Written Assignment 1 – Official Solution

1. Find the domain of the function using interval notation.

$$f(x) = \frac{\sqrt{x-6}}{\sqrt{x-4}}$$

SOLUTION

For the formula to make sense, we need to enforce that $x - 6 \geq 0$ and $x - 4 > 0$. It follows that $x \geq 6$ and $x > 4$. Note that in the denominator we cannot have $x = 4$. We conclude that the domain of the function $f(x)$ can be represented by:

$$Dom(f) = \{x \in \Re\colon\ x \geq 6\},$$

where $\Re$ indicates the set of the real numbers. Using interval notation we have:

$$Dom(f) = [6,\ \infty).$$

2. Sketch a graph of a piecewise function. Write the domain in interval notation.

[Suggestion: for example, go to www.desmos.com/calculator and write

$$y = x^2 \quad \{-1 < x < 1\}$$

and

$$y = 3x - 2 \quad \{1 < x < 3\}$$

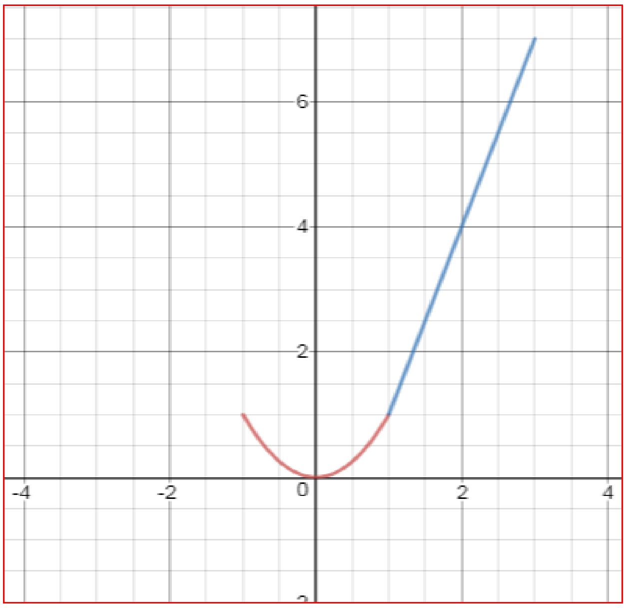
Then choose your own functions and have fun.

SOLUTION

A possible example could be:

$$f(x) = \begin{cases} x^2, & if\ -1 \leq x \leq 1, \\ 3x-2, & if\ 1 \leq x \leq 3. \end{cases}$$

Screenshot of a graph obtained in www.desmos.com/calculator.



3. The cost in dollars of making x items is given by the function $C(x) = 10x + 500$.

a. The fixed cost is determined when zero items are produced. Find the fixed cost for this item.

b. What is the cost of making 25 items?

c. Suppose the maximum cost allowed is $1,500. What are the domain and range of the cost function, $C(x)$?

SOLUTION

a. The fixed cost is simply $C(0) = 10\times 0 + 500 = 500$.

b. Just put 25 in the place of x: $C(25) = 10\times 25 + 500 = 250 + 500 = 750$.

c. $1500 = C(x) = 10x + 500$ implies that $10\,x = 1000$ and, therefore, $x = 100$. We conclude that the domain of the cost function is [0, 100] and the range is [500, 1500].