6 Mostly probabilistic models

Group 1
Author: Min Wu(8603148365, DIT025)
Email:minwuh081@gmail.com

I hereby declare that all solutions are entirely my own work, without having taken part of other solutions.

The number of hours spent: 20hours (Min Wu)
The number of hours has been present in supervision for this module: 0h

(DATA CALIBRATION)
a) Say that you have data from an opinion poll about how people vote. In your poll you happen to have a 53/47 distribution of men and women. However, in the population as a whole you know that the ratio is 50/50. Is it possible to improve the poll result, by taking into account this additional knowledge? If so, how would you suggest to do it?

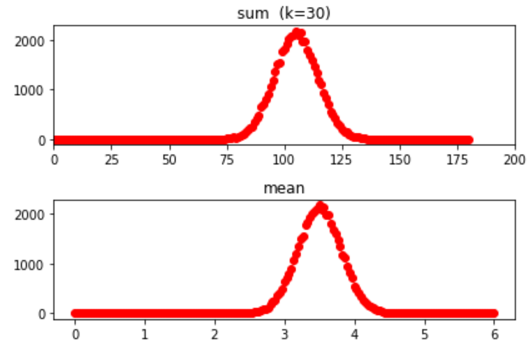
No, I don't think it will a way to improve the poll result by taking into account this information. 53/47 distribution is very close to 50/50. In the problem, there isn't mention how many people in vote either. The variance may exist.

(investigating the abstract)
(DICE SIMULATION)
Have a look at the program dice.py, and study it, including the pseudo-random generator. (you may read about about pseudo-number generation) Simulate the sum of two dice (Monte Carlo simulation = simulation based on random generation). Explain why some values appear to be more probable than others.

Understanding the program dice.py:

1. one dice roll 30 times
2. calculate the sum of those appearing values
3. repeat 1 and 2 50000 times.
4. plot the distribution of each sum

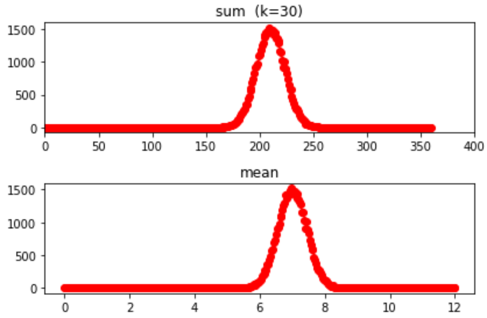
one dice



The number of times vs of the sum of one dice rolled for 30 times (up panel), The number of times vs the value for a dice (down panel). The highest probability is located at 105 which appears 2182 times and its mean value is 3.5.

- the minimal number of one dice is 1, so the minimal sum of one dice rolled for 30 times is 30.
- the maximal number of one dice is 6, so the maximal sum of one dice rolled for 30 times is 180.
- The mean value is in the range of 1-6.

two dice



The number of times vs of the sum of two dice rolled for 30 times (up panel), The number of times vs the value for two dice (down panel). The highest probability is located at 209 which appears 1521 times and its mean value is 7.

The minimal sum of two dices rolled for 30 times is 60 and the maximal sum of two dices rolled 30 times is 360.The mean value is in the range of 1-6.

a)Simulate for more than two dice, i.e. as a function of k. For the sum, what can you observe about its expected value and its variance? What can you observe about the mean? At least, give a qualitative answer, and a more quantitative if you can (eg. exactly what is the expected value of the sum?). (I here assume that you have already refreshed the notion of mean and variance from your other course)

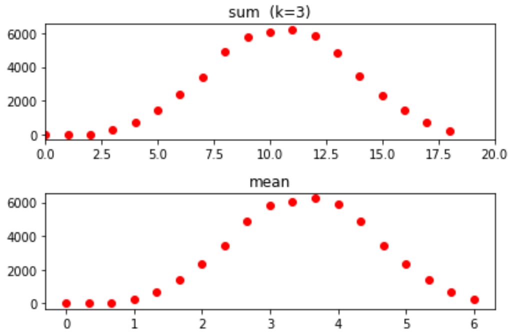
One dice rolls k times,
simulate two dice is related to one dice roll two times of k
simulate three dice is related to one dice roll three times of k.
The number of dice (p) as a function of k is one dice roll pk times.

The expected value is p* one dice sum value and its variance is p*the variance of one dice.

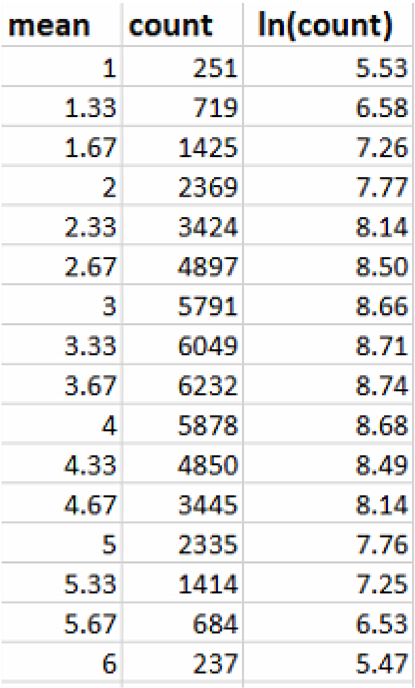
- one dice = 105 the variance ∓ 30 mean 3.5
- two dice = 210 the variance ∓ 60 mean 7.0
- three dice = 315 the variance ∓ 90 mean 10.5
- four dice = 420 the variance ∓ 120 mean 13.5

As for the shape of the curve apparent already for k=3, it is quite possible to discover a formula with the techniques of trying out that you have already practiced in the course. Even if you already know the answer from other courses, try to imagine and propose how the formula could be discovered with simple observations and testing. If you don't know the answer make an initial attempt to guess the formula and/or suggest some ideas, but don't spend a lot of time here unless you really want to. I ask this mostly to make you aware that by trying things out, you would probably after a while find the formula.

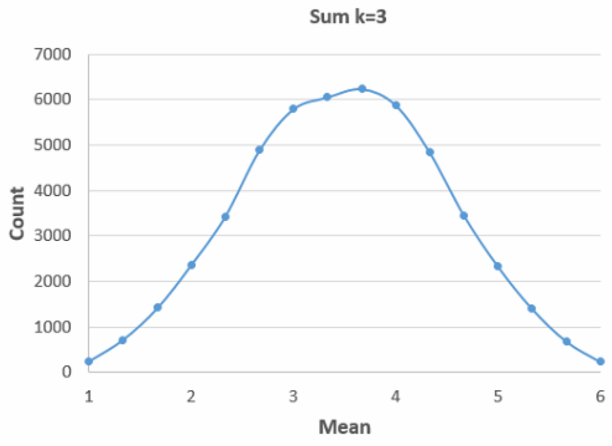
k= 3



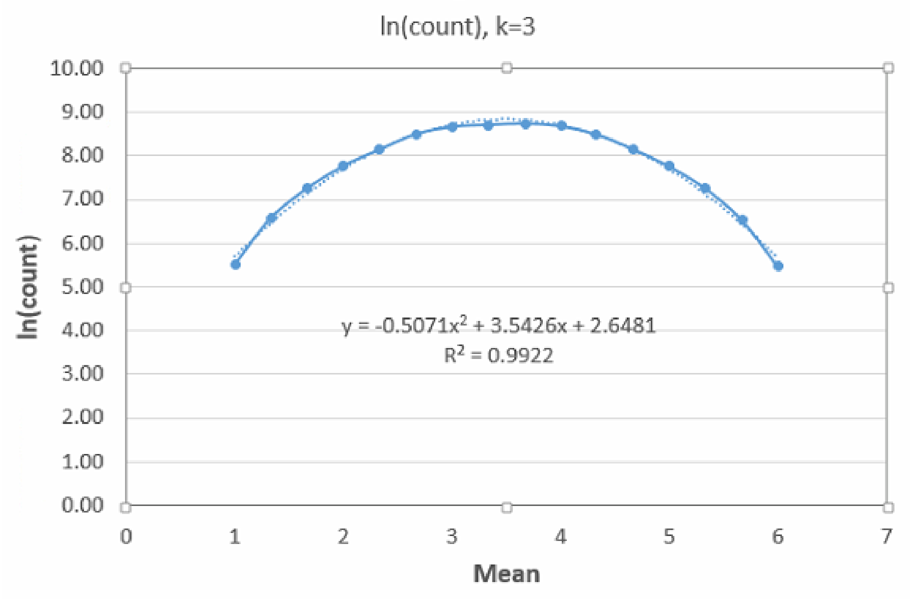
The minimal sum of one dice rolled three times is 3 and the maximal sum of one dice rolled three times is 18. The mean value is in the range of 1-6.



mean	count	ln(count)
1	251	5.53
1.33	719	6.58
1.67	1425	7.26
2	2369	7.77
2.33	3424	8.14
2.67	4897	8.50
3	5791	8.66
3.33	6049	8.71
3.67	6232	8.74
4	5878	8.68
4.33	4850	8.49
4.67	3445	8.14
5	2335	7.76
5.33	1414	7.25
5.67	684	6.53
6	237	5.47



Take the ln(count), so the plot becomes:



The fitted function is -0.5071x^2+3.5426x+2.6481 with R^2 equals 0.9922.

It seems quite good.

The final formula is :

y=exp(-0.51x^2+3.54x+2.65)

(investigating the world)
(STOCHASTIC TRAFFIC SIMULATION)
A classical application of statistical models is simulation of systems where things happen randomly based on certain probability distributions that are chosen to be as realistic as possible. This is called Monte Carlo simulation. See for example these simple traffic simulation demos: demo1, demo2, demo3, wikipedia (there are lots out there).

All I ask you is to take a look so that you have seen it. Comments still welcome!

Those simulations are based on random number generation. Is it in the real life, the driver drive a car from one place to the other place is a random decision?

(RADIOACTIVE DECAY)
Radioactivity can be modelled by assuming that each atom (of a certain radioactive kind) has a given probability per time unit to emit a particle. When the particle has been emitted, the atom is no longer radioactive. Motivate what kind of function you can expect the radioactivity to follow over time.

I think each atom should randomly emit a particle in a certain periodic time and in the end it will close to 0. At beginning all the atoms have radioactivity, the probability to emit a particle is highest, with the time increase, more and more atom is no longer radioactive, the radioactivity may be largely decayed. I guess it will be related to a exponential decay function.

(MEDICAL TEST)

A public screening is done of a group of people to find the persons who have the disease X. This is done with a medical test. As with most medical tests, the test is not 100% reliable. It gives a correct result with a probability of 99% if the person has the disease, and with 97% if the person does not have the disease. Prior to the screening, it has been estimated that about 0.33% of the individuals in the group have the disease.

For a particular person the test has indicated a positive result. What is the probability that the person actually has the disease? (If you cannot solve the problem, at least try to explain why the answer is not simply 0.99 or 0.97!)

Hint: Begin by writing down in mathematical notation what you know from the start. Try also to think what would happen with very extreme or symmetric numbers to investigate and understand the problem (this is a good generally useful technique).

- If the person has a disease, it gives the correct result will be a probability of 99%.
- If the person has no disease, the correct result will be a probability of 97%.
- it has been estimated that 0.33% of the individuals actually in the group

The problem is if the person has a positive result, what is the probability that the person actually has the disease?

if there are 100000 persons in the group, 330 of them actually have disease.327 of them are predicted to be positive which is right and 3 of them predict to be negative which are not right.

99670 person is health, 96680 of them predict to be negative which are right and 2990 of them are predicted to be positive which are not right.

The list is as followed:

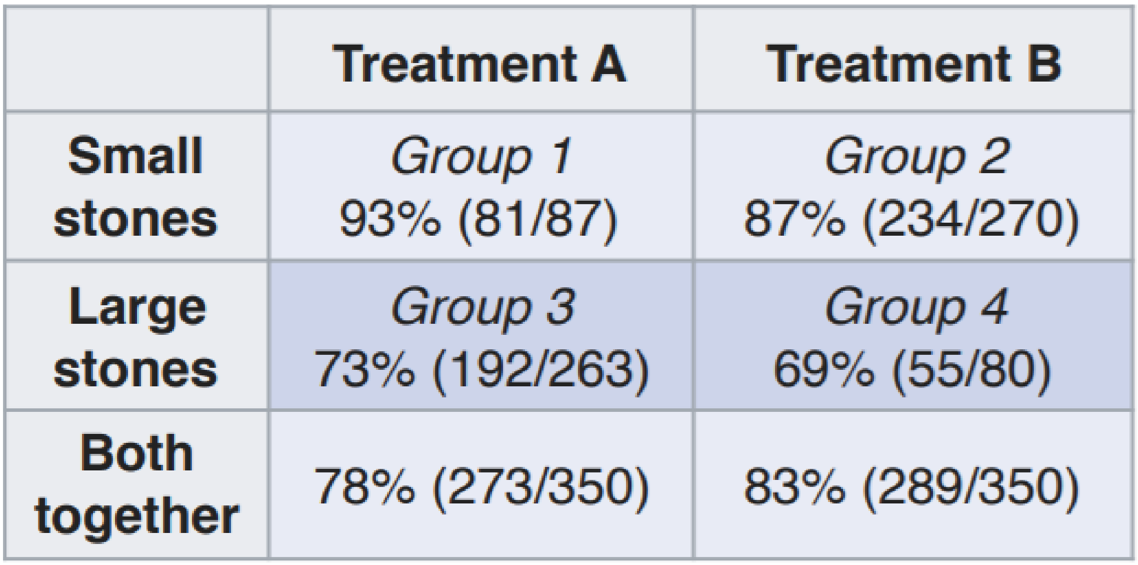
	actual positive	actual negative
predict positive	327 (right)	2990 (wrong)
predict negative	3 (wrong)	96,680 (right)

It turns out that totally 327+2990=3317 samples are predicted to be positive, the correct results is only 9.9%.

Totally, 96683 samples are predicted to be negative, the correct results are 100%.

For a particular person if the test has indicated to be positive, the probability to actually have disease is about 9.9%.

(KIDNEY STONE TREATMENT)
In a study, two treatments for kidney stones were considered (this is real data). Carefully study the table with the success rates and make any observations.



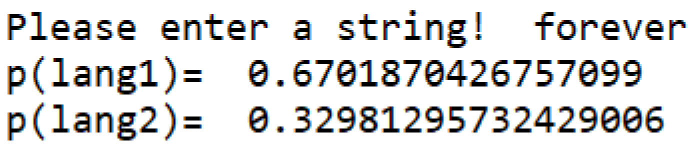
	Treatment A	Treatment B
Small stones	*Group 1* 93% (81/87)	*Group 2* 87% (234/270)
Large stones	*Group 3* 73% (192/263)	*Group 4* 69% (55/80)
Both together	78% (273/350)	83% (289/350)

The success rates for both together are not comparable.
Because for treatment A, the data is largely distributed on large stones which has lower success rates compared to its small stones treatment.

For treatment B, the data is largely taken from the small stones which has higher successful rate. In order to have comparable result, the number of samples for different size of stone should be similar for both treatments.

(LANGUAGE RECOGNITION)
Look at the simple language recognition program in language.py. It uses the probabilities of single letters for two languages (from reference texts lang1.txt and lang2.txt), to calculate if a string is likely to belong to the first or the second language. Investigate and explain!

Investigate



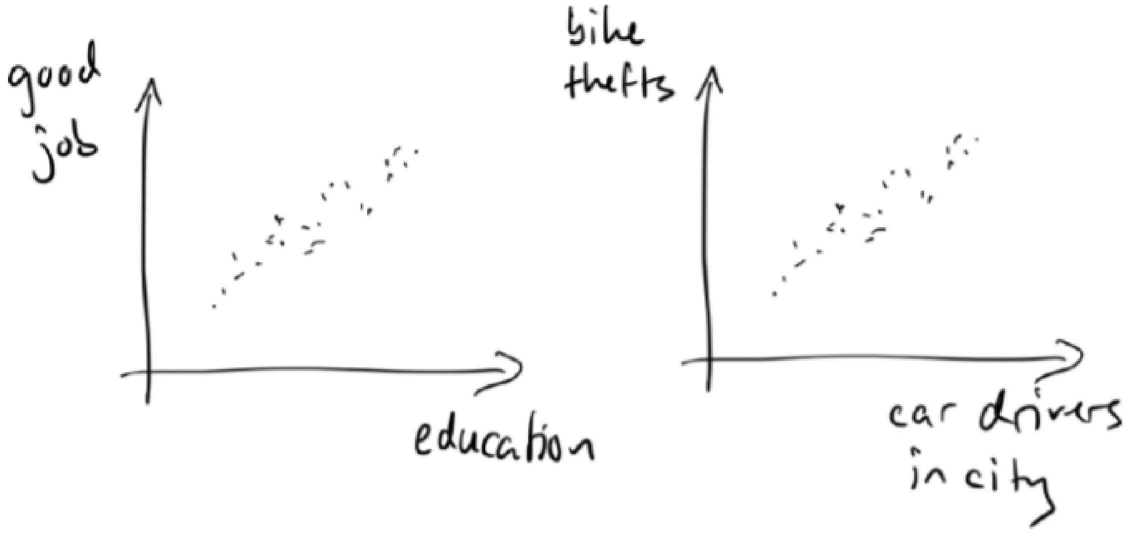
```
Please enter a string!  forever
p(lang1)=  0.6701870426757099
p(lang2)=  0.32981295732429006
```

explain

1. The programme get words from different text written using different language.
2. Calculate the distribution of each characters in a language
3. Read a string
4. compare the characters of the string to the character in a language to predict the word coming from which language.

Comments: the correlation between two character or even more characters are missing in this programme. The results are thus not very believable.

(CORRELATION AND CAUSALITY)
Study the two graphs. What are your observations?



Higher education level, better jobs. Education is correlated with education because high education is required in some good jobs.

More car drivers in the city, more bike thefts. However, the car drivers in the city seem not correlated to the bike thefts. A car driver is not a necessary condition to steal a bike.

(WEATHER PREDICTION)
Consider the issue of predicting the probability of precipitation on a given future day. To your help you have weather statistics from the last five years. Now suppose you want to predict if there will be any precipitation on May 19.
Should you base your prediction on
a) statistics for May 19 during these years,
b) statistics for all days in May during these years
c) statistics for all days during these years?

Before you consider anything else, assume that the probability is simply estimated as the relative frequency of precipitation for all days you choose to include. Motivate your answer, and discuss the difficulties involved in choosing the model. Would the

situation be any different if you had 100 years of weather statistics? Is choosing the model something that necessarily requires human judgement?
Hint: while the specific question is not so difficult to intuitively answer, the general issues behind the question are deep. So think! (I placed the problem in this section, since we are not really asking how to best predict the weather, but rather asking a fundamental question about modelling and prediction)

I will choose statistics for all days in May during these years. The weather seems related to a season, so it will not make sense to statistics for all days in a year. Statistics for May 19 during these years will be so specific to a certain date. The samples will only be 5 if the years are 5. I think May will be in the middle of spring so that I choose statistics for all days in the middle of a season. I could get the weather distributions for May. However, it still hard to evaluate the May 19th.

If I had 100 years of weather statistics. I could try to statistics for May 19th during 100 years.

(NATURE OF RANDOMNESS AND PROBABILITY)
a) What kind of predictions can you draw from stochastic models, compared to when you have a deterministic model (like for example an astronomical model of planetary motion).

Stochastic models are used to identify unknown potential relationships among variables. for a deterministic model, the relationships are already determined.

b) Does randomness exist? If so, what is it?
No. I don’t think the exact randomness exist.

c) Is there a “right” probability e.g. for rain tomorrow? Or even for a die?
No.

(SELF-CHECK)
pass