists.
output: dictionary that represents the function $g \circ f$.
example: given $f = \{0{:}\text{'a'}, 1{:}\text{'b'}\}$ and $g = \{\text{'a'}{:}\text{'apple'}, \text{'b'}{:}\text{'banana'}\}$, return $\{0{:}\text{'apple'}, 1{:}\text{'banana'}\}$.

For procedures in the following five problems, use the following format:

```
def <ProcedureName>(L):
    current = ...
    for x in L:
        current = ...
    return current
```

The value your procedure initially assigns to `current` turns out to be the return value in the case when the input list `L` is empty. This provides us insight into how the answer should be defined in that case. Note: You are not allowed to use Python built-in procedures `sum(·)` and `min(·)`.

Problem 4: `mySum(L)`
Input: list of numbers
Output: sum of numbers in the list

Problem 5: `myProduct(L)`

Submmited

input: list of numbers
output: product of numbers in the list

Problem 6: `myMin(L)`
input: list of numbers
output: minimum number in the list
Hint: The value of the Python expression `float('infinity')` is infinity.

Problem 7: `myConcat(L)`
input: list of strings
output: concatenation of all the strings in `L`

Problem 8: `myUnion(L)`
input: list of sets
output: the union of all sets in `L`.

In each of the above problems, the value of `current` is combined with an element of `myList` using some operation $\diamond$. In order that the procedure return the correct result, `current` should be initialized with the *identity element* for the operation $\diamond$, i.e. the value i such that $i \diamond x = x$ for any value x.

It is a consequence of the structure of the procedure that, when the input list is empty, the output value is the initial value of `current` (since in this case the body of the loop is never executed). It is convenient to define this to be the correct output!

Ungraded problem: Keeping in mind the comments above, what should be the value of each of the following?

1. The sum of the numbers in an empty set.
2. The product of the numbers in an empty set.
3. The minimum of the numbers in an empty set.
4. The concatenation of an empty list of strings.
5. The union of an empty list of sets.

What goes wrong when we try to apply this reasoning to define the intersection of an empty list of sets?

Problem 9: Each of the following asks for the sum of two complex numbers. For each, find the solution.

a. $(3 + 1\mathbf{i}) + (2 + 2\mathbf{i})$

b. $(-1 + 2\mathbf{i}) + (1 - 1\mathbf{i})$

c. $(2 + 0\mathbf{i}) + (-3 + .001\mathbf{i})$

d. $4(0 + 2\mathbf{i}) + (.001 + 1\mathbf{i})$

Combining operations on complex numbers

Problem 10: Write a procedure `transform(a,b, L)` with the following spec:

- *input:* complex numbers a and b, and a list L of complex numbers
- *output:* the list of complex numbers obtained by applying $f(z) = az + b$ to each complex number in L

Next, for each of the following problems, explain which value to choose for `a` and `b` in order to achieve the specified transformation. If there is no way to achieve the transformation, explain.

a. Translate z one unit up and one unit to the right, then rotate ninety degrees clockwise, then scale by two.

b. Scale the real part by two and the imaginary part by three, then rotate by forty-five degrees counterclockwise, and then translate down two units and left three units.

Problem 11: For each of the following, calculate the answer over $GF(2)$.

a. $1 + 1 + 1 + 0$

b. $1 \cdot 1 + 0 \cdot 1 + 0 \cdot 0 + 1 \cdot 1$

c. $(1 + 1 + 1) \cdot (1 + 1 + 1 + 1)$