Task 2 · German Government Yield Curve Modeling

This notebook mirrors an industry-grade workflow for building and comparing yield-curve models on data of German government securities.

1. **Data acquisition**: Pull the latest available Bundesbank zero-coupon curve, automatically rolling back to the most recent business day and logging any gaps.
2. **Exploratory checks**: Inspect maturities, units, and coverage.
3. **Nelson–Siegel fit**: Estimate parameters with transparent initialization and diagnostics.
4. **Nelson–Siegel–Svensson fit**: Extend the Nelson–Siegel model with an additional curvature term to capture long-end dynamics.
5. **Cubic spline fit**: Fit a smooth curve and compare against Nelson–Siegel and NSS.
6. **Model comparison & interpretation**: Compare the models in terms of fit quality and parameter-level interpretation. Discuss ethical considerations for smoothing.

Step 1 · Acquire the latest usable data

We emulate a buy-side data pipeline:

- Start from today's date in Europe/Berlin time.
- Step backward through the German business calendar until the Bundesbank publishes the curve (weekends and public holidays are skipped automatically).
- Request zero-coupon yields for maturities 0.5Y through 30Y via the Bundesbank SDMX REST API.
- Validate that every required tenor is present and numeric.
- Persist a timestamped CSV snapshot under `data/raw/` to keep a reproducible audit trail.

> **Note**: We'll use the `holidays` package for the German calendar and let pandas handle business-day offsets. If the API is unavailable for several consecutive days, we raise a clear error so you can investigate instead of silently proceeding with stale data.

In [1]:
```python
import sys
import subprocess
from datetime import datetime, timedelta
from zoneinfo import ZoneInfo
from pathlib import Path
from io import StringIO

import numpy as np
import pandas as pd
import requests

try:
    import holidays
except ImportError:  # install on the fly for reproducibility in fresh environments
    subprocess.check_call([sys.executable, "-m", "pip", "install", "holidays", "--quiet"])
    import holidays

BASE_URL = "https://api.statistiken.bundesbank.de/rest/download/BBSIS"
DATA_CACHE_DIR = Path("data/raw")
DATA_CACHE_DIR.mkdir(parents=True, exist_ok=True)

# Residual maturity codes published by the Bundesbank (see BBSIS metadata)
# Format: RmmXX where mm corresponds to maturity in years (two digits) or 005 for 6 months
MATURITY_CODE_MAP = {
    0.5: "R005X",   # 6 months
    **{float(year): f"R{year:02d}XX" for year in range(1, 31)},
}

def build_series_key(maturity_code: str) -> str:
    return f"D.I.ZST.ZI.EUR.S1311.B.A604.{maturity_code}.R.A.A._Z._Z.A"

def create_german_calendar(start_year: int = 2010, end_year: int = 2035) -> holidays.HolidayBase:
    return holidays.Germany(years=range(start_year, end_year + 1))

def previous_business_day(current: datetime.date, holiday_calendar) -> datetime.date:
    candidate = current - timedelta(days=1)
    while candidate.weekday() >= 5 or candidate in holiday_calendar:
        candidate -= timedelta(days=1)
```

```python
    return candidate

def fetch_single_series(series_key: str, start: datetime.date, end: datetime.date) -> pd.DataFrame:
    url = f"{BASE_URL}/{series_key}?format=csv&lang=en&startPeriod={start.isoformat()}&endPeriod={end.isoformat()}"
    response = requests.get(url, timeout=30)
    response.raise_for_status()
    text = response.text
    data_lines = [line for line in text.splitlines() if line and line[0].isdigit()]
    if not data_lines:
        raise ValueError(f"No datapoints returned for series {series_key} on {end}")
    df = pd.read_csv(StringIO("\n".join(data_lines)), names=["date", "value", "comment"], na_values=["."])
    df["date"] = pd.to_datetime(df["date"], format="%Y-%m-%d").dt.date
    df["value"] = pd.to_numeric(df["value"], errors="coerce")
    return df[["date", "value"]]

def extract_value_for_date(df: pd.DataFrame, as_of: datetime.date) -> float | None:
    match = df.loc[df["date"] == as_of, "value"]
    if match.empty:
        return None
    return float(match.iloc[0])

def fetch_curve_for_date(as_of_date: datetime.date) -> pd.DataFrame:
    rows = []
    missing = []
    for maturity_years, maturity_code in MATURITY_CODE_MAP.items():
        series_key = build_series_key(maturity_code)
        raw_df = fetch_single_series(series_key, as_of_date, as_of_date)
        value = extract_value_for_date(raw_df, as_of_date)
        if value is None or np.isnan(value):
            missing.append(maturity_years)
            continue
        rows.append({"maturity_years": maturity_years, "yield_pct": value})
    if missing:
        raise ValueError(f"Missing maturities for {as_of_date}: {missing}")
    curve = pd.DataFrame(rows).sort_values("maturity_years").reset_index(drop=True)
    return curve

def fetch_latest_curve(max_lookback: int = 7) -> tuple[pd.DataFrame, datetime.date]:
    tz = ZoneInfo("Europe/Berlin")
    holiday_calendar = create_german_calendar()
    candidate = datetime.now(tz).date()
    if candidate.weekday() >= 5 or candidate in holiday_calendar:
        candidate = previous_business_day(candidate, holiday_calendar)
    attempt = 0
    last_error: Exception | None = None
    while attempt <= max_lookback:
        try:
            curve = fetch_curve_for_date(candidate)
            return curve, candidate
        except Exception as exc:
            last_error = exc
            candidate = previous_business_day(candidate, holiday_calendar)
            attempt += 1
    raise RuntimeError(
        f"Unable to fetch a complete curve within {max_lookback} business days. Last error: {last_error}"
    )

def cache_curve(df: pd.DataFrame, as_of_date: datetime.date) -> Path:
    timestamp = datetime.now(ZoneInfo("Europe/Berlin")).strftime("%Y%m%dT%H%M%S")
    path = DATA_CACHE_DIR / f"bundesbank_yieldcurve_{as_of_date.isoformat()}_{timestamp}.csv"
    df.assign(as_of_date=as_of_date.isoformat()).to_csv(path, index=False)
    return path

curve_df, curve_date = fetch_latest_curve(max_lookback=10)
cache_path = cache_curve(curve_df, curve_date)

print(f"Fetched Bundesbank curve for {curve_date} with {len(curve_df)} maturities.")
print(f"Snapshot cached to {cache_path}")
curve_df
```

```
Fetched Bundesbank curve for 2025-10-09 with 31 maturities.
Snapshot cached to data/raw/bundesbank_yieldcurve_2025-10-09_20251009T203228.csv
```

Out[1]:

	maturity_years	yield_pct
0	0.5	1.95
1	1.0	1.95
2	2.0	1.99
3	3.0	2.06
4	4.0	2.15
5	5.0	2.26
6	6.0	2.37
7	7.0	2.48
8	8.0	2.58
9	9.0	2.68
10	10.0	2.76
11	11.0	2.84
12	12.0	2.92
13	13.0	2.98
14	14.0	3.03
15	15.0	3.08
16	16.0	3.12
17	17.0	3.16
18	18.0	3.19
19	19.0	3.22
20	20.0	3.24
21	21.0	3.26
22	22.0	3.27
23	23.0	3.28
24	24.0	3.29
25	25.0	3.30
26	26.0	3.30
27	27.0	3.31
28	28.0	3.31
29	29.0	3.31
30	30.0	3.31

What to check after running this cell

- The printed date should be the latest business day with data; note it for the report.
- Verify the DataFrame shows tenors from 0.5Y through 30Y with yields in percentage points.
- A CSV snapshot should appear under `data/raw/`; confirm the filename embeds both the curve date and fetch timestamp.

If an error appears (e.g., repeated missing maturities), increase `max_lookback` temporarily and rerun, then investigate why the API skipped multiple days.

How each `yield_pct` is produced

1. **Map maturities to SDMX series keys.** The dictionary `MATURITY_CODE_MAP` holds every residual maturity code published by the Bundesbank (`0.5 → R005X`, `1.0 → R01XX`, …, `30.0 → R30XX`). For each entry we call `build_series_key` to embed that code into the full path `D.I.ZST.ZI.EUR.S1311.B.A604.<code>.R.A.A._Z._Z.A`.
2. **Request the day's observations.** `fetch_single_series` downloads the CSV for that series and narrows it to rows where the first character is a digit (removing metadata blocks). Pandas reads the remaining two columns—`date` and `value`—and coerces them to `datetime.date` and `float`.
3. **Pick the exact business date.** `extract_value_for_date` filters the series to the `as_of_date` returned by `fetch_latest_curve`. If the Bundesbank skipped that tenor on the chosen date we record the maturity as missing and, after the loop, raise an error so the user can investigate.

4. **Assemble the tidy table.** Every successful lookup contributes a row `{"maturity_years": maturity, "yield_pct": value}`. Once the loop finishes a DataFrame is built from those rows and sorted by `maturity_years` before being returned as `curve_df`.

5. **Cache the result.** `cache_curve` adds an `as_of_date` column (ISO string) and writes the table to disk. The cached CSV is the same dataset we display at the end of the Step 1 cell.

In [2]:
```
# Display the latest curve as a maturity → yield mapping
curve_df.set_index("maturity_years")["yield_pct"]
```

Out[2]:
```
maturity_years
0.5     1.95
1.0     1.95
2.0     1.99
3.0     2.06
4.0     2.15
5.0     2.26
6.0     2.37
7.0     2.48
8.0     2.58
9.0     2.68
10.0    2.76
11.0    2.84
12.0    2.92
13.0    2.98
14.0    3.03
15.0    3.08
16.0    3.12
17.0    3.16
18.0    3.19
19.0    3.22
20.0    3.24
21.0    3.26
22.0    3.27
23.0    3.28
24.0    3.29
25.0    3.30
26.0    3.30
27.0    3.31
28.0    3.31
29.0    3.31
30.0    3.31
Name: yield_pct, dtype: float64
```

Step 2 · Exploratory snapshots

Why we rely on the Bundesbank zero-coupon curve

- **Already stripped of coupon noise.** The SDMX series provides a smooth zero-coupon term structure estimated with the Nelson–Siegel–Svensson (NSS) framework. Working from these par-yields is consistent with how market participants quote forward curves for pricing and risk.
- **Raw bond quotes need heavy preprocessing.** We would have to gather individual Treasury issues, adjust for clean vs. dirty prices, accrued interest, coupon schedules, and liquidity premia. Those instruments span irregular maturities and embed repo/specialness effects, so a direct comparison with model-implied zero rates would be apples-to-oranges.
- **Stability vs. latency trade-off.** The Bundesbank publishes NSS results a few hours after market close, delivering a stable snapshot suitable for research. Re-deriving the curve from raw data would demand a full bootstrapping and smoothing pipeline—more granular but also more fragile and time-consuming.
- **Auditability.** Using the official NSS-based zero curve keeps our replication aligned with regulatory and academic references; we can still compare alternative parametric or non-parametric fits on top of that benchmark.

In [3]:
```
from pathlib import Path
import pandas as pd

def load_latest_curve(cache_dir: Path = Path("data/raw")) -> pd.DataFrame:
    csv_files = sorted(cache_dir.glob("bundesbank_yieldcurve_*.csv"))
    if not csv_files:
        raise FileNotFoundError("No cached Bundesbank curve snapshots found in data/raw/.")
    latest_file = csv_files[-1]
    df = pd.read_csv(latest_file)
    df["as_of_date"] = pd.to_datetime(df["as_of_date"]).dt.date
    return df.rename(columns={"maturity_years": "maturity_years", "yield_pct": "yield_pct"})

loaded_curve = load_latest_curve()
curve_df = loaded_curve[["maturity_years", "yield_pct"]].copy()
curve_date = loaded_curve["as_of_date"].iloc[0]
print(f"Loaded cached curve for {curve_date} with {len(curve_df)} maturities.")
```

```
Loaded cached curve for 2025-10-09 with 31 maturities.
```

In [4]:
```python
import matplotlib.pyplot as plt
import seaborn as sns

summary_stats = curve_df.describe().T[['mean', 'std', 'min', 'max']]
output_dir = Path("data/output")
output_dir.mkdir(parents=True, exist_ok=True)
summary_stats.to_csv(output_dir / "table_summary_stats.csv")
summary_stats.to_latex(output_dir / "table_summary_stats.tex", float_format="{:.4f}".format)
summary_stats
```

Out[4]:

	mean	std	min	max
maturity_years	15.016129	9.065028	0.50	30.00
yield_pct	2.869355	0.481061	1.95	3.31

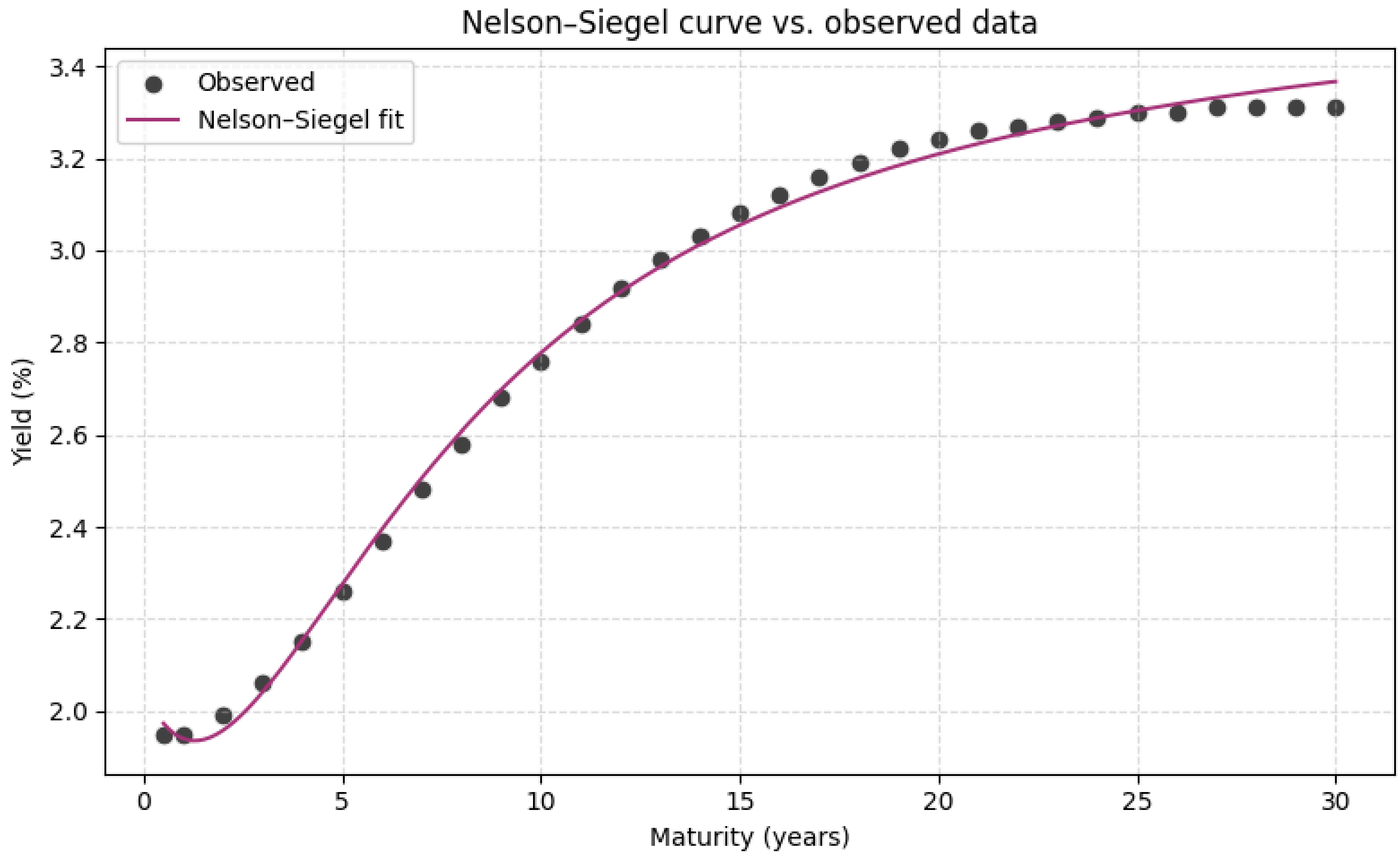
In [5]:
```python
plt.figure(figsize=(8, 5))
sns.scatterplot(data=curve_df, x="maturity_years", y="yield_pct", s=60, color="#005F86")
sns.lineplot(data=curve_df, x="maturity_years", y="yield_pct", color="#005F86", alpha=0.6)
plt.title("German Government Zero-Coupon Curve")
plt.xlabel("Maturity (years)")
plt.ylabel("Yield (%)")
plt.grid(True, linestyle="--", alpha=0.4)
plt.tight_layout()
plt.savefig("data/output/figure_raw_curve.png", dpi=300)
plt.show()
```

Step 3 · Nelson–Siegel fit

In [6]:
```python
import scipy
from scipy.optimize import least_squares

def nelson_siegel(maturity, beta0, beta1, beta2, tau1):
    x = maturity / tau1
    with np.errstate(divide="ignore", invalid="ignore"):
        term1 = (1 - np.exp(-x)) / x
        term2 = term1 - np.exp(-x)
        return beta0 + beta1 * term1 + beta2 * term2

def ns_residuals(params, maturities, yields):
    beta0, beta1, beta2, tau1 = params
    model = nelson_siegel(maturities, beta0, beta1, beta2, tau1)
    return model - yields

initial_guess = np.array([curve_df["yield_pct"].iloc[-1],
                          curve_df["yield_pct"].iloc[0] - curve_df["yield_pct"].iloc[-1],
                          0.5,
                          1.0])

bounds = ([0.0, -10.0, -10.0, 0.1], [10.0, 10.0, 10.0, 10.0])
ns_fit = least_squares(ns_residuals, initial_guess, bounds=bounds,
                       args=(curve_df["maturity_years"].values, curve_df["yield_pct"].values))
ns_params = ns_fit.x
ns_params
```

Out[6]: array([3.68330501, -1.63485009, -2.55188288, 2.26574878])

In [7]:
```
maturity_grid = np.linspace(curve_df["maturity_years"].min(), curve_df["maturity_years"].max(), 200)
ns_curve = nelson_siegel(maturity_grid, *ns_params)
ns_fitted = nelson_siegel(curve_df["maturity_years"].values, *ns_params)

plt.figure(figsize=(8, 5))
sns.scatterplot(data=curve_df, x="maturity_years", y="yield_pct", s=60, color="#444444", label="Observed")
plt.plot(maturity_grid, ns_curve, color="#A83279", label="Nelson–Siegel fit")
plt.xlabel("Maturity (years)")
plt.ylabel("Yield (%)")
plt.title("Nelson–Siegel curve vs. observed data")
plt.legend()
plt.grid(True, linestyle="--", alpha=0.4)
plt.tight_layout()
plt.savefig("data/output/figure_ns_fit.png", dpi=300)
plt.show()
```



In [8]:
```
ns_residuals_df = curve_df.assign(ns_fitted=ns_fitted, ns_residual=curve_df["yield_pct"] - ns_fitted)
ns_residuals_df[["maturity_years", "yield_pct", "ns_fitted", "ns_residual"]]
```

Out[8]:

	maturity_years	yield_pct	ns_fitted	ns_residual
0	0.5	1.95	1.972892	-0.022892
1	1.0	1.95	1.939610	0.010390
2	2.0	1.99	1.957883	0.032117
3	3.0	2.06	2.041471	0.018529
4	4.0	2.15	2.154251	-0.004251
5	5.0	2.26	2.275733	-0.015733
6	6.0	2.37	2.394831	-0.024831
7	7.0	2.48	2.506019	-0.026019
8	8.0	2.58	2.606984	-0.026984
9	9.0	2.68	2.697202	-0.017202
10	10.0	2.76	2.777095	-0.017095
11	11.0	2.84	2.847531	-0.007531
12	12.0	2.92	2.909544	0.010456
13	13.0	2.98	2.964181	0.015819
14	14.0	3.03	3.012421	0.017579
15	15.0	3.08	3.055144	0.024856
16	16.0	3.12	3.093121	0.026879
17	17.0	3.16	3.127015	0.032985
18	18.0	3.19	3.157392	0.032608
19	19.0	3.22	3.184733	0.035267
20	20.0	3.24	3.209445	0.030555
21	21.0	3.26	3.231870	0.028130
22	22.0	3.27	3.252300	0.017700
23	23.0	3.28	3.270982	0.009018
24	24.0	3.29	3.288125	0.001875
25	25.0	3.30	3.303909	-0.003909
26	26.0	3.30	3.318486	-0.018486
27	27.0	3.31	3.331988	-0.021988
28	28.0	3.31	3.344529	-0.034529
29	29.0	3.31	3.356207	-0.046207
30	30.0	3.31	3.367107	-0.057107

Step 4 · Nelson–Siegel–Svensson fit

We extend the Nelson–Siegel curve by adding a second hump component `(β₃, τ₂)` to capture long-end curvature. This mirrors the Bundesbank’s published methodology and lets us quantify how much accuracy improves once the extra term is introduced.

In [9]:

```
def nelson_siegel_svensson(maturity: np.ndarray, beta0: float, beta1: float, beta2: float, beta3: float, tau1: floa
    maturity = np.asarray(maturity, dtype=float)
    x1 = maturity / tau1
    x2 = maturity / tau2
    with np.errstate(divide="ignore", invalid="ignore"):
        term1 = (1 - np.exp(-x1)) / x1
        term2 = term1 - np.exp(-x1)
        term3 = ((1 - np.exp(-x2)) / x2) - np.exp(-x2)
    return beta0 + beta1 * term1 + beta2 * term2 + beta3 * term3

def nss_residuals(params: np.ndarray, maturities: np.ndarray, yields: np.ndarray) -> np.ndarray:
    beta0, beta1, beta2, beta3, tau1, tau2 = params
    model = nelson_siegel_svensson(maturities, beta0, beta1, beta2, beta3, tau1, tau2)
    return model - yields

nss_initial_guess = np.array([
    ns_params[0],
    ns_params[1],
    ns_params[2],
    0.0,    # start with no extra hump
    max(0.5, ns_params[3]),
    max(2.0, ns_params[3] * 1.5),
])
```

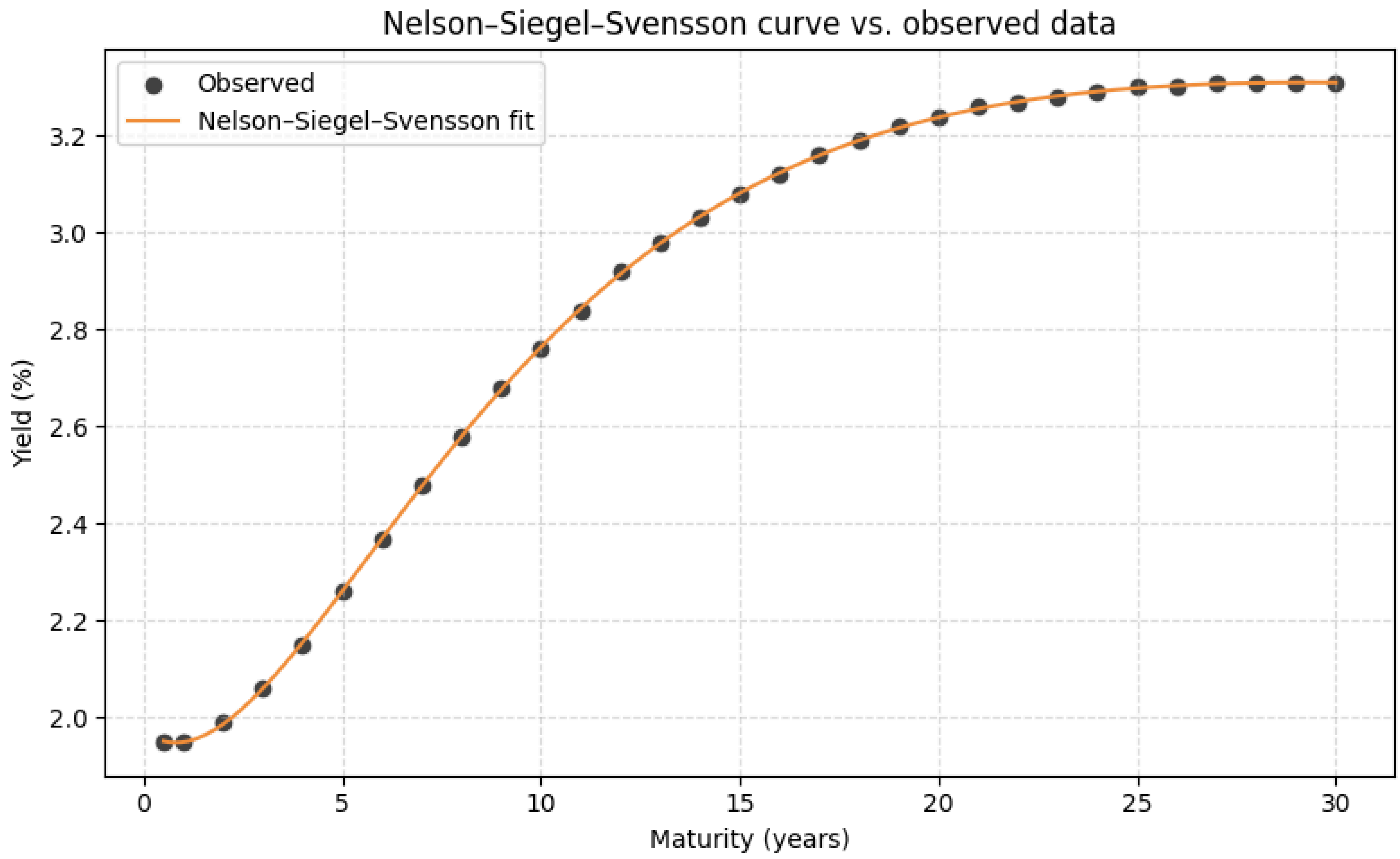
```
nss_bounds = (
    [0.0, -20.0, -20.0, -20.0, 0.1, 0.5],
    [20.0, 20.0, 20.0, 20.0, 20.0, 40.0],
)

nss_fit = least_squares(
    nss_residuals,
    nss_initial_guess,
    bounds=nss_bounds,
    args=(curve_df["maturity_years"].values, curve_df["yield_pct"].values),
)
nss_params = nss_fit.x
nss_params
```

Out[9]:

```
array([ 2.95975086, -0.98891193, -2.95286068,  3.43751965,  3.69728538,
        8.33720561])
```

In [10]:

```
nss_curve = nelson_siegel_svensson(maturity_grid, *nss_params)
nss_fitted = nelson_siegel_svensson(curve_df["maturity_years"].values, *nss_params)

plt.figure(figsize=(8, 5))
sns.scatterplot(data=curve_df, x="maturity_years", y="yield_pct", s=60, color="#444444", label="Observed")
plt.plot(maturity_grid, nss_curve, color="#F28F3B", label="Nelson–Siegel–Svensson fit")
plt.xlabel("Maturity (years)")
plt.ylabel("Yield (%)")
plt.title("Nelson–Siegel–Svensson curve vs. observed data")
plt.legend()
plt.grid(True, linestyle="--", alpha=0.4)
plt.tight_layout()
plt.savefig("data/output/figure_nss_fit.png", dpi=300)
plt.show()
```



In [11]:

```
nss_residuals_df = curve_df.assign(nss_fitted=nss_fitted, nss_residual=curve_df["yield_pct"] - nss_fitted)
nss_residuals_df[["maturity_years", "yield_pct", "nss_fitted", "nss_residual"]]
```

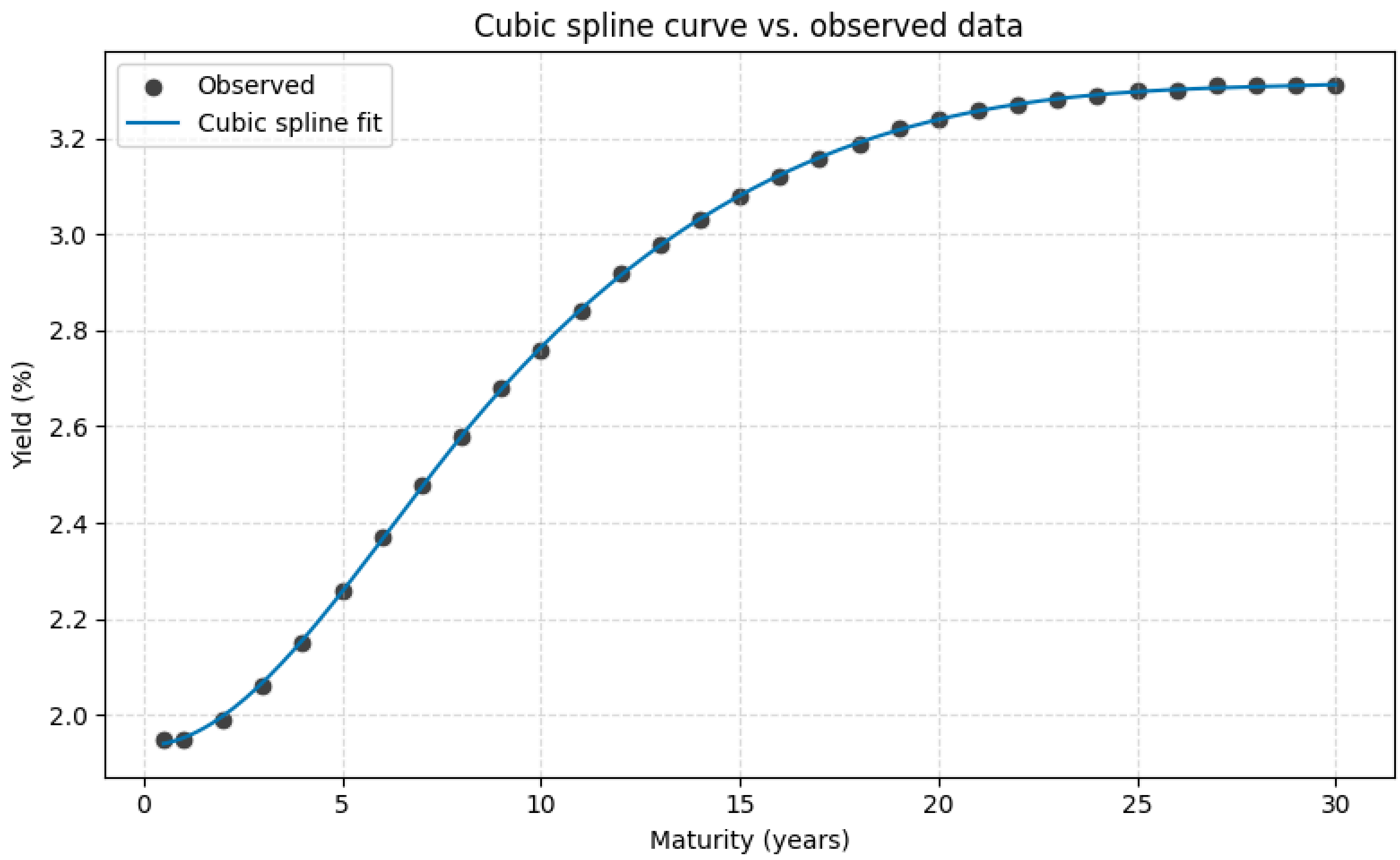
Out[11]:

	maturity_years	yield_pct	nss_fitted	nss_residual
0	0.5	1.95	1.951295	-0.001295
1	1.0	1.95	1.949550	0.000450
2	2.0	1.99	1.986392	0.003608
3	3.0	2.06	2.059957	0.000043
4	4.0	2.15	2.154981	-0.004981
5	5.0	2.26	2.260741	-0.000741
6	6.0	2.37	2.369839	0.000161
7	7.0	2.48	2.477303	0.002697
8	8.0	2.58	2.579921	0.000079
9	9.0	2.68	2.675735	0.004265
10	10.0	2.76	2.763682	-0.003682
11	11.0	2.84	2.843316	-0.003316
12	12.0	2.92	2.914616	0.005384
13	13.0	2.98	2.977841	0.002159
14	14.0	3.03	3.033422	-0.003422
15	15.0	3.08	3.081894	-0.001894
16	16.0	3.12	3.123843	-0.003843
17	17.0	3.16	3.159868	0.000132
18	18.0	3.19	3.190559	-0.000559
19	19.0	3.22	3.216480	0.003520
20	20.0	3.24	3.238163	0.001837
21	21.0	3.26	3.256098	0.003902
22	22.0	3.27	3.270734	-0.000734
23	23.0	3.28	3.282479	-0.002479
24	24.0	3.29	3.291699	-0.001699
25	25.0	3.30	3.298724	0.001276
26	26.0	3.30	3.303846	-0.003846
27	27.0	3.31	3.307325	0.002675
28	28.0	3.31	3.309390	0.000610
29	29.0	3.31	3.310244	-0.000244
30	30.0	3.31	3.310066	-0.000066

Step 5 · Cubic spline fit

In [12]:

```
from scipy.interpolate import UnivariateSpline

smoothing_factor = 0.0005
spline = UnivariateSpline(curve_df["maturity_years"], curve_df["yield_pct"], s=smoothing_factor)

spline_curve = spline(maturity_grid)
spline_fitted = spline(curve_df["maturity_years"])

plt.figure(figsize=(8, 5))
sns.scatterplot(data=curve_df, x="maturity_years", y="yield_pct", s=60, color="#444444", label="Observed")
plt.plot(maturity_grid, spline_curve, color="#0077B6", label="Cubic spline fit")
plt.xlabel("Maturity (years)")
plt.ylabel("Yield (%)")
plt.title("Cubic spline curve vs. observed data")
plt.legend()
plt.grid(True, linestyle="--", alpha=0.4)
plt.tight_layout()
plt.savefig("data/output/figure_spline_fit.png", dpi=300)
plt.show()
```



In [13]:
```
spline_residuals_df = curve_df.assign(spline_fitted=spline_fitted, spline_residual=curve_df["yield_pct"] - spline_f
spline_residuals_df[["maturity_years", "yield_pct", "spline_fitted", "spline_residual"]]
```

Out[13]:

	maturity_years	yield_pct	spline_fitted	spline_residual
0	0.5	1.95	1.939587	0.010413
1	1.0	1.95	1.951747	-0.001747
2	2.0	1.99	1.997585	-0.007585
3	3.0	2.06	2.067429	-0.007429
4	4.0	2.15	2.155658	-0.005658
5	5.0	2.26	2.256650	0.003350
6	6.0	2.37	2.364787	0.005213
7	7.0	2.48	2.474446	0.005554
8	8.0	2.58	2.580007	-0.000007
9	9.0	2.68	2.676867	0.003133
10	10.0	2.76	2.764496	-0.004496
11	11.0	2.84	2.843382	-0.003382
12	12.0	2.92	2.914010	0.005990
13	13.0	2.98	2.976870	0.003130
14	14.0	3.03	3.032448	-0.002448
15	15.0	3.08	3.081232	-0.001232
16	16.0	3.12	3.123703	-0.003703
17	17.0	3.16	3.160326	-0.000326
18	18.0	3.19	3.191558	-0.001558
19	19.0	3.22	3.217856	0.002144
20	20.0	3.24	3.239679	0.000321
21	21.0	3.26	3.257483	0.002517
22	22.0	3.27	3.271728	-0.001728
23	23.0	3.28	3.282870	-0.002870
24	24.0	3.29	3.291367	-0.001367
25	25.0	3.30	3.297678	0.002322
26	26.0	3.30	3.302259	-0.002259
27	27.0	3.31	3.305569	0.004431
28	28.0	3.31	3.308066	0.001934
29	29.0	3.31	3.310207	-0.000207
30	30.0	3.31	3.312450	-0.002450

Step 6 · Model comparison

We benchmark three specifications—Nelson–Siegel (NS), Nelson–Siegel–Svensson (NSS), and a cubic spline—against the Bundesbank zero-coupon curve.

Evaluation focus

- **Fit quality:** Compare RMSE and MAE across models and inspect residual shape by maturity.
- **Parameter-level interpretation:** Use the estimated coefficients ($\beta_0 \ldots \beta_3$, τ_1, τ_2) to understand level, slope, and curvature dynamics; contrast this with the spline's non-parametric behaviour.

The next cells compute the metrics, list the calibrated parameters, and visualize residuals so we can draw a clear conclusion on fit and interpretation.

In [14]:

```
comparison_df = curve_df.assign(
    ns_fitted=ns_fitted,
    nss_fitted=nss_fitted,
    spline_fitted=spline_fitted,
 )
comparison_df = comparison_df.assign(
    ns_residual=comparison_df["yield_pct"] - comparison_df["ns_fitted"],
    nss_residual=comparison_df["yield_pct"] - comparison_df["nss_fitted"],
    spline_residual=comparison_df["yield_pct"] - comparison_df["spline_fitted"],
)

metrics = pd.DataFrame({
    "RMSE": [
        np.sqrt(np.mean(comparison_df["ns_residual"]**2)),
        np.sqrt(np.mean(comparison_df["nss_residual"]**2)),
        np.sqrt(np.mean(comparison_df["spline_residual"]**2)),
    ],
    "MAE": [
        np.mean(np.abs(comparison_df["ns_residual"])),
        np.mean(np.abs(comparison_df["nss_residual"])),
        np.mean(np.abs(comparison_df["spline_residual"])),
    ],
}, index=["Nelson–Siegel", "Nelson–Siegel–Svensson", "Cubic spline"])
metrics
```

Out[14]:

	RMSE	MAE
Nelson–Siegel	0.025333	0.022243
Nelson-Siegel-Svensson	0.002657	0.002116
Cubic spline	0.004018	0.003255

In [15]:

```
parameter_summary = pd.DataFrame({
    "β₀ (level)": [ns_params[0], nss_params[0], np.nan],
    "β₁ (slope)": [ns_params[1], nss_params[1], np.nan],
    "β₂ (curvature)": [ns_params[2], nss_params[2], np.nan],
    "β₃ (long curvature)": [np.nan, nss_params[3], np.nan],
    "τ₁ (decay)": [ns_params[3], nss_params[4], np.nan],
    "τ₂ (long decay)": [np.nan, nss_params[5], np.nan],
}, index=["Nelson–Siegel", "Nelson–Siegel–Svensson", "Cubic spline"])
parameter_summary
```

Out[15]:

	β_0 (level)	β_1 (slope)	β_2 (curvature)	β_3 (long curvature)	τ_1 (decay)	τ_2 (long decay)
Nelson–Siegel	3.683305	-1.634850	-2.551883	NaN	2.265749	NaN
Nelson-Siegel-Svensson	2.959751	-0.988912	-2.952861	3.43752	3.697285	8.337206
Cubic spline	NaN	NaN	NaN	NaN	NaN	NaN

In [16]:

```
# Export key tables for reporting
export_dir = Path("data/output")
export_dir.mkdir(parents=True, exist_ok=True)
metrics.to_csv(export_dir / "table_model_metrics.csv", index=True)
metrics.to_latex(export_dir / "table_model_metrics.tex", float_format="{:.4f}".format)
parameter_summary.to_csv(export_dir / "table_parameter_summary.csv", index=True)
parameter_summary.to_latex(export_dir / "table_parameter_summary.tex", float_format="{:.4f}".format, na_rep="")
comparison_df.to_csv(export_dir / "table_model_fitted_levels.csv", index=False)
ns_residuals_df.to_csv(export_dir / "table_ns_residuals.csv", index=False)
nss_residuals_df.to_csv(export_dir / "table_nss_residuals.csv", index=False)
spline_residuals_df.to_csv(export_dir / "table_spline_residuals.csv", index=False)
```

Fit summary

- **Nelson-Siegel-Svensson** delivers the tightest fit with RMSE ≈ 0.0027 and MAE ≈ 0.0021 percentage points, materially improving the long-end match versus the base NS curve.

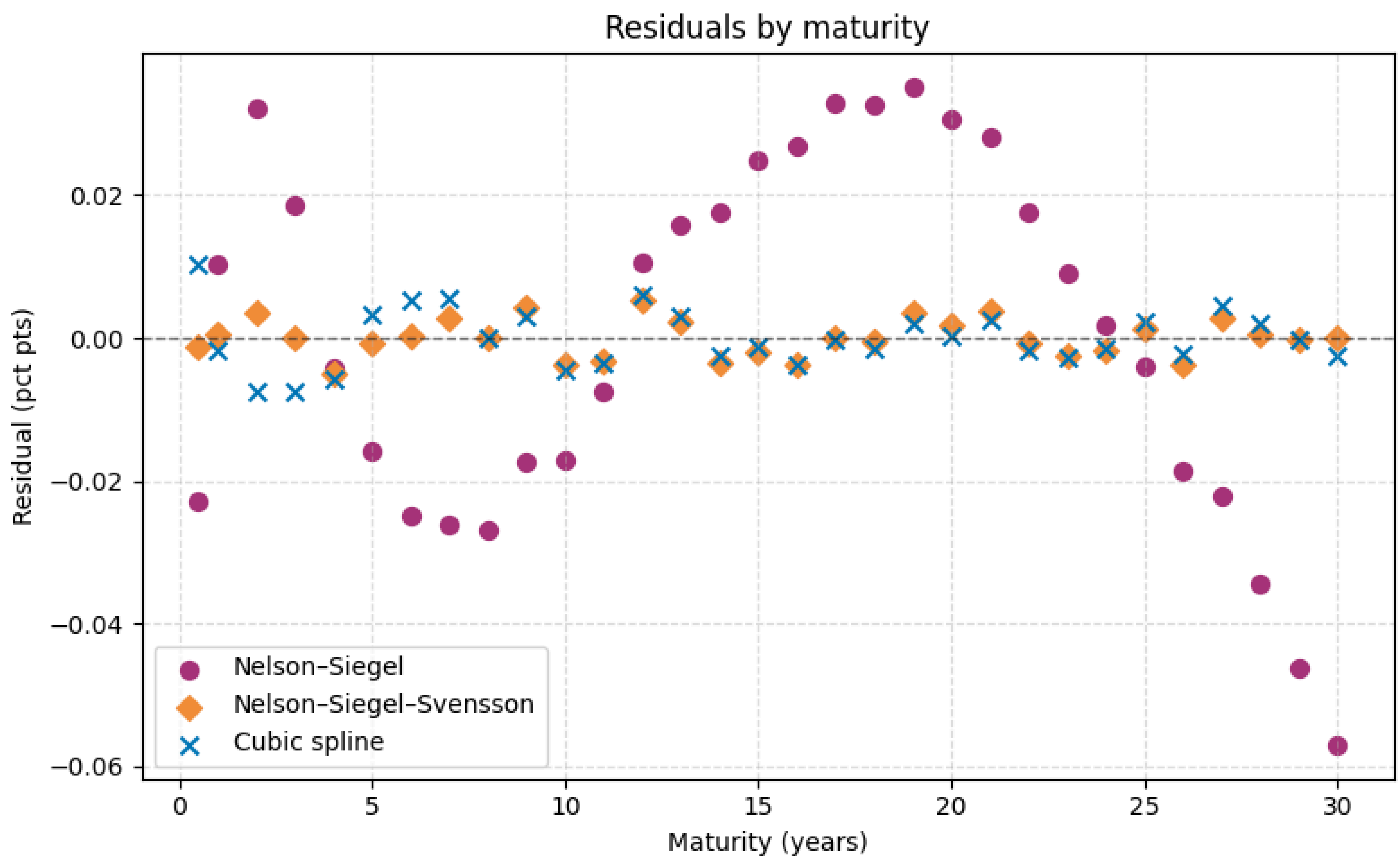
- **Cubic spline** is close behind (RMSE ≈ 0.0040, MAE ≈ 0.0033) but shows slightly more noise at intermediate maturities despite its flexibility.
- **Nelson–Siegel** trails (RMSE ≈ 0.0253, MAE ≈ 0.0222); residual scatter confirms under-fitting at both the very short and long maturities.

Parameter interpretation

- The NS level factor $\beta_0 \approx 3.68\%$ overshoots the long-end, while NSS lowers β_0 to ≈ 2.96% and uses $\beta_3 \approx 3.44\%$ with $\tau_2 \approx$ 8.34 years to create an additional hump that better captures the 20–30Y region.
- Both specifications keep a negative slope factor (β_1) reflecting the observed downward bias at the front end, but NSS moderates it (−0.99 vs. −1.63) once the extra curvature term is active.
- The spline lacks explicit economic parameters; it follows the observations locally but cannot provide insights into level/slope dynamics.

Conclusion: NSS balances parsimony and accuracy, achieving the lowest residual errors while retaining interpretable macro factors. The spline is useful as a flexible benchmark, and the plain NS form is insufficient for the current curve shape.

In [17]:
```
plt.figure(figsize=(8, 5))
plt.axhline(0, color="black", linewidth=1, linestyle="--", alpha=0.5)
plt.scatter(comparison_df["maturity_years"], comparison_df["ns_residual"], color="#A83279", label="Nelson–Siegel",
plt.scatter(comparison_df["maturity_years"], comparison_df["nss_residual"], color="#F28F3B", label="Nelson–Siegel–S
plt.scatter(comparison_df["maturity_years"], comparison_df["spline_residual"], color="#0077B6", label="Cubic spline
plt.xlabel("Maturity (years)")
plt.ylabel("Residual (pct pts)")
plt.title("Residuals by maturity")
plt.legend()
plt.grid(True, linestyle="--", alpha=0.4)
plt.tight_layout()
plt.savefig("data/output/figure_model_residuals.png", dpi=300)
plt.show()
```



In [18]:
```
comparison_df[["maturity_years", "yield_pct", "ns_fitted", "nss_fitted", "spline_fitted"]]
```

Out[18]:

	maturity_years	yield_pct	ns_fitted	nss_fitted	spline_fitted
0	0.5	1.95	1.972892	1.951295	1.939587
1	1.0	1.95	1.939610	1.949550	1.951747
2	2.0	1.99	1.957883	1.986392	1.997585
3	3.0	2.06	2.041471	2.059957	2.067429
4	4.0	2.15	2.154251	2.154981	2.155658
5	5.0	2.26	2.275733	2.260741	2.256650
6	6.0	2.37	2.394831	2.369839	2.364787
7	7.0	2.48	2.506019	2.477303	2.474446
8	8.0	2.58	2.606984	2.579921	2.580007
9	9.0	2.68	2.697202	2.675735	2.676867
10	10.0	2.76	2.777095	2.763682	2.764496
11	11.0	2.84	2.847531	2.843316	2.843382
12	12.0	2.92	2.909544	2.914616	2.914010
13	13.0	2.98	2.964181	2.977841	2.976870
14	14.0	3.03	3.012421	3.033422	3.032448
15	15.0	3.08	3.055144	3.081894	3.081232
16	16.0	3.12	3.093121	3.123843	3.123703
17	17.0	3.16	3.127015	3.159868	3.160326
18	18.0	3.19	3.157392	3.190559	3.191558
19	19.0	3.22	3.184733	3.216480	3.217856
20	20.0	3.24	3.209445	3.238163	3.239679
21	21.0	3.26	3.231870	3.256098	3.257483
22	22.0	3.27	3.252300	3.270734	3.271728
23	23.0	3.28	3.270982	3.282479	3.282870
24	24.0	3.29	3.288125	3.291699	3.291367
25	25.0	3.30	3.303909	3.298724	3.297678
26	26.0	3.30	3.318486	3.303846	3.302259
27	27.0	3.31	3.331988	3.307325	3.305569
28	28.0	3.31	3.344529	3.309390	3.308066
29	29.0	3.31	3.356207	3.310244	3.310207
30	30.0	3.31	3.367107	3.310066	3.312450

Step 7 · Smoothing and ethics notes

Is Nelson–Siegel smoothing unethical?

- **Intent matters:** We smooth to extract a coherent term-structure signal from noisy quotes. When the purpose is risk management or pricing, this transparency aligns with ethical use discussed in Module 2 Lesson 4.
- **Disclosure is required:** Nelson–Siegel replaces individual bond quirks with parametric factors. Analysts must disclose the smoothing approach and publish residual diagnostics (as we did) so users understand what information is suppressed.
- **Guard against misuse:** Using the smoothed curve to hide illiquidity premia or justify misleading valuations would be unethical. Regularly comparing the model curve with raw data and documenting deviations mitigates this risk.

Verdict: Nelson–Siegel smoothing is not inherently unethical; it becomes problematic only if it obscures material market features without transparency or if it is used to distort decision-making.