Written Assignment 2 – Official Solution

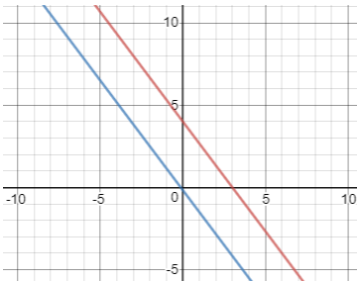
1. Determine whether the lines given by the equations below are parallel, perpendicular, or neither.

$a.\begin{cases} 3y + 4x = 12 \\ -6y = 8x + 1 \end{cases}$ $b.\begin{cases} 3y + x = 12 \\ -y = 8x + 1 \end{cases}$ $c.\begin{cases} 4x - 7y = 10 \\ 7x + 4y = 1 \end{cases}$

[Suggestion: go to www.desmos.com/calculator, write the two equations and try to conclude the answer. Then find a rigorous algebraic solution.]

SOLUTION

a. Screenshot of a graph obtained in www.desmos.com/calculator.



For an algebraically rigorous solution, we must first organize the system as follows:

$$\begin{cases} 3y + 4x = 12 \\ -6y - 8x = 1 \end{cases}$$

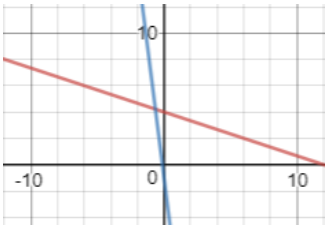
Multiplying the first equation by – 2 yields:

$$\begin{cases} -6y - 8x = -24 \\ -6y - 8x = 1 \end{cases}$$

We easily conclude that a point (x, y) belonging to the first straight line does not belong to the second straight line, and therefore the straight lines are parallel, since the straight lines are in the same plane.

Another solution: the slope of the first straight line is $\frac{8}{-6} = -\frac{4}{3}$ and the slope of the second straight line is the same

b. Screenshot of a graph obtained in www.desmos.com/calculator.



For an algebraically rigorous solution, we must first organize the system as follows:

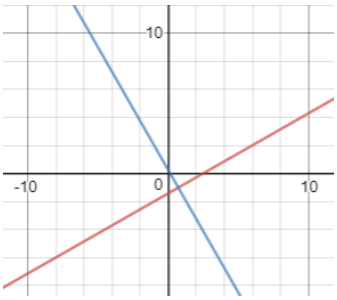
$$\begin{cases} 3y + x = 12 \\ -y - 8x = 1 \end{cases}$$

Multiplying the second equation by 3 gives us:

$$\begin{cases} 3y + x = 12 \\ -3y - 24x = 3 \end{cases}$$

We easily conclude that the system has a single solution given by point $x = -\frac{15}{23}$ and $3y = 12 - x \rightarrow y = 4 - \frac{x}{3} = 4 - \frac{-\frac{15}{23}}{3} = 4 + \frac{15}{69} = \frac{4\times69+15}{69} = \frac{291}{69}$ and therefore the lines are concurrent, that is, they have one point in common, not perpendicular.
Another solution: the slope of the first line is $\frac{-1}{3}$ and the slope of the second line is -8, that is, are not equal and one is not the opposite of the other.

c. Screenshot of a graph obtained in www.desmos.com/calculator.



For an algebraically rigorous solution, we first notice that the system is already organized

$$\begin{cases} 4x - 7y = 10 \\ 7x + 4y = 1 \end{cases}$$

and the slopes are $\frac{4}{7}$ e $\frac{-7}{4}$, that is, one is the opposite of the other. In other words, $\frac{4}{7} \times \frac{-7}{4} = -1$ which means that the straight lines are perpendicular.

2. A ball is thrown in the air from the top of a building. Its height, in meters above ground, as a function of time, in seconds, is given by

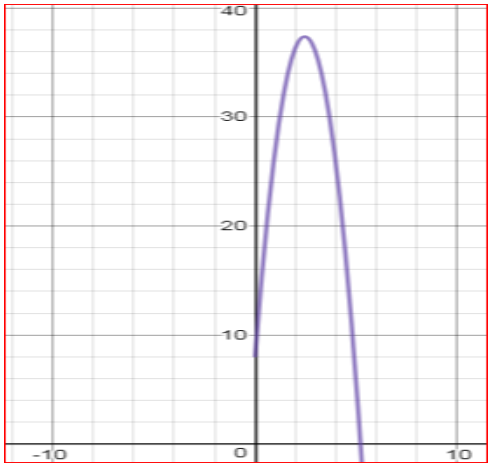
$$h(t) = -4.9t^2 + 24t + 8$$

What is the height of the building? What is the maximum height reached by the ball? How long does it take to reach maximum height?
[Suggestion: for example, go to www.desmos.com/calculator and write

$$y = -4.9x^2 + 24x + 8$$

and observe what is the maximum height. Then find a rigorous algebraic solution.]

SOLUTION
Screenshot of a graph obtained in www.desmos.com/calculator typing
$y = -4.9x^2 + 24x + 8 \quad \{0 < x < 5.3\}$



The height of the building is 8 meters:

$h(0) = -4.9 \times 0^2 \ + \ 24 \times 0 \ + \ 8 = 8$

The ball reaches the maximum height at the horizontal coordinate of the vertex

$$x = \frac{-b}{2a} = \frac{-24}{2(-4.9)} = \frac{-24}{-9.8} = 2.45$$

seconds.

The maximum height is reached at the vertical coordinate of the vertex of the parabola, that is:

$$h\left(\frac{-\mathrm{b}}{2\mathrm{a}}\right) = -4.9 \times 2.45^2 \ + \ 58{,}80 \ + \ 8 = -4.9 \times 6{,}00 \ + \ 58{,}80 \ + \ 8 = 37.40$$

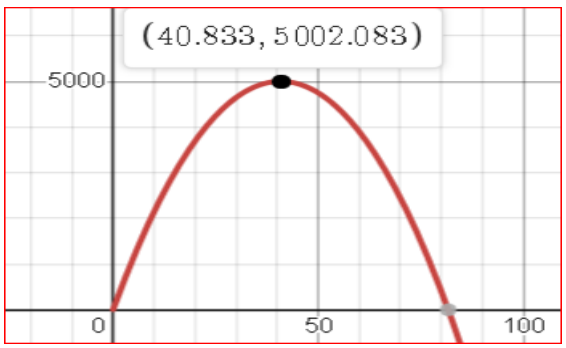
3. A farmer finds that if she plants 75 trees per acre, each tree will yield 20 bushels of fruit. She estimates that for each additional tree planted per acre, the yield of each tree will decrease by 3 bushels. How many trees should she plant per acre to maximize her harvest?

SOLUTION

Our function "bushels" is described by

$B(n) = n\big(20 - 3(n - 75)\big) = n(-3n + 245)$

if *n* is the number of trees per acre.

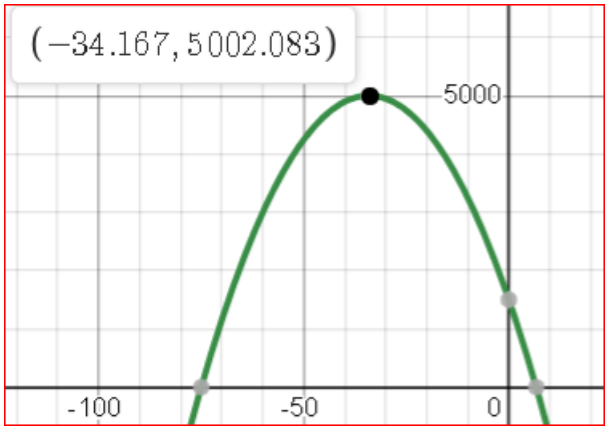


The graph show that she should plant 41 trees per acre to maximize her harvest.

Or

$$B(n) = (20 - 3n) \times (75 + n)A$$

where *n* is the number of additional trees she plants per acre, and *A* is the quantity of acres.
We take *A* = 1 for the number of acres.
Screenshot of a graph obtained in www.desmos.com/calculator for



We see that to maximize her harvest she should plant less than 75 trees per acre.
Observing the graph we see that the maximum occurs around *n* = – 34.

Let´s make a table:

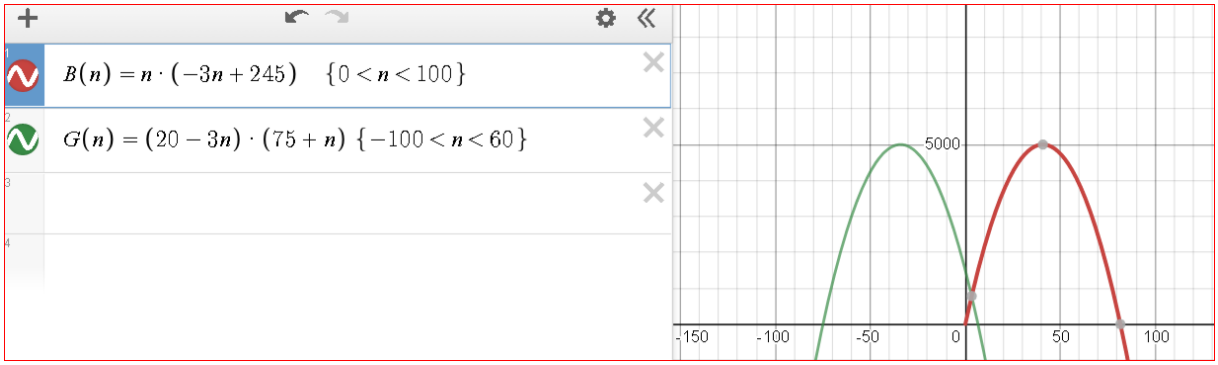
n	20 – 3*n*	75 + *n*	*B*
-40	140	35	4900
-35	125	40	5000
-30	110	45	4950

For an algebraically rigorous solution, we calculate the midpoint of the two roots:

$$\frac{-75 + \frac{20}{3}}{2} = \frac{\frac{-225 + 20}{3}}{2} = \frac{-205}{6} = -34.17$$

Therefore, she should plant $75 - 34 = 41$ trees per acre to obtain the maximum of 5002 bushels.

n	20 – 3*n*	75 + *n*	*B*
-34	122	41	5002
-35	125	40	5000


1
$B(n) = n \cdot (-3n + 245) \quad \{0 < n < 100\}$
2
$G(n) = (20 - 3n) \cdot (75 + n) \ \{-100 < n < 60\}$
3
4
5000
-150
-100
-50
0
50
100