3 Functions, equations and geometry II

Group 1
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I hereby declare that all solutions are entirely my own work, without having taken part of other solutions.
The number of hours spent: 20hours (Min Wu)
The number of hours has been present in supervision for this module: 3h

(keeping track)
CONSUMER TEST RANKING)
Suggest a systematic approach for ranking different products, e.g. computers, dishwashers etc., to determine the best buy. You may need to create your own concrete examples to develop your ideas, and to explain how you are thinking.

For Example, I want to buy a washing machine.

Some possible factors for buying a washing machine.
Price,
Color,
Brand,
Energy,
Guarantee,
Function,
Users' feedback.

The first thing I would take into account is the other users' feedback. It is important to know the machine's defects which probably would be missing in the advertisement.

Second, I would think about the price, the price is not only about how much I need to buy the machine but also I would take into account how often I would use the machine.

- For a washing machine, I probably would use at least once per week. I would think about the electricity cost per month.
- it is not easy to change a washing machine, it will cost a lot for delivery and to throw it away, it is also not easy. so I am aiming to use if for a long periodic time. I would take into account the guarantee so I don't need to pay extra to buy insurance for it.

The final cost will be:
Assume I am going to use the machine for 10 years.
Cost per year=(price for the machine + electricity cost per month * 10 years + delivery cost + insurance cost per year * 10 years) / 10 years.

Third, I would want the machine to have both washing and drying functions, so that it saves my space and also it will save time.

(investigating the abstract)
(THE ARITHMETIC AND GEOMETRIC MEAN)
a) Investigate the relationship between the arithmetic mean and geometric mean. Begin to try out in an experimental way.

arithmetic mean $\frac{x_1 + x_2 + \cdots + x_n}{n}$

geometric mean $\sqrt[n]{x_1 x_2 \cdots x_n}$

Try some numbers they are in the same magnitude.
a=[23, 45, 67, 55, 74]
arithmetic mean a=52.8
geometric mean a=49.0

Try the same numbers but add one number with 2 order magnitude larger.
a=[23, 45, 67, 55, 74, 2450]
arithmetic mean a= 452.3
geometric mean a=94

When all numbers are distributed in the same magnitudes, the results of arithmetic mean are similar to the results of the geometric mean. When the numbers are distributed in a different magnitude, the results of arithmetic mean are increased dramatically compared to the results of the geometric mean.

b) If you found some interesting property, try to prove it. Otherwise you will not know if it is always true or not! HINT try simple cases first!

- There are 100 numbers and all in one order of magnitude.

1<n1<10,
1<n2<10,
.
.
1<n100<10
Arithmetic mean:
1<n1+n2+..n100<1000
1<(n1*n2*..n100)^(1/100)<10
Geometric mean

$1<n_1*n_2*..n_{100}<10^{100}$
$1<(n_1+n_2+..n_{100})/100<10$

When all 100 numbers in one order of magnitude, the results of arithmetic mean and geometric mean are similar.

- There are 100 numbers, 99 of 100 are all in one order of magnitude, but 1 of 100 is in three order of magnitude.

$1<n_1<10$,
$1<n_2<10$,
.
.
$1<n_{99}<10$
$1000<n_{100}<10000$
Arithmetic mean:
$1009<n_1+n_2+..n_{100}<10990$
$10.09<(n_1+n_2+..n_{100})/100<109.9$
Geometric mean
$10^3<n_1*n_2*..n_{100}<10^{(102)}$
$0<(n_1*n_2*..n_{100})^{(1/100)}<10.47$

- There are 100 numbers, 98 of 100 are all in the one order of magnitude, but 2 of 100 is in three order of magnitude.

$1<n_1<10$,
$1<n_2<10$,
. .
. .
$1<n_{98}<10$
$1000<n_{99}<10000$
$1000<n_{100}<10000$
Arithmetic mean:
$2098<n_1+n_2+..n_{100}<20980$
$20.98<(n_1+n_2+..n_{100})/100<209.8$
Geometric mean
$10^6<n_1*n_2*..n_{100}<10^{(106)}$
$0<(n_1*n_2*..n_{100})^{(1/100)}<11.5$

When all 99 or 98 number in one order of magnitudes, one or two numbers are in the higher order of magnitudes, the results of arithmetic mean are getting larger than geometric mean.

- There are n numbers, a of n are all in the x order of magnitude and b of n is in y order of magnitude.

$10^x<na_1<10^{(x+1)}$,
.
.
$10^x<na <10^{(x+1)}$

$10^y<nb1<10^{(y+1)}$

.

.

$10^y<nb<10^{(y+1)}$

Arithmetic mean:

$a*10^x+b*10^y<na1+..na+nb1+nb<a*10^{(x+1)}+b*10^{(y+1)}$

$(a+b)*10^x+b*10^{(y-x)}<sum<(a+b)*10^{(x+1)}+b*10^{(y-x)}$

$10^x+(b/(a+b))*10^{(y-x)}<\text{Arithmetic mean}<10^{(x+1)}+(b/(a+b))*10^{(y-x)}$

Geometric mean

$10^{ax}*10^{by}<na1*..na*nb1*nb<10^{a(x+1)}*10^{b(y+1)}$

$10^{(ax+by)}<na1*..na*nb1*nb<10^{(a(x+1)+b(y+1))}$

$10^{((ax+by)/(a+b))}/\text{Geometric mean}<10^{(a(x+1)+b(y+1)/(a+b))}$

$10^{((ax+by)/(a+b))}/\text{Geometric mean}<10^{(a(x+1)+b(y+1)/(a+b))}$

$10^{((ax+by)/(a+b))}/\text{Geometric mean}<10^{((ax+by)/(a+b)+1)}$

if $x=y$, n numbers are in the same order of magnitude

$10^x<\text{Arithmetic mean}<10^{(x+1)}$

$10^x<\text{Geometric mean}<10^{(x+1)}$

Arithmetic mean and Geometric mean are in the same range.

if $y>x$, n numbers are in the different order of magnitdue

$10^x+(b/(a+b))*10^{(y-x)}<\text{Arithmetic mean}<10^{(x+1)}+(b/(a+b))*10^{(y-x)}$

$10^{((ax+by)/(a+b))}/\text{Geometric mean}<10^{((ax+by)/(a+b)+1)}$

In the terms of Arithmetic mean, right side minus the left side, I can get the gap which is $[10^{(x+1)}+(b/(a+b))*10^{(y-x)}]-[10^{(x+1)}+(b/(a+b))*10^{(y-x)}]=10^{(x+1)}-10^x$, with the x increased the number will be dramatically increased.

In the terms of Geometric mean, the difference between right side and left side is calculated by $[10^{((ax+by)/(a+b)+1)}]/[10^{((ax+by)/(a+b))}] = 10$ which is independent on x and y.

(investigating the world)

(EMPIRICAL CURVE FITTING)

For two related physical variables the following relationship has been measured:

time	distance
88.0	57.9
224.7	108.2
365.3	149.6
687.0	228.07

time	distance
4332	778.434
10760	1428.74
30684	2839.08
60188	4490.8
90467	5879.13

Investigate and suggest a mathematical equation describing the relation between these two quantities. We ask you to NOT use a graph drawing calculator or other software, with automatic curve fitting. We want you to experience the problem of

manually searching for a function yourselves. Preferably use the Mathematica functions suggested below for plotting. Otherwise, feel free to use your creativity.

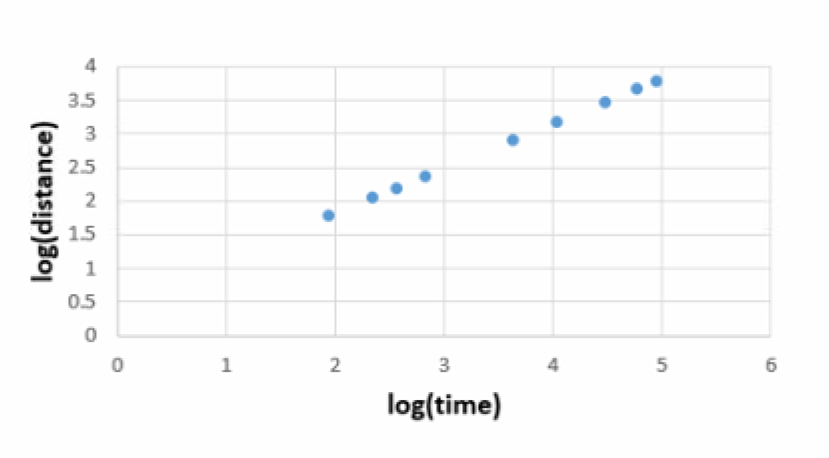
Explain how you did to find the model, and motivate your choice. Is the fit of your equation good? How can deviations from the known table entries be justified?

time=[88.0, 224.7, 365.3, 687.0, 4332, 10760, 30684 ,60188, 90467]
distance=[57.9, 108.2, 149.6, 228.07, 778.434, 1428.74, 2839.08, 4490.8, 5879.13]

The numbers in the time and distance list are increased dramatically. Firstly, I will try logarithm of all those numbers in order to get the order of magnitude.

log(time)=[1.94, 2.35, 2.56, 2.84, 3.64, 4.03, 4.49, 4.78, 4.96]
log(distance)=[1.76, 2.03, 2.18, 2.36, 2.89, 3.16, 3.45, 3.65, 3.77]

Then I plot the curve for log(time) and log(distance):



It is likely a linear relationship between log(time) and log(distance) so I try y=ax+b

In order to get this linear function, I will take two coordinates, one is from the first four points and the other form the last five points.

P1(2.56, 2.18) and P2 (4.03, 3.16)

a=(3.16-2.18)/(4.03-2.56)=0.98/1.47=0.67

y=0.67x+b

put P1 back into the function 2.18=0.67*2.56+b b=0.47

y=0.67x+0.47

log(time)=x so time= 10^x

log(distance)=y so distance=10^y

final solution is **Distance=exp(2/3exp(Time)+0.47)**

To evaluate the errors I will take the root mean square of (real value - expected value)

$$error = \sqrt{1/n \sum (y1_{real} - y1_{expected})^2}$$

Expected log(distance)=[1.77, 2.05, 2.19, 2.37, 2.91, 3.17, 3.48, 3.67, 3.79]

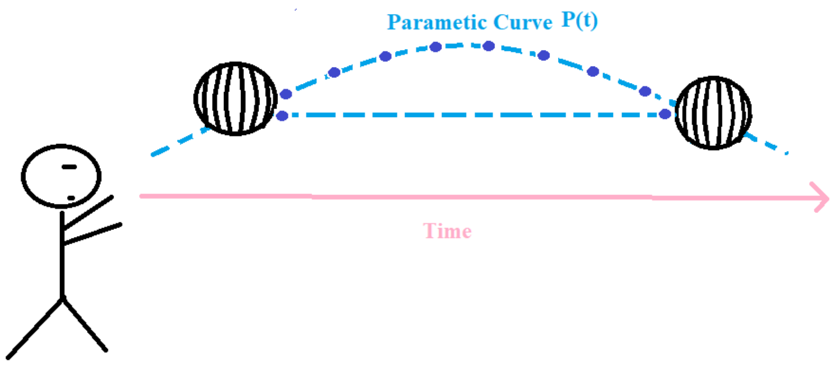
Error= 0.016 which is quite good.
(designing)
(CURVE FITTING AND COMPUTER GRAPHICS)
This is a reading exercise.

- **A short note on curve fitting**
- **Bezier curves in computer graphics. The Bezier curves were first invented by Pierre Bézier at Renault (automotive modelling). This text is quite characteristic for the use of mathematics in computer science and engineering, and also gives a (limited) impression of the area of computer graphics and curve modelling. When the text talks about a point, it refers to a two or three dimensional vector with the x,y and z coordinates. Such vectors can easily be added and multiplied by a scalar (a single number). Ask if you need further explanation! See also the short note on vector arithmetic at the end of this module. Make an attempt to read and understand the text, especially the first part. Don't worry if you have difficulties in understanding some parts - simply try to read and see how far you get and what difficulties you run into. Technical texts are never easy, and whatever it is that you manage to understand is good!**

As the answer in a concise way explain the parts you understand. Remember how we discussed about the unit conversions. In whatever you explain, do not only explain something around the issue, but try to explain the core of the matter as clearly as possible. Doing this forces you to think about what the main concepts really are.

When modeling systems, curves,and curved surfaces can be obtained by equations. Through those equations, the sets of triangles are then created and sent down the pipeline to be rendered. More triangles are turned, more detailed modeling is presented. The points in the triangles are used to form a curved surface and needed in animation to generate a smooth tessellation.

Parametric Curves
Parametric curves are used to describe the pathways for animations. This contains both the position and orientation.



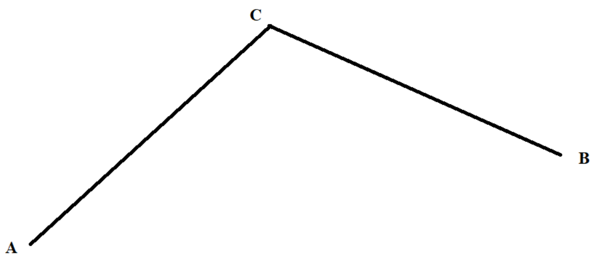
The points of a parametric curve are generated by a function of a parameter T, which generates the points P(t). In the more extremely short time, two points that are more close to each other which can generate a more smooth moving pathway.

Bezier Curves

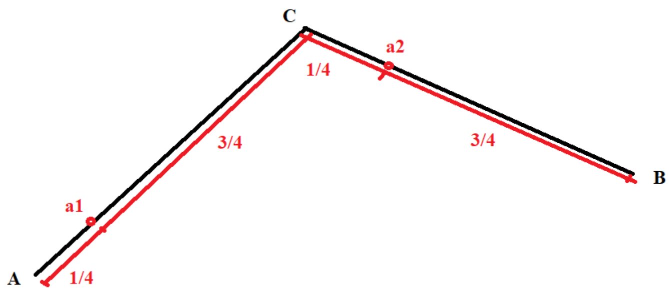
Bezier Curves are obtained based on repeatedly applying the linear interpolating between several control points.

A simple example of Bezier Curve

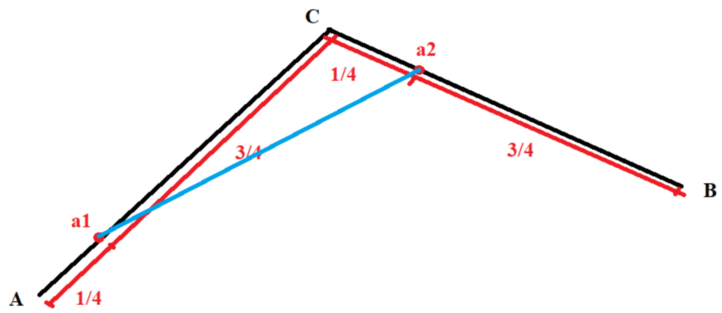
1. There are three points A,B,C, draw two lines connecting AC and BC respectively.



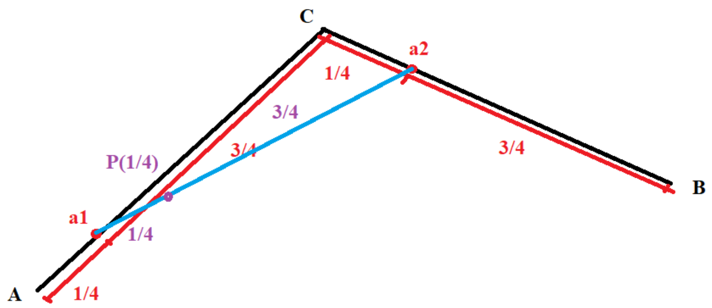
2. To find p(¼) then I could add two extra points a1 and a2 on the lines, let Aa1 = ¼ AC and Ba2=3/4BC



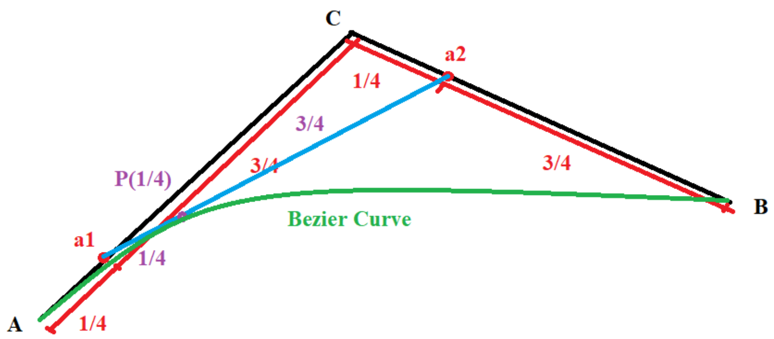
3. Connect a1 and a2



4. find a point P(¼) on the line a1a2 which a1P(¼)=¼ a1a2



5. Plot the curve pass A, B and p(¼) to obtain Bezier Curve



P(¼)=(1-(¼))a1+(¼)a2
a1=(1-(¼))A+(¼)C
a2=(1-(¼))B+(¼)C
P(¼)=(1-(¼))^2A+2(1-(¼))(¼)C+(¼)^2B

With more control points applied, more curves are generated. With the control points n, after the linear interpolation applied k times, there will be P_n^k intermediate points produced. The generalized function for Bezier Curves is:

$$\mathbf{p}_i^k(t) = (1-t)\mathbf{p}_i^{k-1}(t) + t\mathbf{p}_{i+1}^{k-1}(t), \quad \begin{cases} k = 1 \dots n, \\ i = 0 \dots n-k. \end{cases}$$

By using Bernstein Polynomials, the Bezier Curves can be obtained through an algebraic formula.

In the term of rational Bezier Curves, it takes a weighted sum of the Bernstein polynomials in the function.

In the term of bounded Bezier Curves, the region between the curve and the straight line between the first and last control points is defined. Only the pixels which are located inside of the region will be reserved.

In the term of Continuity and Piecewise Bezier Curves, The Bezier curve can be smoothed by a composite curve formed from several curve species.

Cubic Hermite Interpolation is used if the curves haven't a smooth construction for example, rendering hair.

(REASONING - DIFFERENT KINDS OF REASONING)
Try to characterize what kind of reasoning is used in the following conclusions:
• I have parked my bike here many times, so I think it's safe.
plausible reasoning
• All men are mortal. Socrates is a man. Therefore, Socrates in mortal.
deductive reasoning
• The lawn is wet. I suppose it must have rained.
plausible reasoning
• From the figure we can see that the area of the circle is clearly less than 4r^2, and also more than 2r^2. So let's say 3*r^2.
deductive reasoning
• We can see in this graph that those with more education are much less likely to become unemployed in the future. So therefore, we should encourage everyone to get a proper education.
plausible reasoning
• Our state has the highest crime rate in the country. And our city has the highest crime rate in the state. So we have the highest crime rate in the country.
plausible reasoning
• If there are no more than five possibilities and I check all of them, then I will know the answer.
deductive reasoning
• My very healthy grandfather smokes two packages a day and he is 95 years old => smoking is harmless.
plausible reasoning
• It is raining. I have an umbrella => I will take the umbrella.

plausible reasoning

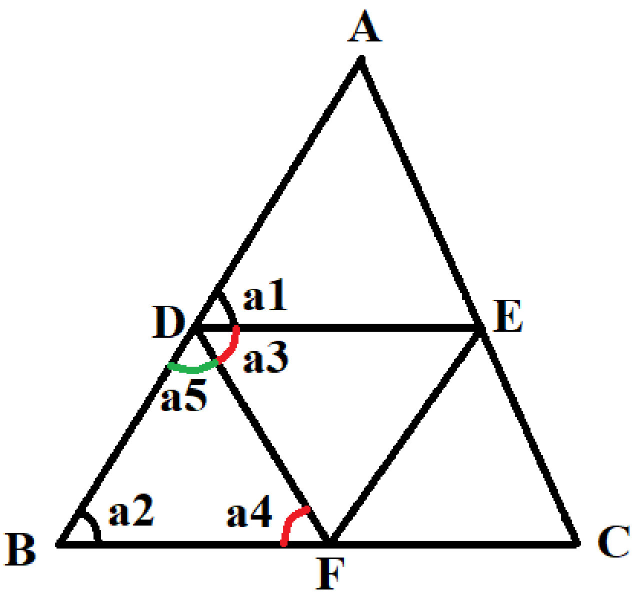
The most important distinction is between deductive reasoning (where the conclusion is certain given the premises), and plausible reasoning (where the conclusion may be probable but is not certain), so primarily try to categorize the reasoning in these two groups. Common forms of plausible reasoning are inductive and abductive reasoning (not exhaustive).

(REASONING - NATURE OF PROOFS)

A proof can be seen as a deduction from known (or assumed) premises (input) to a conclusion (output). So it's like a function in this respect - from what we already know to something new that we didn't know. The most basic assumptions we begin with are called axioms (obvious truths), but often other generally known results are accepted as "input" to other proofs. What premises the proof is using is an important aspect of the proof, and that can be quite different for different proofs of the same thing! It can sometimes be difficult to sort out exactly what basic assumptions a proof depends on since we often tend to use what we already know quite freely.

a) For a nice example, see this discussion on proofs for the sum of the angles in a triangle. Answer this question by explaining in your own words what the premises are in the two proofs presented.

- The first proof:



There is a triangle ∆ABC here, draw another triangle ∆DEF inside of ∆ABC with the line DE paralleled with BC.

because:

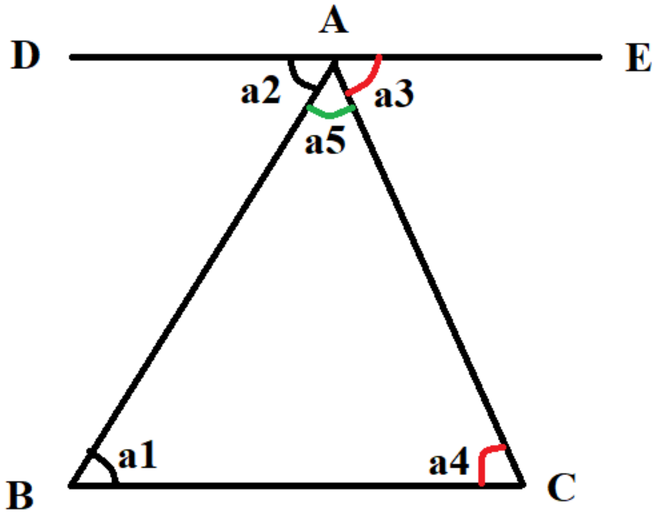
1. <a1+<a3+<a5=180degree
2. DE//BC

   so <a1=<a2 and <a5=<a4

so <a2+<a4+<a5=180 degree

Thus the sum of angles in a triangle is equal to 180 degree

- The second proof



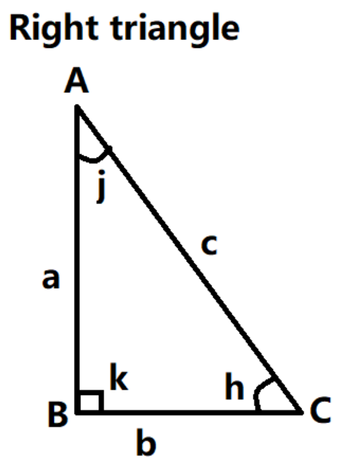
There is a triangle △ABC, draw a line DE paralleled with BC.
Because:

1. <a2+<a5+<a3=180degree
2. DE//BC
   <a2=<a1 and <a3=<a4

so <a1+<a5+<a4=180degree.
Thus the sum of angles in a triangle is equal to 180 degree.

(MODELLING/REASONING - ORIGIN AND MOTIVATION OF FORMULAS)
This problem is to make you aware of where formulas come from and to what extent you should believe in them. For each equation below discuss:
a) how do we know it is true (or at least useful),
and b) is it exact or is it some kind of approximation)?

The equations: • a^2+b^2=c^2 (Pythagoras' theorem)
motivated formulation
It is exactly true for a right triangle, seeing the figure below. <k is 90 degree and a^2+b^2=c^2



proof:
because:

1. $\sin j=b/c$
2. $\cos j=a/c$
3. $(\sin j)^2+(\cos j)^2=1$

so:

$(b/c)^2+(a/c)^2=1$
$b^2+a^2=c^2$

• population= C * a^t (C and a are constants, t is the time in years)
Constant C and a are a kind of approximation. In order to know if the equation is true, so an error needs to be estimated based on the real data and expected data
• F= G m1 m2 / r^2 (gravity between two bodies)
It is used to calculate the force of gravitational attraction between any two massive bodies. constant G is a kind of approximation.
• stock index= 2045+0.0034 t (trend analysis)
motivated
The constant 2045 and 0.0034 is a kind of approximation, error thus needs to be estimated based on the real data and the calculated data.
• 100*weight+length < 320 (max allowed parcel size for a postal service)
It is an exact. It is not an equation based on equivalent, it is a range of 0-320. If it is not allowed to have a parcel size larger than 320. It means all the size must smaller than 320.
• #presentStudents + #absentStudents = #allStudents (for a class)
It is an exact.
• air drag force = C* v^2 * A (air drag for a moving object, C is a constant depending on the shape but not the size, v velocity, A cross section area)
motivated model
The constant C is a kind of approximation.
If you can, try to categorize the expressions in different groups. Spend a moderate time on this exercise, a short comment on each equation and maybe a few general observations is sufficient.
It is always a kind of approximation when constants are enclosed into a certain equivalent equation.

Reflection

I. (SUPERVISION AND FOLLOW-UP LECTURE)

a) Did you have your checkpoint meeting for this module?

No

b) Did both of you attend the compulsory follow-up lecture? If you already talked to us about this, please explain.

Yes, I attend the compulsory follow-up lecture.

c) If you were asked to talk to a supervisor about the main submission, who did you talk to?

No

Reflect on your experiences from working with the module and try to make the most out of them. You are also encouraged to discuss your experiences with other groups.

If you reflect around individual problems (which is good), try to also draw general conclusions that may be helpful for you going forward in this course and long-term.

(Time spent in the reflection is time well spent, as it maximizes the learning from the significant effort you already made when working with the problems.)

As the answer, give some well motivated points summarizing your reflections.

I spent 15 hours and tried my best to solve those problems in module 2. I went to supervision three times in the week.

The first task in module 2 is quite straightforward to me. I need to give a suggestion using a systematic approach for ranking a product. I gave an example of buying a washing machine. I gave some factors and calculator the cost per year for the purchasing. I want to buy a water mashing with the lowest costing per year. I didn't specify a decision of choosing.

For the second task, I started from some values and calculated the arithmetic mean and geometric mean. I got some feeling about the arithmetic mean is larger than the geometric mean and proved it. During the following-up lecture, I got one more property that geometric mean can not deal with the value smaller than zero. Furthermore, I got some examples of their physical meaning.

The third task, the values increase dramatically both for distance and time. I took logarithm for distance and time and then got a linear relationship. Then I calculated the square average mean value for the error. It turned out quite good results. I am happy with my results. I learned another method in the follow-up lecture by assuming and adjust the parameter for the power to get a good fitting.

The fourth task, I think I solve it quite good after reading the references and tried a simple example myself.

The sixth task, I can correctly distinguish those statements between plausible reasoning and deductive reasoning.

The seventh task, I can solve it using my junior learned knowledge which is quite straight forward. But In the following-up lecture, Euclid's method is more mathematical prove. It does not depend on the sum of the total angle of the triangle is 180 degree.

The eighth task, It is quite straightforward to me. I realize there some statements I could understand and k aware of where formulas come from and to what extent how I could believe in them. But in others, I could not aware of where formulas come from and I could not believe them.

Summarize

In this module,

I learned how I could ranking a product bought in daily life. I need first figure out what is my goal for example, I want to have a longest using time or I want to have a lowest annually cost. Then I could take into accounts those factors which may be play rolls in the purchasing.

Furthermore, I learned the properties of arithmetic mean and geometric mean, arithmetic mean is always larger than the geometric mean. The geometric mean can not deal with the values smaller than zero. The geometric mean is better suitable for proportional growth and exponential growth, financial indexes and noise-canceling filter in image processing.

I also further learned how to fit a curve together with the evaluation of the fitting results by square mean derivations.

I learned Bezier Curves and its derivatives, for example, rational Bezier Curves, bounded Bezier Curves, Continuity and Piecewise Bezier Curves. Bezier Curves is a parametric curve widely applied in computer graphics and related field.

I also learned different reasoning, for example, plausible reasoning and deductive reasoning, aware of where conclusions come from and to what extent you should believe in them. If a proof takes a known input to a conclusion, it is a deduction and if a proof takes most basic assumptions or generally known results to a conclusion, it is plausible reasoning.

In the last part of this module, I learned the difference between modeling and reasoning which help me to analysis where the formulas they come from and to what extent I should believe in them.