5 Mostly dynamic systems

Group 1
Author: Min Wu(8603148365, DIT025)
Email:minwuh081@gmail.com
I hereby declare that all solutions are entirely my own work, without having taken part of other solutions.
The number of hours spent: 20hours (Min Wu)
The number of hours has been present in supervision for this module: 3h
(keeping track)
(investigating the abstract)
(ACHILLES AND THE TORTOISE)
- Have you heard about Achilles and the tortoise?
- No, what of it?
- Well, it is the hero Achilles and the tortoise.
The tortoise says that he can win a race against Achilles if he just gets a head start.
- Why, that sounds crazy, the tortoise must be much slower.
- Yes, but listen. When the race starts Achilles is say 15 metres behind the tortoise. And then the race begins. - Ok?
- So then they both begin to run, Achilles will quickly run these 15 meters to the place where the tortoise started, but then the tortoise has also moved ahead say 1 metre.
- Yes,… - So the tortoise is still ahead. And when Achilles has run another metre to where the tortoise was before, the tortoise has moved yet some distance forward. And this repeats itself, since whenever Achilles runs to the point where the tortoise was before, the tortoise will have moved ahead yet again. So Achilles will never be able to overtake the tortoise!
- But this is nonsense. Of course Achilles will win
- he runs faster!
- So it may seem, but what about the argument?
- I don't care much about arguments, just use your common sense and you will be fine. Forget arguments, they are not to be trusted. It's practice that counts, every sensible person knows that.
- But I find it interesting. I want to think about it. There must be something wrong with the argument, but I haven't yet figured out what it is. I think it can be explained and that it will all agree with common sense, this is what I am using when I am thinking. And I am not going to give up until I really understand…
- Well, good luck, I'll go and watch a movie.

Can you sort this out? Is it possible to discover any interesting mathematics from this? Investigate and explain as well as you can!

Achilles runs faster than tortoise resulting in Achilles will probably win the race if the distance of the race is long enough. Assume after t minuter, Achilles will meet tortoise means the distance of Achilles movement = tortoise movement + 15 meters.

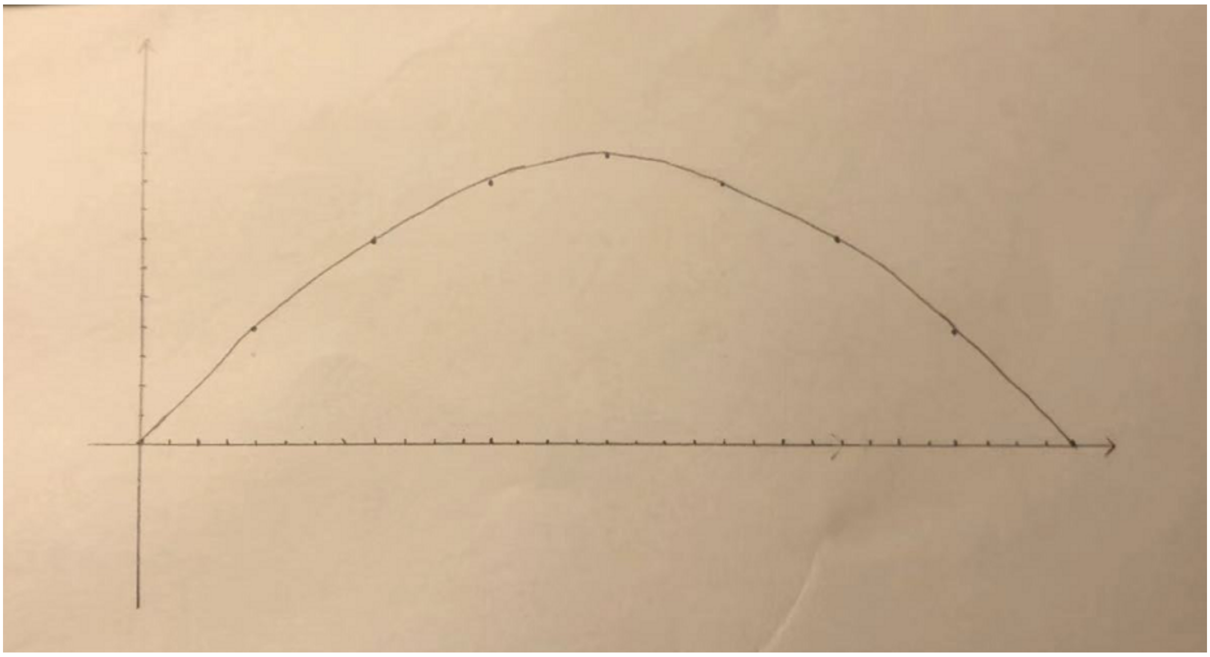
- Achilles runs faster means the speed of Achilles' movement is larger than the speed of tortoise's movement assume the speed of Achilles' movement is v1 and the speed of tortoise's movement is v2.
- When Achilles meets tortoise, the distance of Achilles' movement is 15 meters larger than tortoise because at the starting point Achilles is 15 meters behind the tortoise.
- The time t for Achilles movement equals to tortoise's movement. After t minuter, Achilles meets tortoise.

Mathematical equation:

$$v1*t=v2*t+15$$

(investigating the world)
(THROW BALL)
Do the following very simple theoretical experiment. Draw the trajectory of a thrown ball in the following way
. Start in (0,0) and draw a dot there. Now work in steps:
• in the next step move 4 steps to the right, and 4 steps up. Draw a dot.
• then move 4 steps to the right and 3 steps up. (This is because the gravity of the earth changes the vertical speed, but not the horizontal speed. So decrease the vertical speed by one in every step!)
• continue with the same pattern and see what happens!

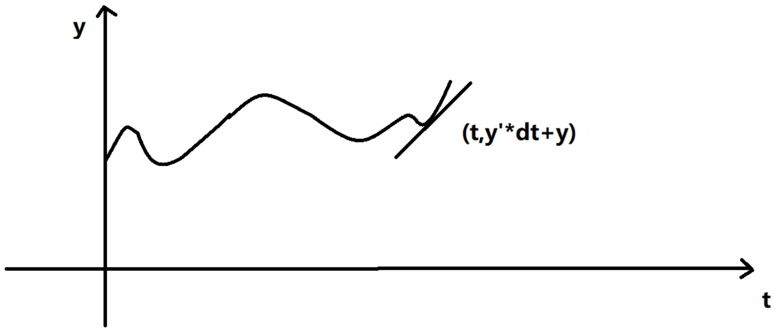
The trajectory of a thrown ball is in the following figure according to the steps above.



(WHALES AND KRILL)
Here we will investigate a simple model for the population dynamics of whales and krill, where krill is assumed to be the main source of food for the whales. We consider the following model, described by the differential equations
k’ = (a-bw) k
w’ = (-m+nk) w
where k is the krill population (as a function of the time in years), w is the whale population, and a, b, m and n are positive constants. The two equations describe the rate of change k’ for the krill, and the rate of change w’ for the whales - also called the derivatives of k and w, with respect to the time. Together, the two equations determine the dynamic behaviour of this system, for any given initial values for k and w.

a) Investigate the program krill.py for simulating this system. Make sure you fully understand the program, and how the differential equations written above are used in the simulation. Study the behaviour of the system, and try to explain what you see and why. It is a good idea to enter different initial values to see what happens. There Is also a second so called parametric plot that you can activate and look at.

The relationships of populations change rates between krill and whale:
krill’= (0.2-0.0001*whale)krill
whale’=(-0.5+0.000001*krill)*whale



krill(t+h) = krill(t) + krill’(t)*h
whale(t+h) = whale(t) +whale’(t)*h

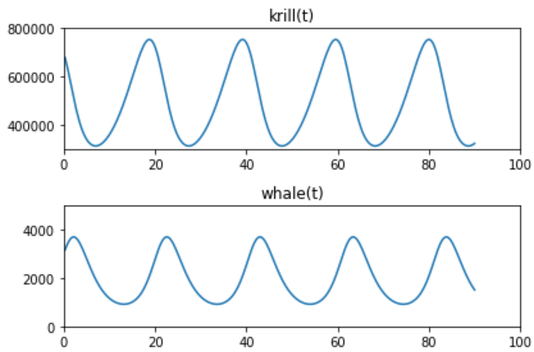
where h is the time increased.
The current populations of krill depend on the previous populations of krill together with the current populations of whale.

- The next whale depends on the current populations of whale and the current populations of krill.
- The current rates of populations change depend on the previous points of krill and whales’ populations.
- The initial points of whales, krill and time are 3 000, 700 000 and 0.
- The numbers of whales vs the number of years show a periodic relationship same as the number of years vs krill.

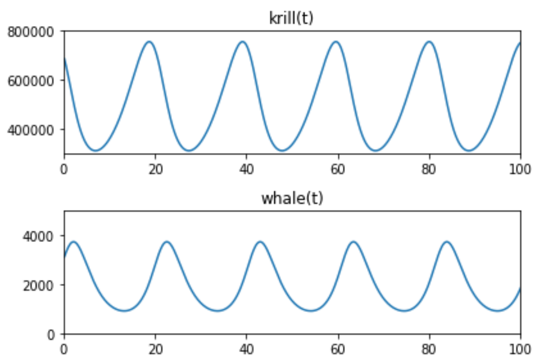
Test different initial points for h, the populations of whale and the populations of krill

Different h:

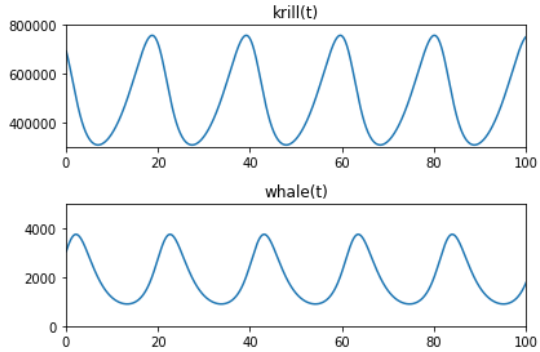
h=0.3



h=0.2



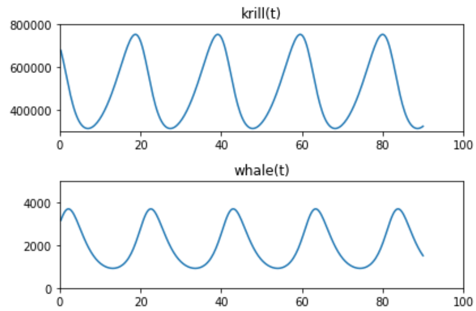
h=0.1



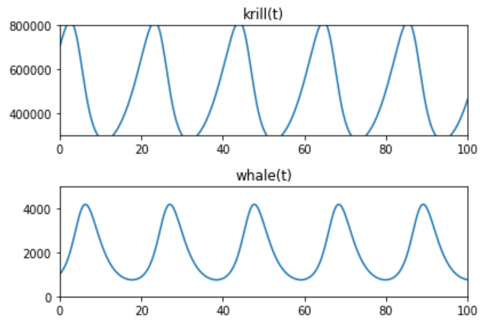
Conclusion: the plots are independent on h

Different initial populations values of whale and krill

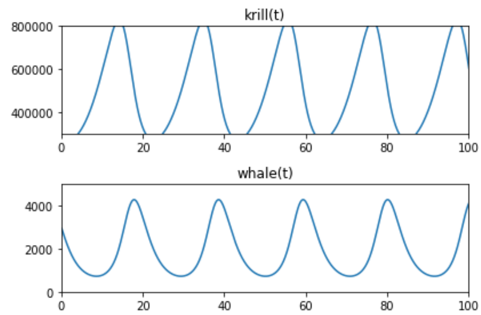
krill 700000 and whale 3000



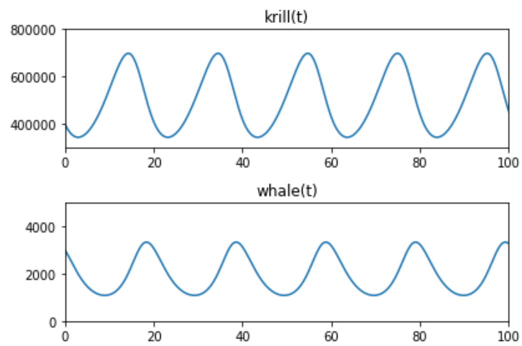
krill 700000 and whale 1000



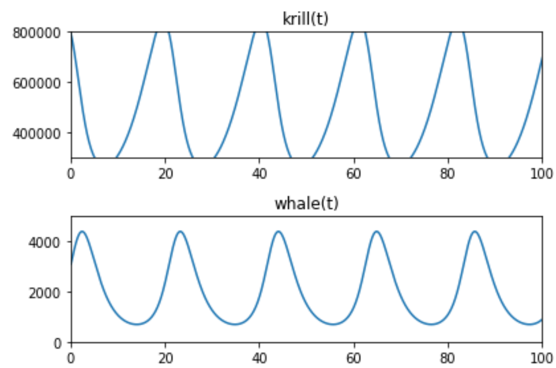
krill 700000 and whale 4000



krill 400000 and whale 3000



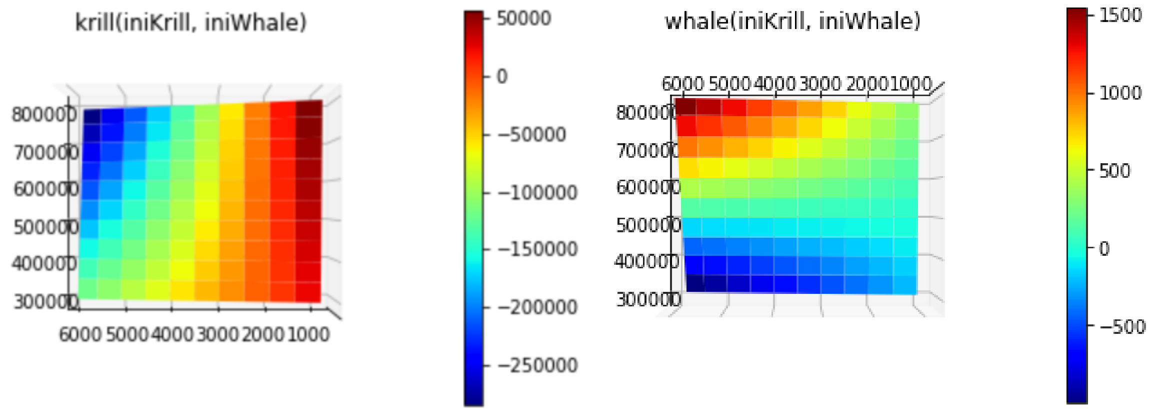
krill 800000 and whale 3000



Conclusion: the initial values of whale and krill determine the change rates of the populations of whale and krill.

Parametric plot

Different initial points of whale (x-axis) and krill (y-axis) populations vs the change rates of krill populations (left panel) and the change rates of whale populations (right panel).



Relatively large change rates of populations obtained for whale and krill, when:

- Relatively large populations of krill together with relatively large populations of whale
- Relatively small populations of whales together with relatively large populations of krill
- Relatively small populations of krills together with relatively large populations of whale

b) Can you find any equilibrium points for this system with the help of the simulation and some thinking?

krill'= (0.2-0.0001*whale)krill

whale'=(-0.5+0.000001*krill)*whale

The equilibrium points are when krill' and whale' equal to 0

So the equilibrium points, when:

1. 0.2-0.0001*whale=0
2. krill=0
3. -0.5+0.000001*krill=0
4. whale=0

the populations of whale: 0 or 2000 and the populations of krill: 0 or 500 000.

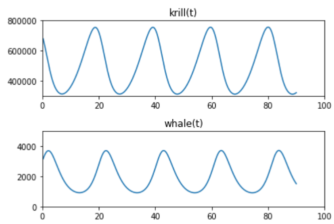
d) Investigate the effect of krill fishing on these populations.To model this, we can add a term -rk to the equation for k', where r<a. Try out different values of r, simulate and discuss your observations.

The krill fishing effect added, so the krill’ become:

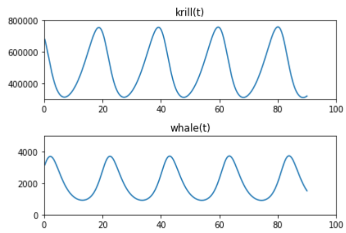
krill’= (0.2-0.0001*whale)krill-rk

Plots with different r:

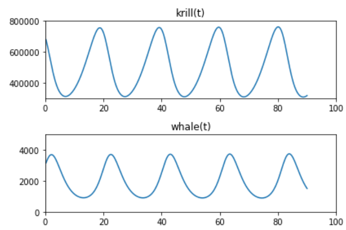
r=0.05



r=0.1

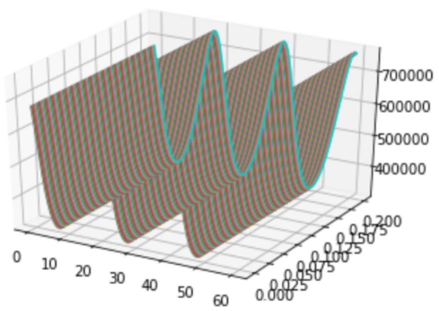


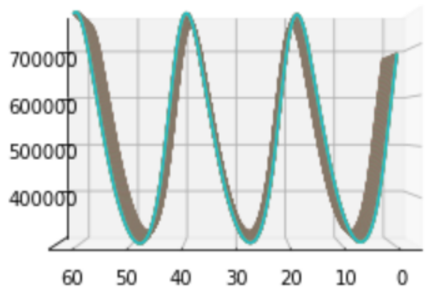
r=0.15



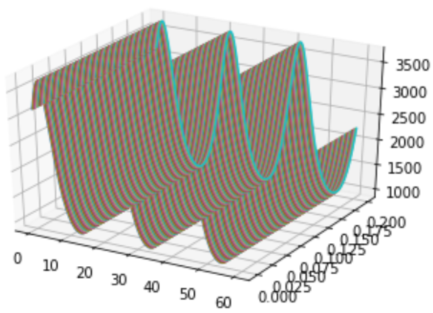
Parametric plot:

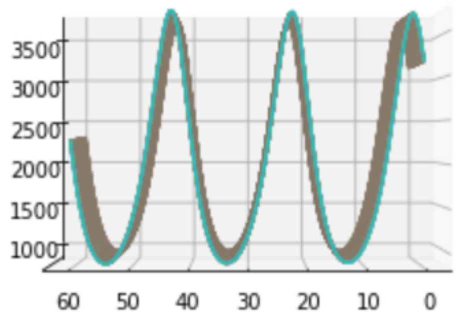
The 3d plot (time(x), r(y) and the populations of krill(z)) is shown in the figure below.





The 3d plot of time(x)vs r(t) vs the populations of whale(z) is shown in the figure below:





Conclusion: the plots are indicating that the krill fishing effect has no influence in the populations of whale and krill.

(BOUNCING BALLS PROGRAM)

This Java program simulates bouncing balls in a box. First run the program. Then read the program and try to understand it. Explain the mathematics involved. The entire model is in the method step in Model.java so focus your efforts there.

Ball object :

Constructor:

It is to define the property of each ball, it contains:

- The size of the ball (radius).
- The velocity of the ball with a 2d vector (x-direction and y-direction)
- The mass of the ball.
- The position of the ball with a 2d vector (x-direction and y-direction)

The other attributes:

- A force with a 2d vector (x-direction and y-direction)
- Acceleration with a 2d vector (x-direction and y-direction)

Move object:

Constructor:

- define the move area's width and height.
- initialize a "balls" list containing three different balls(different size, different velocity, and mass)

Function:

Step:

input: time derivative (delta T)

three for loops:

1. The first for loop:

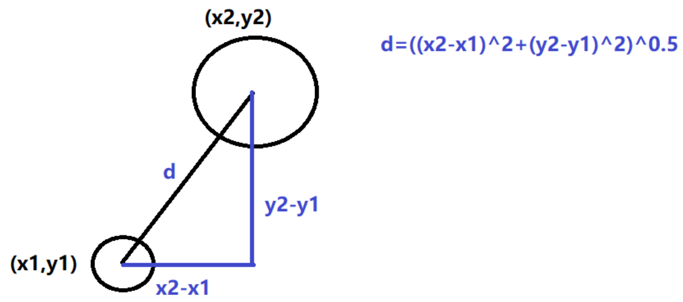
is to define the force on each ball if the position of the ball touches the walls. The initial force is zero:

- the radius > the positions to the right and left walls, then the x-axis force become 3000*(radius - distances to the right and left wall)
- the radius > the position to up-side wall, then the y-axis force become 3000*(radius - distances to the up-side wall)
- the radius > the position to the down wall, then the y-axis force will also depend on the direction of the velocity

The final y-axis force also takes gravity into account.

2. The second for loop**:**

is to define the force if the balls touch each other.



The distance between two balls equals to d using the Pythagorean theorem.

if the sum radius of each two balls is larger than the distance d, it indicates the two balls collision. The force equals 3000* (radius of ball1+radius of ball2 - d).

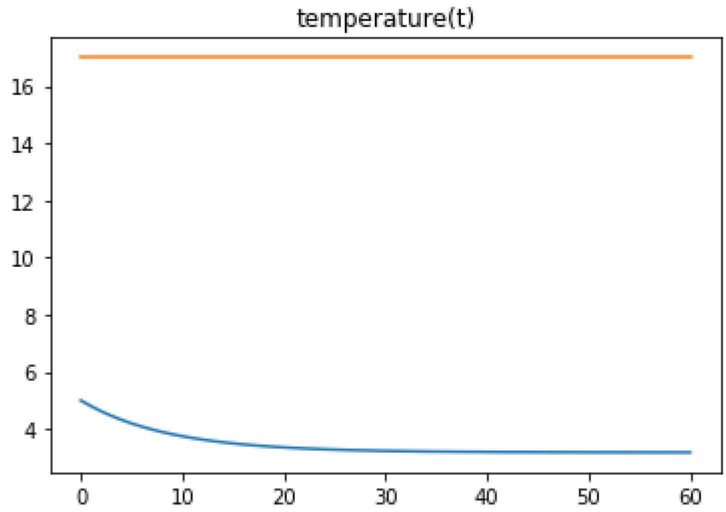
3. **The third for loop**

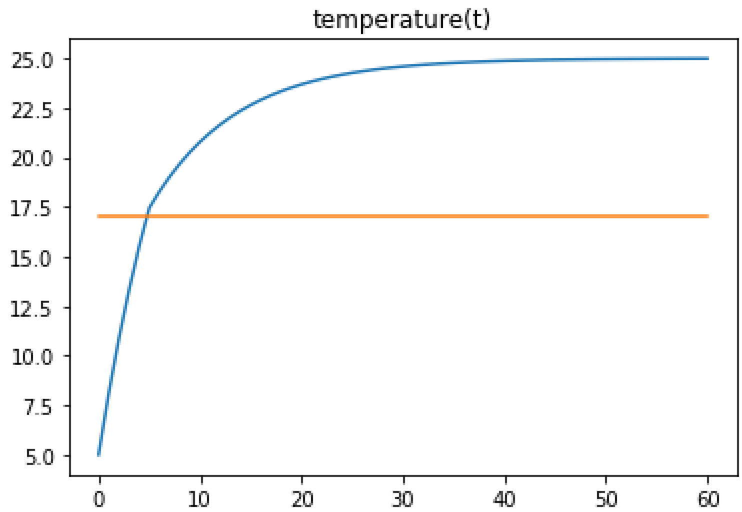
is to translate force into acceleration using Newtons law F=ma. and then compute new velocity and position from the current acceleration and velocity of the ball.

(TEMPERATURE CONTROL SYSTEM)

The program control.py simulates a simple model for the temperature in a room. Initially the model implements a 1000W heater fan with simple thermostat regulation. Due to particular storage requirements for a special kind of paint, it is desired to have an excellent temperature regulation at 17 degrees Celsius. If possible, this should work in the range of -5 to +25 degrees of outside temperature.

a) Try to understand the program, run it and make any observations.

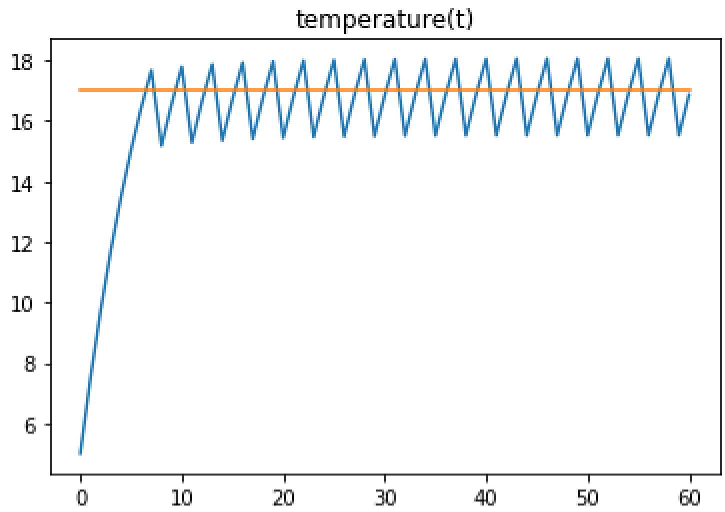




- The red line is constant which is desire room temperature (17 degrees).
- The blue line is the actual temperature in the room. As time increased, the room temperature is decreased and reach a stable state.
- It indicates that the actual temperature can not reach the desired room temperature when the room temperature is 5 and the outside temperature is -5 even the fan keeps running all the time. (up panel)
- It indicates that the actual temperature can not reduce the desired room temperature when the room temperature is 5 and the outside temperature is 25 even the fan stop running all the time. (down panel)

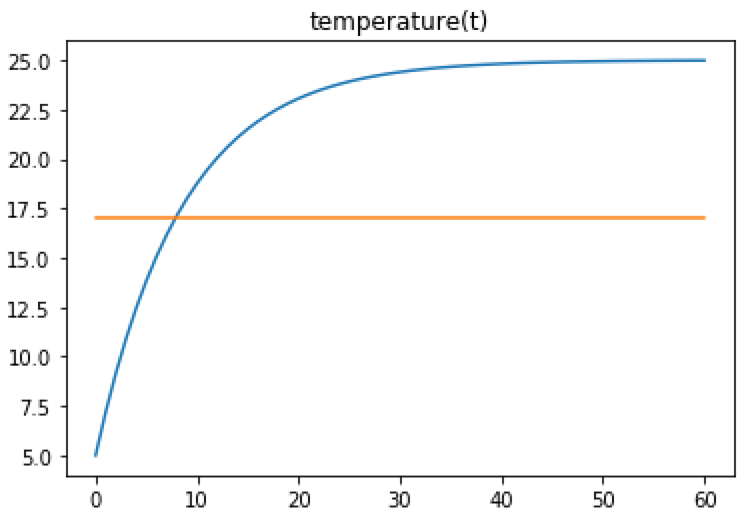
b) Improve the design of the regulator to meet the objectives as well as possible. The possibilities you have in your design is to dimension the max power of the heating fan, and to implement the regulator, which is allowed to set the power of the fan to any value once every minute.

Change the initial point of maxpower
try maxpower=4000, and the room temperature is 5 together with the outside temperature is -5 degree.



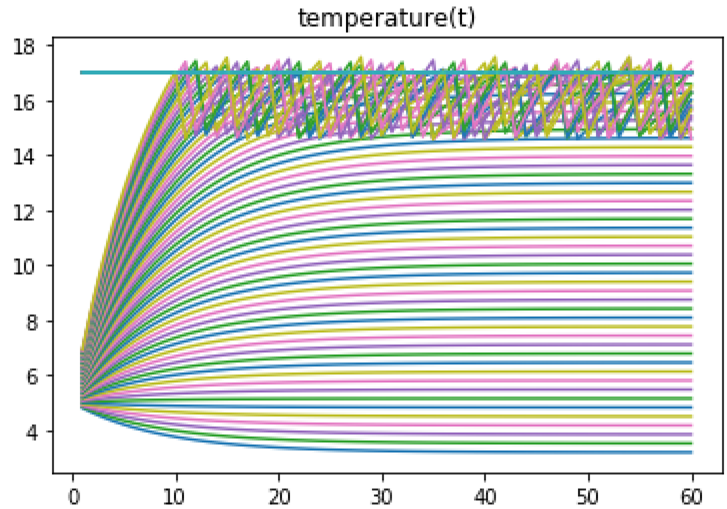
The required temperature is reached when time around 8 minutes and then the actual temperature is up and down around the required temperature.

try maxpower is 0 and the room temperature is 5 together with the outside temperature is 25 degrees.

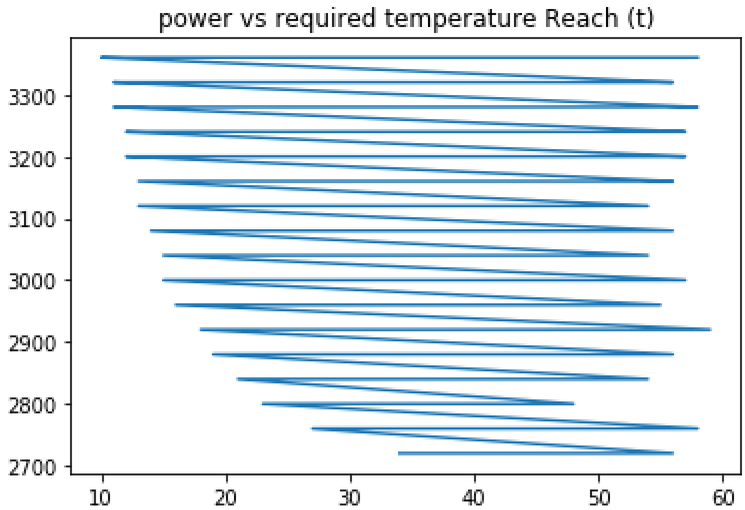


It is not possible to keep the room temperature to the required temperature if the outside temperature is 25 even the heating machine doesn't work at all. After around 12 minutes, the room temperature is larger than the required temperature.

Parameteric plots

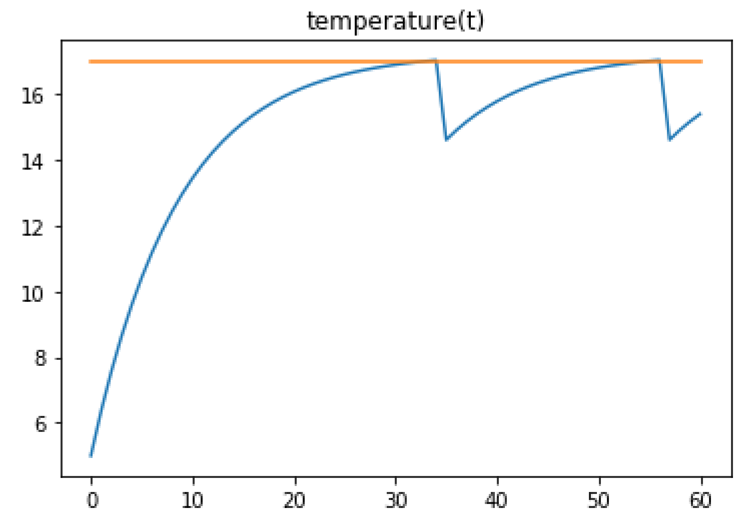


The time vs actual temperature with different initial points of maxpower (different color) when the outside temperature is -5 degree and the room temperature is 5 degrees.

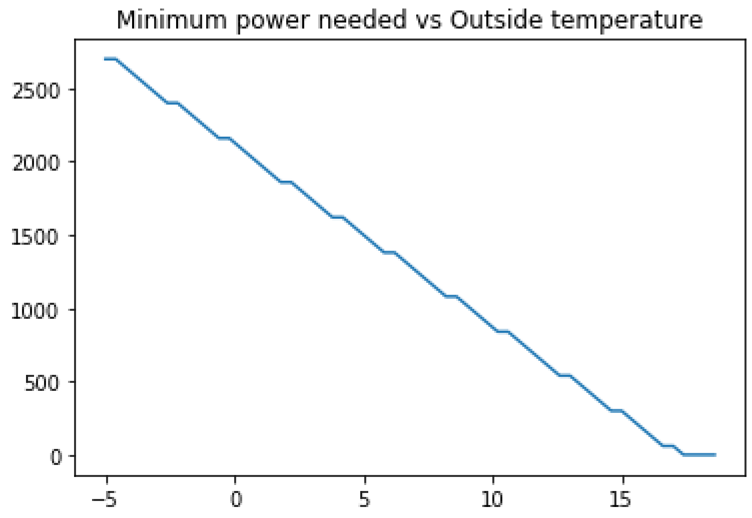


The maxpower vs the time reaches the required temperature.

The larger maxpower, the quicker the required temperature reach. The minimum of the maxpower needed to reach the required temperature is about 2720, see the following.

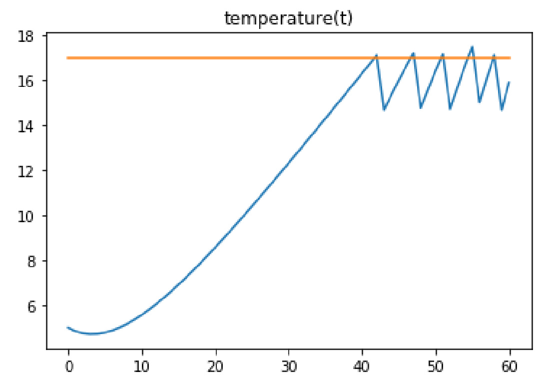


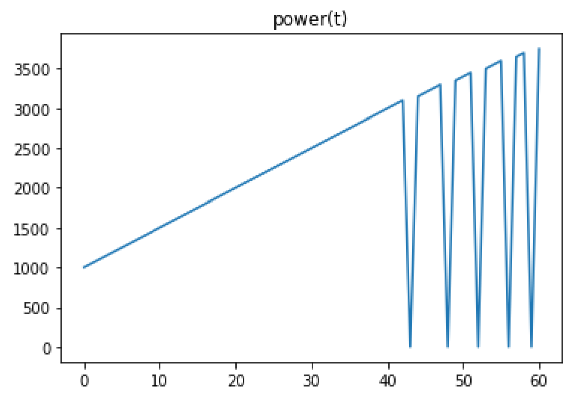
The outside temperature vs minimum maxpower needed to reach the required temperature if the room temperature is 5 is shown in the figure below.



For the regulator, it can change the power in a minute.
The rate to change the power is initially 5 per minute.
so:
power=power + 50*1minute

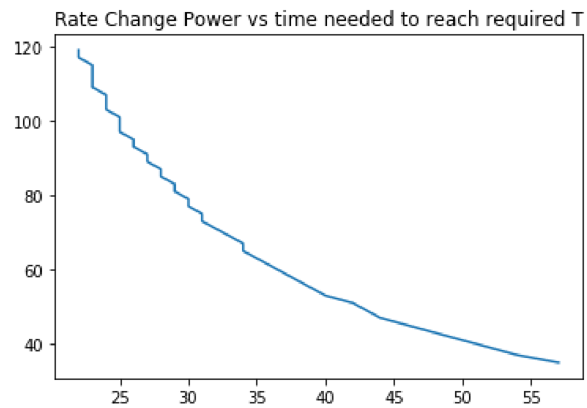
The actual temperature vs time (left panel) is presented in the figures below and the power vs time (right panel)



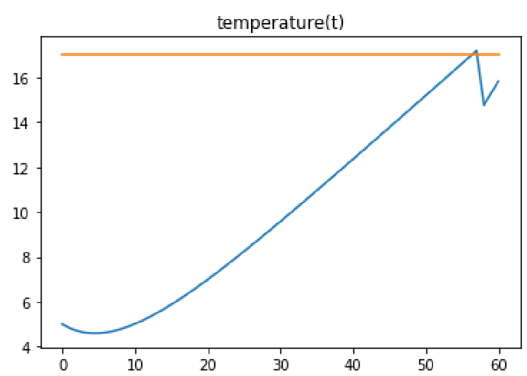


Parametric plot

To test different rates:



The different rates vs the minimal time needed to reach the required temperature are plotted.The minimal change rate of power is 35 per minute and 57 minutes are needed to increase the room temperature to the required temperature, seeing the figure below.



(MATHEMATICAL MODELLING OF DYNAMIC SYSTEMS)

Now when you you see the similarity between the program in the previous module for generating some well-known mathematical functions using the rate-of-change/derivative, and these different simulations of real-world systems, what are your thoughts?

That mathematical modeling of those dynamic systems is quite similar in these different simulations of real-word.

(SOCRATIC QUESTIONING)
In this course an important method of supervision is to ask questions, and often you are then able to answer these questions yourselves. Now temporarily place yourself in the role of the supervisor, and suggest one or a few good supervision questions in each of the following student situations:

a) We have looked at this table for a while, but we haven't learned any method to find a curve for such tables (the problem with the planets, can be solved without knowing about the least square method).

- Can you plot the data according to the values in the table?
- Begin with some simple relationship for example (linear) to see how far the curve can match the values in the table.

b) We tried with the log function, and think it is about right but maybe not perfect. Is that the answer? (planets again)
How to determine if the function is not right or perfect, how large the errors?
d) We don't know how to solve this problem, and are stuck. What should we do? (any problem)
What is the problem?
Why I can not solve the problem (some factor needed, or some other knowledge needed)
c) We have written a constraint here - is it correct? (emergency care problem)
Test the results according to the real situation.
e) We just want to check if we are on the right track… (any problem)
Go back again to problem and think about what the actual problem is.
f) Why do you think I am asking you to suggest these questions? (this is not an imagined student question but a question directly to you!)
Maybe you want tome can supervise myself to find a way to solve the problems in the future.