An Analytical Method to Extract the Physical Parameters of a Solar Cell From Four Points on the Illuminated J–V Curve

H. Saleem and Shreepad Karmalkar

***Abstract*—For a variety of solar cells, it is shown that the single exponential J–V model parameters, namely—ideality factor η, parasitic series resistance R_s, parasitic shunt resistance $R_{\rm sh}$, dark current J_0, and photogenerated current $J_{\rm ph}$ can be extracted simultaneously from just four simple measurements of the bias points corresponding to $V_{\rm oc}$, $\sim 0.6V_{\rm oc}$, $J_{\rm sc}$, and $\sim 0.6J_{\rm sc}$ on the illuminated J–V curve, using closed-form expressions. The extraction method avoids the measurements of the peak power point and any dJ/dV (i.e., slope). The method is based on the power law J–V model proposed recently by us.**

***Index Terms*—Parameter extraction, single exponential J–V model, solar cell.**

I. Introduction

THIS LETTER discusses the extraction of the five physical parameters of the single exponential current density—voltage (J–V) equation of illuminated solar cells given by

$$J = J_{\rm ph} - J_0 \left\{\exp\left[(V + JR_s)/\eta V_t\right] - 1\right\} - (V + JR_s)/R_{\rm sh}. \quad (1)$$

Here, the physical parameters are the following: $J_{\rm ph}$—photogenerated current density, J_0—dark current density, R_s, $R_{\rm sh}$—parasitic unit area resistances, and η—ideality factor. The knowledge of these parameters of fabricated solar cells is necessary for cell array simulation and process optimization. The five-point analytical extraction method [1] needs elaborate apparatus, and accurate measurement of $dJ/dV|_{V=V_{\rm oc}}$, $dJ/dV|_{J=J_{\rm sc}}$ and the peak power point, which is difficult. Direct measurement too is elaborate, and is possible only when the J–V curve has distinct regions, where parameters other than the one being extracted can be assumed to have negligible effect [1], [2]. Parameter extraction by curve fitting requires measurement of a large number of points, prior knowledge of the parameters of interest, i.e., proper initial guesses, and extensive computation [1]–[4].

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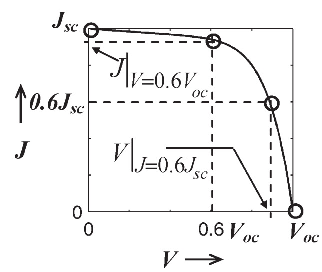

Fig. 1. Four measured points (open circles) on the illuminated J–V curve (line) employed in this letter for extracting the physical parameters, namely—η, R_s, $R_{\rm sh}$, J_0, and $J_{\rm ph}$. If the points $V|_{J=0.6J_{\rm sc}}$ and $J|_{V=0.6V_{\rm sc}}$ are too close, as may happen for some poor quality cell, other points may be used provided the constants in (3), (4), (14) and (15) are modified appropriately.

Recently [5], we presented an explicit power law J–V model of illuminated solar cells that allowed extraction of the peak power point and fill-factor from just four simple measurements of $V_{\rm oc}$, $J_{\rm sc}$, $V|_{J=0.6J_{\rm sc}}$, and $J|_{V=0.6V_{\rm sc}}$ (see Fig. 1). In this letter, we show how the physical parameters η, R_s, $R_{\rm sh}$, J_0, and $J_{\rm ph}$ can also be extracted simultaneously from the same four measurements using closed-form expressions, avoiding the difficult measurements of dJ/dV slopes and peak power point. The method is presented after a review of the relevant equations of the power law J–V model from our earlier work [5].

II. Power Law Model [5]

The J–V curve of illuminated solar cells is represented by the explicit power law function

$$j = 1 - (1-\gamma)v - \gamma v^m, \qquad \text{where } j = J/J_{\rm sc}, \quad v = V/V_{\rm oc}. \quad (2)$$

The linear term of this equation captures the slow fall in current with voltage near the short-circuit point, and the power law term captures the rapid fall near the open-circuit point. While $V_{\rm oc}$ and $J_{\rm sc}$ are measured directly, γ and m are calculated from the measured $V|_{J=0.6J_{\rm sc}}$ and $J|_{V=0.6V_{\rm sc}}$ as

$$\gamma \approx (j|_{v=0.6} - 0.4)/0.6 \quad (3)$$

$$m = \log\left[(0.4 - (1-\gamma)v|_{j=0.6})\,\gamma^{-1}\right] / \log \quad v|_{j=0.6}. \quad (4)$$

The approximation involved in writing (3) is good for $0.6^m \ll 0.6$. The choice of $\nu = 0.6$ and $j = 0.6$ has been explained in

TABLE I

DETAILS OF CELLS (ILLUMINATION $\sim$ 1 SUN) CONSIDERED FOR PARAMETER EXTRACTION. THE CELLS OF A GIVEN MATERIAL ARE LISTED IN DESCENDING ORDER OF QUALITY OR FF, WHICH VARIES IN THE RANGE $0.45 \leq$ FF ≤ 0.81. THE RANGES OF VARIOUS PARAMETER VALUES ARE THE FOLLOWING: PHYSICAL PARAMETERS—$1.23 \leq \eta \leq 2.45$, $0.16 \leq R_s \leq 8.59\ \Omega \cdot \mathrm{cm}^{-2}$, $0.6\ \mathrm{pA} \cdot \mathrm{cm}^{-2} \leq J_0 \leq 1.37\ \mu\mathrm{A} \cdot \mathrm{cm}^{-2}$, $90\ \Omega \cdot \mathrm{cm}^2 \leq R_{\mathrm{sh}} \leq 7.9\ \mathrm{k}\Omega \cdot \mathrm{cm}^2$, $7.94 \leq J_{\mathrm{ph}} \leq 36.6\ \mathrm{mA} \cdot \mathrm{cm}^{-2}$, AND $15 \leq T \leq 90\ ^{\circ}\mathrm{C}$; $0.450 \leq V_{\mathrm{oc}} < 1.011$ V; MODEL PARAMETERS—$4.37 \leq m \leq 20.2$ AND $0.512 \leq \gamma \leq 0.987$. THE PHYSICAL PARAMETERS OF CELLS # 2, 3, 5, 6, 7, 9, 15, AND 19 ARE REPRODUCED FROM LITERATURE; THE PARAMETERS FOR OTHER CELLS WERE GENERATED BY US TO MATCH THE NUMERICALLY CALCULATED J–V CURVE TO THE DATA FROM LITERATURE

S. no.	T °C	η	R_s Ω-cm^2	R_{sh} kΩ-cm^2	J_0 nA-cm^{-2}	$J_{ph} \approx J_{sc}$ mA-cm^{-2}	V_{oc} (V)	m	γ	Fill Factor		Ref.	Cell Material
										Num	Extr		
1	27	1.34	0.16	0.70	1.51	35.3	0.588	14.7	0.974	0.760	0.760	[6]	Silicon
2	27	1.51	0.28	4.00	27.5	25.6	0.537	12.0	0.987	0.737	0.737	[7]	
3	50	1.38	0.47	6.48	29.4	26.3	0.527	11.0	0.987	0.729	0.724	[1]	
4	15	1.80	0.49	1.02	53.0	30.0	0.590	10.6	0.969	0.713	0.708	[8]	
5	60	1.56	1.02	0.86	217	26.3	0.523	8.30	0.950	0.670	0.657	[1]	
6	34	1.72	2.06	0.67	233	21.8	0.520	6.99	0.922	0.635	0.614	[7]	
7	90	1.49	1.22	0.69	1372	26.3	0.459	6.94	0.914	0.627	0.609	[1]	
8	27	1.26	0.62	0.09	1.83	22.0	0.522	13.0	0.733	0.597	0.596	[9]	
9	25	1.75	6.13	3.82	56.8	31.6	0.596	4.37	0.707	0.495	0.456	[4]	
10	27	1.23	0.22	0.79	26.8	36.6	0.451	11.6	0.977	0.730	0.727	[10]	CuInSe
11	27	1.82	2.11	0.09	947	42.0	0.490	5.51	0.765	0.510	0.534	[11]	
12	27	2.05	1.08	0.40	199	28.7	0.629	8.43	0.921	0.658	0.645	[10]	CuInGaSe
13	27	1.78	0.70	0.48	2.81	21.4	0.727	12.7	0.926	0.713	0.709	[12]	CuInS
14	27	1.44	0.85	7.91	6×10^{-4}	8.45	0.869	20.2	0.987	0.811	0.810	[13]	InP
15	27	2.18	0.74	1.51	0.45	28.0	1.011	14.1	0.974	0.760	0.754	[14]	GaAs / Ge
16	27	1.40	0.41	0.25	6.73	27.1	0.548	12.3	0.914	0.701	0.698	[2]	a-SiC:H/c-Si
17	27	2.45	1.28	1.13	46.4	23.0	0.827	9.73	0.955	0.696	0.686	[15]	CdTe
18	27	1.49	1.07	0.39	24.5	16.8	0.516	10.1	0.909	0.674	0.667	[16]	Al/SiO$_2$/Si
19	27	2.31	8.59	0.20	13.6	7.94	0.755	9.44	0.512	0.452	0.450	[17]	Polymer

[5]. Alternately, γ, m, V_{oc}, and J_{sc} can be estimated from the physical parameters using

$$\gamma \approx 1 - (V_{\mathrm{oc}}/J_{\mathrm{sc}} R_{\mathrm{sh}}) \tag{5}$$

$$m \approx \frac{V_{\mathrm{oc}}/\eta V_t}{1 + \theta\, \gamma (J_{\mathrm{sc}} R_s / \eta V_t)} \tag{6}$$

$$J_{\mathrm{sc}} \approx J_{\mathrm{ph}} (1 + R_s/R_{\mathrm{sh}})^{-1} \tag{7}$$

$$V_{\mathrm{oc}} \approx \eta V_t \ln \left\{ \left[J_{\mathrm{ph}} - \eta V_t \ln(J_{\mathrm{ph}}/J_0)/R_{\mathrm{sh}} \right] / J_0 \right\}. \tag{8}$$

In (6), θ is an empirical parameter. The normalized peak power voltage v_p is given by

$$v_p \approx (m+1)^{-1/m} - 0.05(1-\gamma). \tag{9}$$

The peak power current density j_p and FF $= v_p j_p$ are obtained from (2) and (9).

III. FOUR-POINT EXTRACTION METHOD

The equations of our method are derived starting from (10) and (12) of the five-point method, given in [1], for η and R_s, respectively. In these equations, we approximate $R_{\mathrm{sho}} = dV/dJ|_{V=V_{\mathrm{oc}}} \approx R_{\mathrm{sh}}$ and express it in terms of γ using (5), eliminate $R_{\mathrm{so}} = dV/dJ|_{J=J_{\mathrm{sc}}}$ using (6) and [1, eqs. (11) and (12)], and calculate (v_p, j_p) using (2), (3), (4), and (9). This is how our power law model eliminates the measurements of dJ/dV slopes and peak power point.

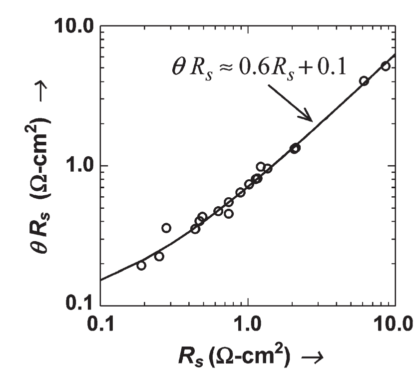

Fig. 2. Fitting function employed to derive a simple expression for R_s extraction; open circles represent the calculations for the various cells of Table I.

After some algebraic manipulation, we can write [1, eq. (10)] in terms of our power law model parameters m and γ as

$$\eta = \frac{V_{\mathrm{oc}}}{V_t} \left\{ \frac{v_p + j_p (R_{\mathrm{so}} J_{\mathrm{sc}}/V_{\mathrm{oc}}) - 1}{m \ln v_p + j_p/\gamma} \right\} \tag{10}$$

where

$$\frac{R_{\mathrm{so}} J_{\mathrm{sc}}}{V_{\mathrm{oc}}} = \frac{R_s J_{\mathrm{sc}}}{V_{\mathrm{oc}}} + \frac{\eta V_t}{\gamma V_{\mathrm{oc}}}. \tag{11}$$

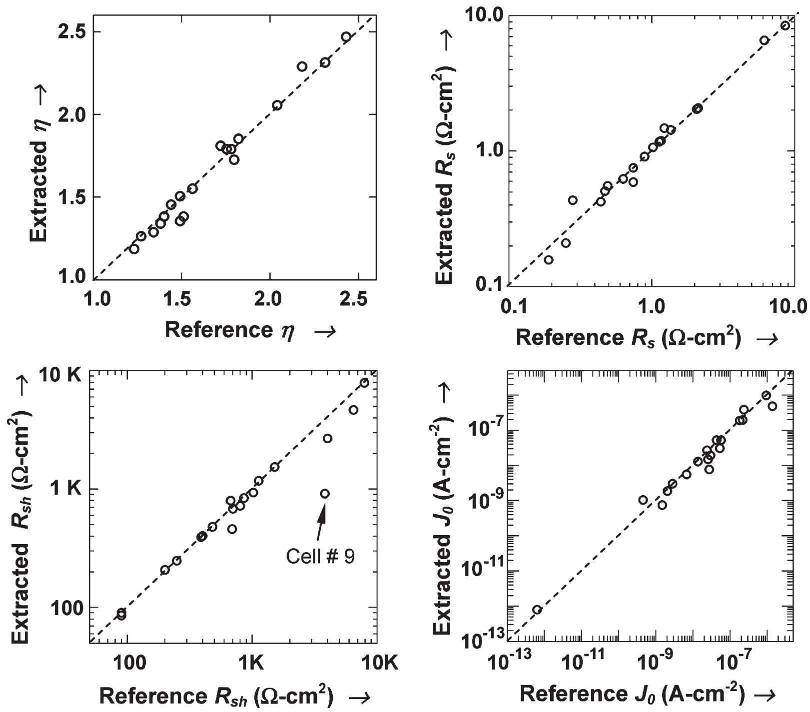

Fig. 3. Plot of the extracted values of η, R_s, $R_{\rm sh}$, and J_0 versus their reference values listed in Table I. The line represents the ideal situation in which extracted and reference values are equal. The rms error in extracted values is 3.8% for η, 16% for R_s, 14% for $R_{\rm sh}$ (excluding cell # 9), 47% for J_0 (which is low enough to correctly predict the order of magnitude of J_0), and 0.14% for $J_{\rm ph}$.

The above equation (11) is obtained by combining [1, eqs. (11) and (12)]. Furthermore, we get

$$R_s = \frac{V_{\rm oc}}{\theta\,\gamma\, m\, J_{\rm sc}}\left(1-\frac{\eta m V_t}{V_{\rm oc}}\right) \tag{12}$$

by transforming (6) of our model. Substituting (11) and (12) into (10), we have

$$\eta = \left(\frac{V_{\rm oc}}{mV_t}\right)\left(\frac{\theta[\gamma m/j_p][1-v_p]-1}{\theta[\gamma m/j_p]\ln(1/v_p)-1}\right). \tag{13}$$

To decide the value of θ, we calculated this empirical parameter for as many as 19 solar cells [1], [2], [4], [6]–[17], operating at about one-sun illumination, and having different materials, temperatures, and quality ranging from poor to good ($0.45 \le$ FF ≤ 0.81). The physical parameters of these cells are listed in Table I. For each cell, the values of $V_{\rm oc}$, $J_{\rm sc}$, $V|_{J=0.6J_{\rm sc}}$, and $J|_{V=0.6V_{\rm oc}}$ were calculated numerically using (1), the values of γ, m, v_p, and j_p were estimated from (2), (3), (4), and (9) and substituted into (13) to solve for θ. These calculations lead to the approximation $\theta \approx 0.77 j_p/\gamma$, whose substitution in (13) yields

$$\eta \approx \left(\frac{V_{\rm oc}}{mV_t}\right)\left(\frac{0.77m(1-v_p)-1}{0.77m\ln(1/v_p)-1}\right). \tag{14}$$

The value of η varies in the range $0.3(V_{\rm oc}/mV_t)$ to $0.9(V_{\rm oc}/mV_t)$ for the cells considered.

We can obtain R_s by substituting (14) into (12), in which the value of θ is chosen so as to minimize the extraction error. This value of θ was determined by evaluating the $R_s\theta$ product in (12) for the cells of Table I, using η given by (14), and γ, m, $V_{\rm oc}$, and $J_{\rm sc}$ determined above. These calculations, shown in Fig. 2, can be fitted using the linear function $\theta\, R_s \approx 0.6R_s + 0.1\ \Omega\cdot{\rm cm}^2$, whose substitution in (12) leads to

$$R_s \approx \left(\frac{V_{\rm oc}}{0.6\gamma\, m\, J_{\rm sc}}\right)\left(1-\frac{\eta m V_t}{V_{\rm oc}}\right) - 0.1\ \Omega\cdot{\rm cm}^2. \tag{15}$$

The values of R_s calculated using (15) vary between $0.1(V_{\rm oc}/\gamma\, mJ_{\rm sc})$ and $0.56(V_{\rm oc}/\gamma\, mJ_{\rm sc})$.

Writing (1) at the measured $V_{\rm oc}$ point, and setting $J_{\rm ph} - V_{\rm oc}/R_{\rm sh} \approx \gamma J_{\rm sc}$, we solve for J_0 as

$$J_0 \approx \gamma\, J_{\rm sc} e^{-V_{\rm oc}/\eta V_t}. \tag{16}$$

To solve for $R_{\rm sh}$, we write (1) at the measured $0.6V_{\rm oc}$ point and then use (7) to get

$$j|_{v=0.6} = 1 - \frac{0.6V_{\rm oc}}{J_{\rm sc}R_{\rm sh}} - \frac{J_0}{J_{\rm sc}}\exp\left(\frac{0.6V_{\rm oc}+j|_{v=0.6}J_{\rm sc}R_s}{\eta V_t}\right). \tag{17}$$

Substituting for $j|_{v=0.6}$ in terms of γ from (3), and $J_0/J_{\rm sc}$ from (16), $R_{\rm sh}$ is solved for as

$$R_{\rm sh} \approx \frac{V_{\rm oc}}{J_{\rm sc}}\left[1-\gamma-\frac{\gamma}{0.6}\times\exp\left(\frac{[0.4+0.6\gamma]J_{\rm sc}R_s - 0.4V_{\rm oc}}{\eta V_t}\right)\right]^{-1}. \tag{18}$$

The $R_{\rm sh}$ extracted as above tends to be more accurate than that extracted using (5), i.e. $R_{\rm sh} = (V_{\rm oc}/J_{\rm sc})(1-\gamma)^{-1}$, which is more sensitive to the errors introduced in γ by the neglect of the 0.6^m term in the denominator of the RHS of (3).

Parameters are extracted in the following sequence: γ and m from measured $V_{\rm oc}$, $J_{\rm sc}$, $V|_{J=0.6J_{\rm sc}}$, and $J|_{V=0.6V_{\rm sc}}$ using (3) and (4); $FF = v_p j_p$ using (2) and (9); η using (14); R_s using (15); J_0 using (16); $R_{\rm sh}$ using (18); and $J_{\rm ph}$ using (7).

IV. Results and Conclusion

We have validated the above procedure for the cells listed in Table I. For each cell, values of $V_{\rm oc}$, $J_{\rm sc}$, $V|_{J=0.6J_{\rm sc}}$, and $J|_{V=0.6V_{\rm sc}}$ were generated numerically using (1) and the parameters of Table I. Treating these values as measured four-point data, the values of physical parameter were extracted and compared with the reference values listed in Table I, as shown in Fig. 3. In all cases except $R_{\rm sh}$ of cell # 9, the errors in the extracted values are acceptable considering the method's simplicity and wide applicability to the variety of devices considered in this letter. Cell # 9 has a low FF but a high $R_{\rm sh}$. For such devices, the parameter m is low enough to invalidate the assumption $0.6^m \ll 0.6$ underlying (3), so that γ is underestimated, which lowers the $R_{\rm sh}$ extracted using (18). An improved estimate of γ taking into account the 0.6^m term in (3) improves the extracted $R_{\rm sh}$.

In conclusion, it has been shown that the five physical parameters of the SE model of a variety of illuminated solar cells can be extracted simultaneously using closed-form expressions and four simple J–V measurements of bias points corresponding to $V_{\rm oc}$, $\sim 0.6V_{\rm oc}$, $J_{\rm sc}$, and $\sim 0.6J_{\rm sc}$, avoiding any dJ/dV (slope) and peak power point measurements.

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