Module 2 Functions, equations and geometry I

Group 1
Author: Min Wu(8603148365, DIT025)
Email:minwuh081@gmail.com

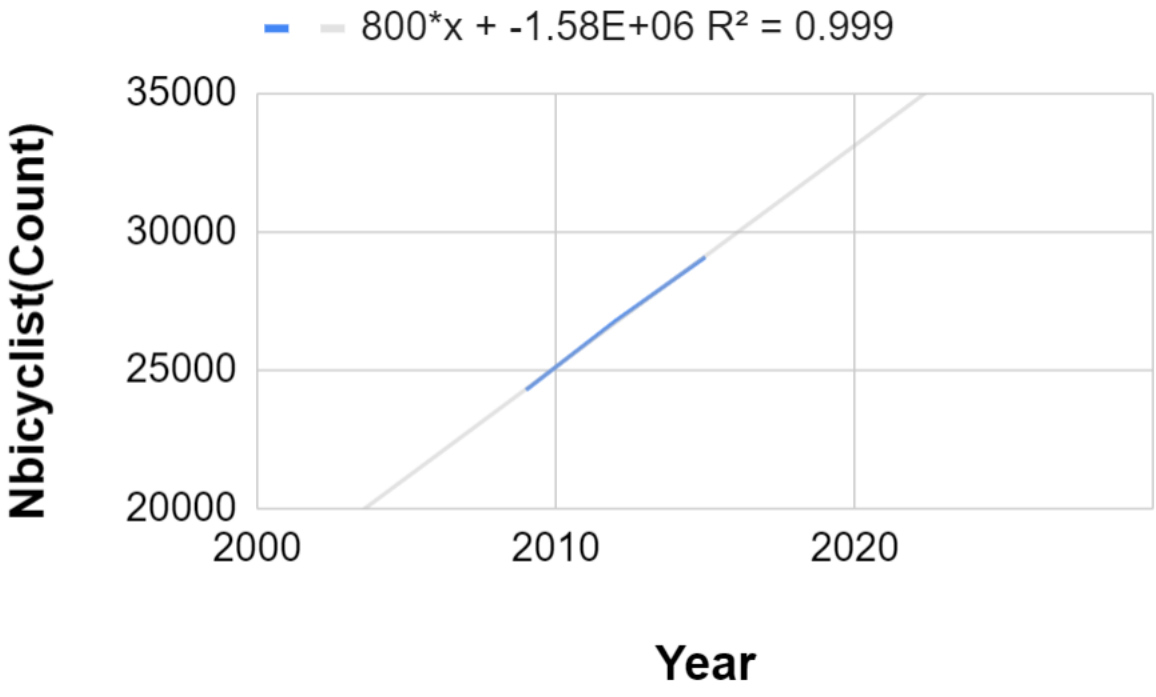
I hereby declare that all solutions are entirely my own work, without having taken part of other solutions.

The number of hours spent: 20hours (Min Wu)
The number of hours has been present in supervision for this module: 5h

(keeping track)
(SIMPLE FORECAST)

In a city, the number of daily bicyclists have been estimated every third year, see the table. a) The city needs an estimate for 2013, to compare with other data for 2013. Suggest an estimate for 2013 and explain how you were thinking to find it. b) For the planning project "CITY 2020", the city also wishes an estimate for the year 2020. Propose such an estimate, explain your thinking, and propose what you want to say to anyone who will use your estimate.

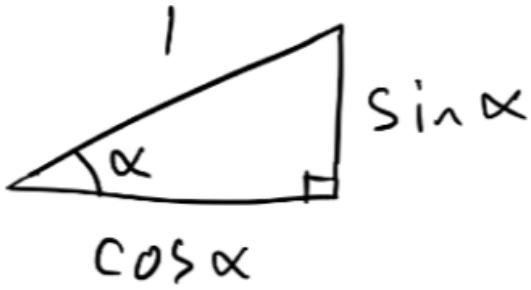
2009	24 300
2012	26 800
2015	29 100



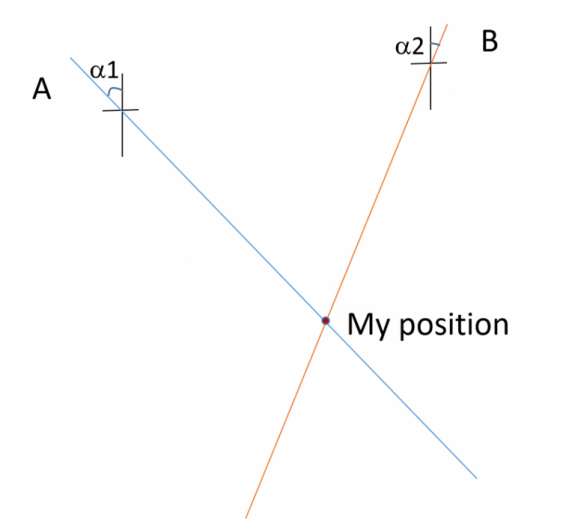
By the fitting to linear regression, the function is obtained Nbicyclist=800*(year)-1.58E+0.6 with R^2 equals to 0.999. R-square is a goodness-of-fit measure for linear regression models

which indicate the strength of the relationship between the model and the dependent variable on a convenient 0 – 1 scale. Higher the number, better the fitting is.
For year 2013, I can estimate that the number of the daily bicyclists will be about 30 400.
For year 2020, I can estimate that the number of the daily bicyclists will be about 30 600.

(HOMING)
When you approach land in a ship, you can establish your position on the nautical chart by using a compass to measure the angles to two different landmarks with a known position. This is a classical way to ensure a safe approach.



a) Explain how the position of the ship can be calculated analytically, in sufficient detail so that it can easily be implemented in a computer program.



- The angle (a1) of position A, shown in the picture, is known, same to the angle (a2) of position B, then I can make two linear expressions of position A and B according to the angles.

  Position A: $y = (1 / tan\alpha 1) * x + bA$
  Position B: $y = (1 / tan\alpha 2) * x + bB$

- I know the coordinates A (x1, y1) and B (x2,y2) so I can solve the constant bA and bB

$$y1 = (1/tan\alpha 1) * x1 + bA$$
$$bA = y1 - (1/tan\alpha 1) * x1$$
$$y2 = (1/tan\alpha 1) * x2 + bB$$
$$bB = y2 - (1/tan\alpha 1) * x2$$

- My position is the cross point of two linear expressions. So I can solve my position by the equation $(1/tan\alpha 1) * x + bA = (1/tan\alpha 2) * x + bB$, then put back the x to get y. I can have my position (x,y)

(investigating the abstract)
(INVESTIGATE MATHEMATICAL FUNCTIONS THROUGH CHANGE)
Have a look at the program function.py. It creates some well-known mathematical functions in a simple way, by generating them from left to right based on their rate of change (the derivative). If you cannot run yourself, here is the output.
a) Carefully study the mechanism of the program and try to understand it. Give some comments on the mathematical aspects of the program.

The relationships between y and x are linked to different functions. However, in the program, the points for y-axis are obtained using a recursion function. Downstream y only depends on its upstream which is independent on x.

b) If you have the graph of a (one-variable) function, how can you geometrically construct the inverse of the function?
I could set x to y-axis and y to x-axis. for example, gplot(x,y) will become gplot(y,x).

Are any of the inverses of the functions shown by the program familiar to you? (I ran the program with an Anaconda installation of Python 3 on my Mac. It needs the matplotlib library.)

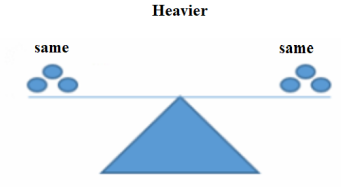
- Proportional function y=ax the inverse of the function is x=y/a
- Linear function y=ax +b, the inverse of the function is x=(y-b)/a
- Quadratic function y=a(x+b)^2 + c, the inverse of the function is x=((y-c)/a)^0.5-b
- Exponential function y=exp(x), the inverse of the function is x=lny.
- Exponential function y=exp(-x), the inverse of the function is x=-lny.
- Trigonometric sine function y=sin(2(pi)fx+w): x=(arcsin(y)-w)/2(pi)f
- Trigonometric cosine function cos(2(pi)fx+w): x=(arccos(y)-w)/2(pi)f

(TWELVE BALLS PROBLEM)
This is a classical mathematical puzzle. You have twelve balls with identical appearance. However, one of the balls is slightly lighter or heavier than the others. You also have a balance scale. How many weighings do you need to find the odd ball, and determining if it is lighter or heavier. Explain how.
3-4 times weighings

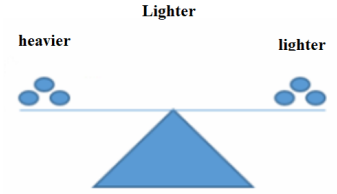
HINT: make a serious attempt, and report the best solution you found in a limited time.

1. put every 6 balls on each side of a balance scale, the 6 balls which have the lighter and heavier weight are recorded. Assume the results are shown as followed:

2. The heavier 6 balls are divided into two groups and put them on each side of a balance scale.
    - If the two groups have the same weight which indicates that there is a lighter ball exist in the lighter 6 balls.

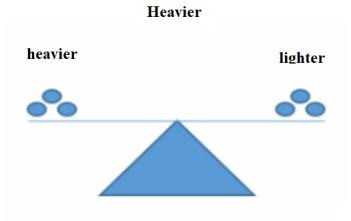


The lighter group with 6 balls divided into two groups and put them on each side of a balance scale.



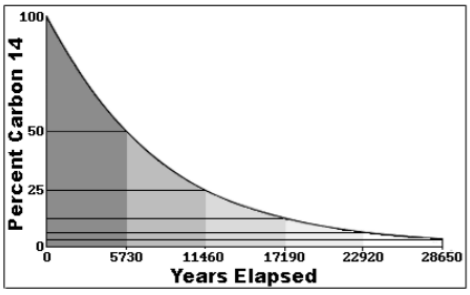
pick the three-ball in the lighter group and randomly choose two balls from these three balls and put one on each side of a balance scale. if two balls have the same weight, then the third one is the lighter or heavier one, if two balls have different weight, then the lighter or heavier one is determined

- If the two groups have different weight, which indicates that there is a heavier ball exist.

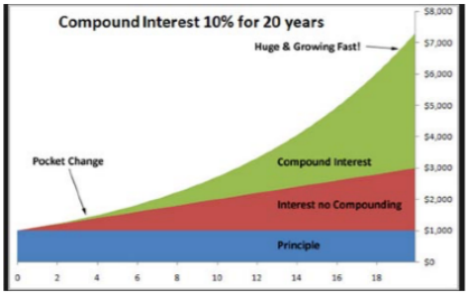


pick the three-ball in heavier group and randomly choose two balls from these three balls and put one on each side of a balance scale. if two balls have the same weight, then the third one is the lighter or heavier one, if two balls have different weight, then the lighter or heavier one is determined

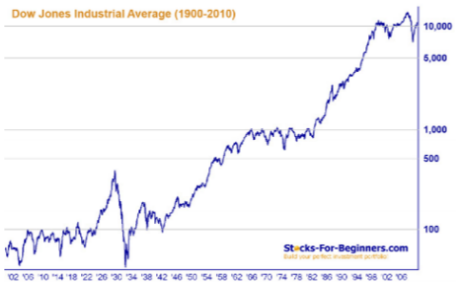
(investigating the world)
(IDENTIFY FUNCTIONS IN THE REAL WORLD)
Have a look at the different tables and diagrams (see separate page at the end), and try to link to mathematical functions you know and/or saw in the previous exercise.



When x = 0, y = 100.
Year > =0 and percent Carbon 14 in the range [0:100]
As the year increased to positive infinity, percent Carbon 14 close but not equal to 0
Neg exponential function y=100exp(-x).

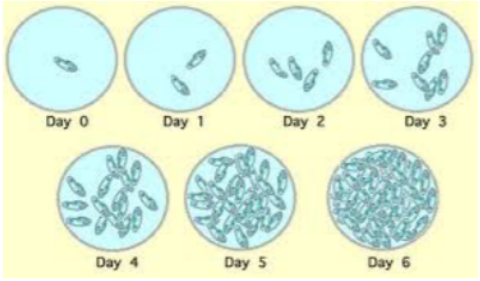


The principle is 1000 which is independent of the number of years.
y=1000
The interest no compounding is a proportional function + principle
y=100x+principle
Compound interest is a quadratic function + principle
y=142,5x^2+principle

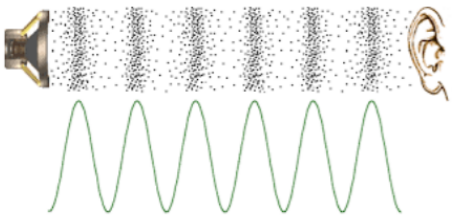


The relationship is linked to a systematic function, the different ranges of x values are linked to different linear functions.

The height of the sun is linked to a sine function which depends on the time in a day h(t)=sin(2(pi)ft+w).



The relationship between the number of the cells and days is linked to the function: the number of the cells = 2^day.



The strength of the sound is related to a sine function depending on the distance between ear and trumpet.

The height of the water is linked to a quadratic function depending on the distance of how far the water can reach.

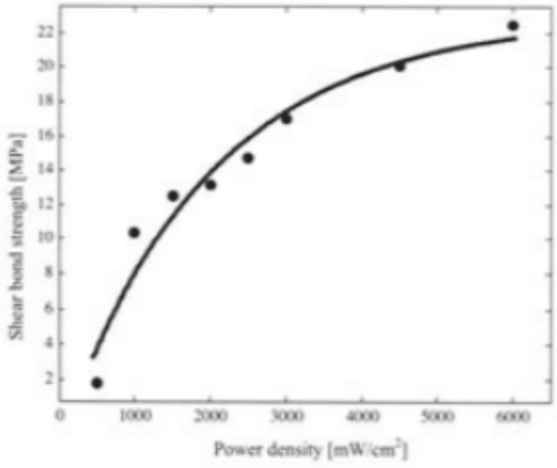
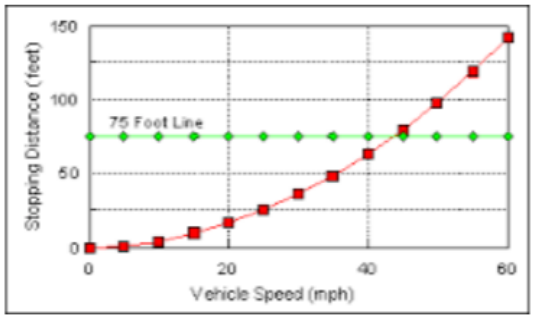


Fig. 1 – Shear bond strength as a function of power density. An exponential model (see Table 4) describes the relationship between shear bond strength and power density. The best-fit curve based on this model shows that shear bond strength enters a region of saturation at a power density of approximately 4000 mW/cm².

The relationships between shear bend strength and power density is linked to a exponential model.

The relationship between the height of the bridge and the length of the bridge is linked to a quadratic function.



The relationship between vehicle speed and stopping distance is linked to a quadratic function.

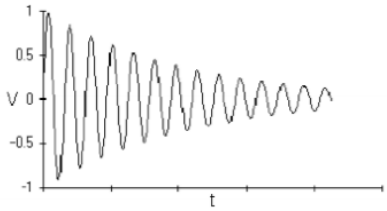
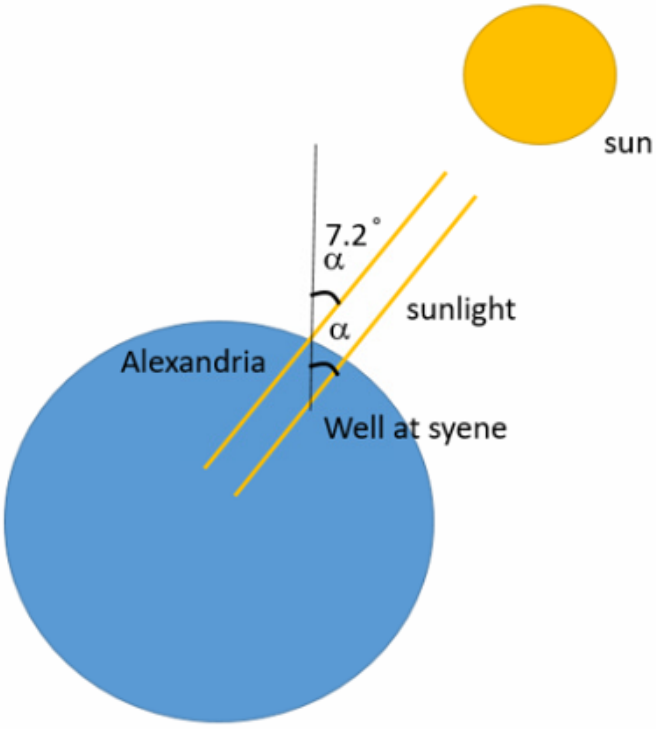


Figure 5
The voltage signal induced in the receiver coil versus time.

The relationship between the voltage signal and the time is related to a sine function*Neg exponential function.

(SIZE OF THE EARTH)
Almost 2300 years ago, Erathostenes estimated the size of the Earth in the following way. He had heard that on the day of summer solstice the sun lit the bottom of the wells in the city of Syene, 800 km south of his home city Alexandria. On the same day, Erathostenes measured the angle to the sun to be 7 degrees and 12 minutes (a minute is a 1/60 of a degree). Figure out and explain his calculation.



1. The sunlights are paralleled.
2. The angle between well at Syene and Alexandria is 7.2 degree
3. The angle 7.2 degree in a circle 360 degree is 7.2/360 = 0.02
4. The distance between Alexandria and Syene is 800km
5. Assume Earth is a perfect round, the distance of 800km between Alexandria and Syene is take up 0.02 in a circle of the total size of the Earth
6. The total size of the earth is 800/0.02=40 000.

(design)
(RENEWABLE ELECTRIC ENERGY SYSTEM)
Suggest a cost-aware design for a simple renewable electricity system for a small holiday cottage in the countryside, not connected to the electric grid. Motivate your design. Briefly comment also on any important practical considerations. Some information:
• Smaller solar panels cost about 2000 SEK per 100W, and a solar charge regulator about 1000 SEK.
• Solar panel yield in Sweden.
• Wind generators (they also provide some energy yield estimates for each product)
• Batteries cost about 2000 SEK for a good 12V 100Ah battery. You should normally use only half of the capacity or the battery will more quickly degrade over time.
• A good 12V refrigerator will need about 1A.
• Obviously don't search for ready-made cottage packages for sale, the task is for you to suggest the design. However if you wish to look up other data that you find relevant you may do so. Make sure that you end with an unambiguous instruction for how to dimension the system, according to your thinking.

HINT even if you feel that you lack some facts, you can either try to work without assuming anything (more complicated), or simply make some assumptions for example about how much energy you need, about the weather etc. (easier), and continue with the design. Details and numbers can always be changed afterwards, and continuing with the analysis can still provide a lot of insight.

1.The solar cells
Electricity production from solar cells is entirely determined by the angle of incidence of the solar rays towards the solar cells. The electricity to be obtained also depends on which month in a year. Assume the holiday cottage will be used during May to September. The cost by using solar panels = (2000/100)w+1000 and can generate 162Ah and 100W solar panels

2. Wind generators
Windcharger 12V Rutland 504)costs 4600SEK and can generate 100Ah

3. Batteries
2000 SEK for 50Ah

Assume 12V:
The solar cell costs ((2000/100)*100+1000)/162 =18.5kr/Ah
The wind generator costs 4600/100=46kr/Ah
Battery costs 2000/50 = 40kr/Ah

The solar cell will be a better choice for a holiday cottage (opened in May-September) in the countryside, it will costs the lowest compared to the other electricity generators.

(DIFFERENT LEVELS OF UNDERSTANDING)
Are there any differences in how deeply you understand the following statements. We are asking about your personal understanding! For example, do you understand what the statement says or not?
Do you also understand why the statement is true?
• Two parallel lines never meet
The statement is not true if two parallel lines stay in a ball, for example, two longitudes are parallel but they are crossed at Antarctic and Arctic
• The order of multiplication doesn't matter.
The statement is true when the multiplications are done among real numbers. But for example two matrix the order of multiplication does matter.

$$A = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \quad \text{and} \quad B = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

AB is not equal to BA.
• There are infinitely many prime numbers.
It is true.
• The average is

$$A = \frac{1}{n} \sum_{i=1}^{n} x_i$$

The average value of a set of numbers is equal to the sum of those values divided by the number of values.

• You can calculate W with the formula W=C*m^2 where m is the mass and C= 0.00236481

The statement seems missing units, I can't understand what W means. I didn't understand the function. I didn't find the physical meaning linked to this function.

W is the • The formula Summarize your thoughts.

$$\operatorname{div}\left(\operatorname{curl}\vec{F}\right) = \frac{\partial}{\partial x}(z) + \frac{\partial}{\partial y}(2yz) + \frac{\partial}{\partial z}(y - z^2) = 2z - 2z = 0$$

I could understand the mathematical derivation, but I could understand physical meaning.

Please write out any statements you refer to for easier marking.

The mean square derivation is

$$MSD = 1/n \sum_{i=1} (x_i - \overline{x})^2$$

Mean square deviation is calculated by the sum of the square of each value minus the average value divided by the number of values.The mean square deviation shows how much the distributions around the average value leading to indicating the convergence of a set of numbers. Higher the mean square derivation, lower the convergence is.

(MATHEMATICAL THINKING LECTURE) Did you find anything particularly relevant for you in the mathematical thinking lecture? (last Thursday - see the lecture notes).

In the mathematical thinking lecture, making the largest triangle in a circle is particularly relevant to me. It is useful when I want to cut out the cake with the largest triangle from a round cake in order not to waste so much.

(mathematical knowledge)
(EUCLID'S ELEMENTS)
Euclid's Elements, written about 2300 years ago, is one of the most successful textbooks of all time. Browse the book and give some brief impressions! What do you think the purpose of the book is?

The book presents a lot of definitions . Those definitions indicate a lots of relationships and rules in the nature science.

How is the mathematical knowledge presented?

The mathematical knowledge is not presented or delivered by mathematical derivations as usual but using a lot of geometries to show relationships between those angles and distances through descriptions.

What are the similarities and differences to a modern-day maths textbook?

In the modern-day maths, the textbook includes also a lot of mathematical definitions which are quite similar to Euclid's Elements. However, in the modern-day maths textbook, the mathematical derivations are presented using mathematical symbols and equations instead of descriptions. It become easier to read and understand the logical relationships behind.
(You can find the proof of Thales' theorem in Chapter 3, proposition 31. Pythagoras' theorem is in Book 1, proposition 47. The later parts with number theory are also interesting.) Web version of Euclid's elements Typeset version of Euclid's elements.

Reflection:

I. (SUPERVISION AND FOLLOW-UP LECTURE)

a) Did you have your checkpoint meeting for this module?

Yes. I had the checkpoint meeting for this module.

b) Did both of you attend the compulsory follow-up lecture? If you already talked to us about this, please explain.

Yes, I attend the compulsory follow-up lecture.

c) If you were asked to talk to a supervisor about the main submission, who did you talk to?

I talked to TA. Shaun McMurray

Reflect on your experiences from working with the module and try to make the most out of them. You are also encouraged to discuss your experiences with other groups.

If you reflect around individual problems (which is good), try to also draw general conclusions that may be helpful for you going forward in this course and long-term.

(Time spent in the reflection is time well spent, as it maximizes the learning from the significant effort you already made when working with the problems.)

As the answer, give some well motivated points summarizing your reflections.

I spent 15 hours and tired my best to solve those problems in module 2. I went to supervision three times in the week.

The first task in module 2 is quite straightforward to me. I could plot those points with year vs daily bicyclists in Excel and then fit the data by a linear function together with R^2 which gave an indication about how good the fittings. Then I could estimate the daily bicyclists for 2013 and 2020.

The second task is a bit hard for me. I get confused about the questions. For example, which information I could get for known positions. So I went to the Tuesday supervision. In the supervision, I knew that two coordinates for two known positions I could use to solve the problem. But I still get very confused about how to solve the problem. I tried to use cosine and sine functions to get the distance between the known positions and my position. But it seems not to work. After I discussed them with TA, I realize that I was so stuck in solving the questions. I should try something which happens in real life and should not solve the questions such as doing the mathematic homework. Finally, I realized that I could solve the position by two linear equations and cross point will be my current position.

The third task, I was a bit lost because I misunderstood the questions. I thought I should find out and summarize the characters for those functions. But actually, I should focus on the programming methodology. It is quite different from the normal function programming which focuses on the relationship between different arguments instead the function is solved using recursion function in which, the downstream point only depends on its upstream point.

The fourth task, I think I solve it quite good by not only the description but also the figures. But in the follow-up lecture, I was very surprised by other peoples' solvation. For example, the statistical method if the person can be extremely lucky. It was beyond my thought and very interesting to see from other peoples' solvation.

The fifth task, I can solve most relationships by using different functions, but the last one I was a bit stuck I find it is likely to be linked to a sine function but degraded. After I went to supervision, I get a hint about it could be solved not only on one function. Then I realized that it could be the negative exponential function multiply the sine function.

The sixth task, I can solve it but I miss some steps in the solution, so it is not quite to be understandable by TA which I should be more careful in the future work.

The seventh task, I was a bit hard for me in the beginning, because I was not familiar with solar panel and wind generator, but I tried to solve the problems as much as I can by looking up the information through the websites. It turns out a quite reasonable analysis and good solvation in the end.

The eighth task, after I think about the statements and discuss with TA, I realize there some statements I could understand and know it is true, but in others, I could not understand and not know why it is true. It is very important for me to realize that I might be dangerous to make a conclusion if I just know it is true but I don't know the proof. It might turn out wrong or unsystematic conclusions based on them.

The ninth task, by read the book and compare it to the modern book, I find out the similarity and differences. In the old mathematical book, it has a lot of proofs, theorems, definitions and axioms which is similar to the modern book, however, there is no formulas, no coordinate systems and pure math which is much different from the modern mathematical book.

III. (HOW WELL DID YOU SOLVE THE PROBLEMS?)

Give a single assessment for the whole module and motivate with a sentence or two. This is for your own practice.

Use the scale "insufficient/sufficient/good/very good", or a combination such as "between good and very good" or "good or very good". Use the grading criteria we have suggested, or clearly motivate your own.

(We as teachers will then set the grade for this module. We think it is better if you are able to make a fair assessment rather than an inflated one.)

For the whole module, I did it quite well by trying to solve those questions myself and discussing with TA and getting hints from them. I learned and progressed and realize the defect when solving the problems. I thus made a very good performance in this module.