# PHYS 512 - Problem Set 4 Solutions

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## 1 Question 1

With the fit params originally in the code, $\chi^2 = 15267.937968222595$ for 2501 degrees of freedom which is not quite good. This can also be seen in the plot of the fit itself:

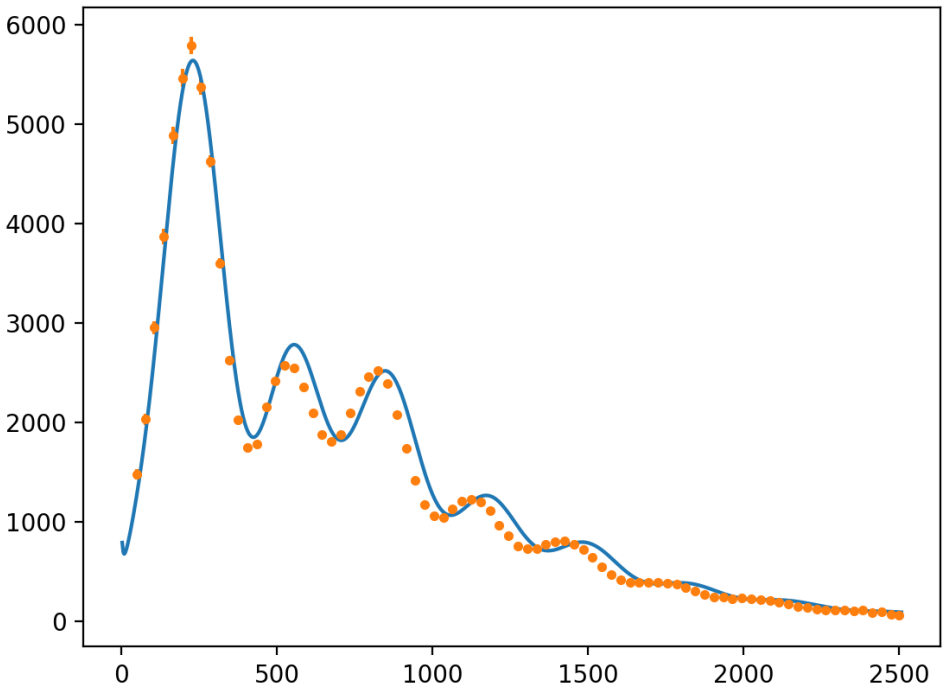


Figure 1: Fit using initial params.

With the new parameters given, the $\chi^2 = 3272.2033778089576$ for 2501 degrees of freedom. This is still quite terrible, but not as bad as before.

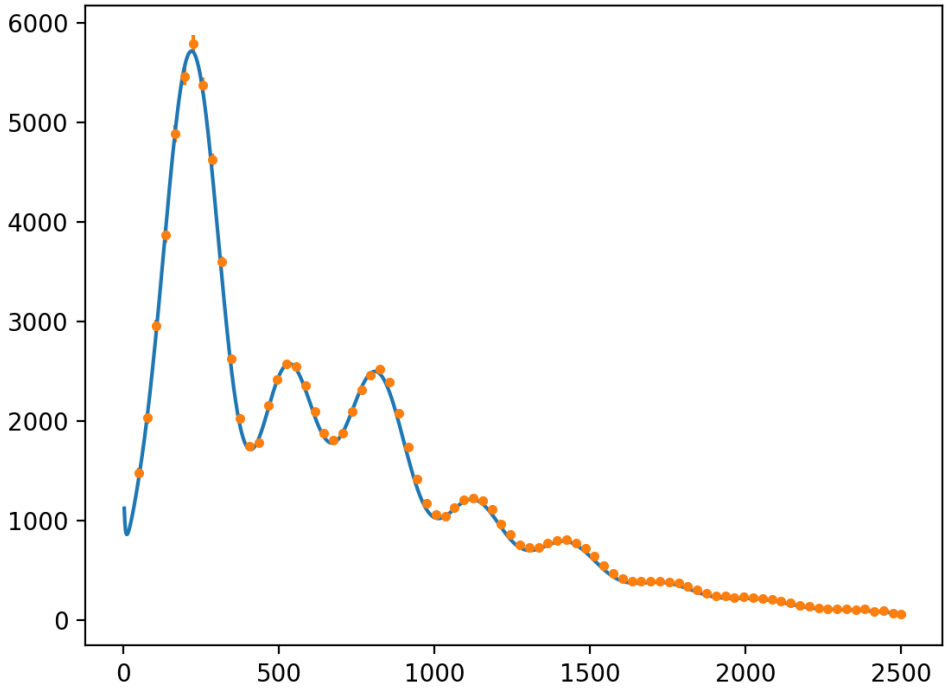


Figure 2: Fit using new params.

## 2 Question 2

Using fitter, we get the best fit parameters are:

| Parameter | Values |
|---|---|
| $H_0$ | 69.1705365 ± 1.27354302 |
| $\Omega_b h^2$ | 2.25387347e-02 ± 2.52671795e-04 |
| $\Omega_C h^2$ | 1.15855025e-01 ± 2.79812113e-03 |
| $\tau$ | 9.77886961e-02 ± 2.83323042e-02 |
| $A_s$ | 2.26553685e-09 ± 1.23078773e-10 |
| $n_s$ | 9.77073974e-01 ± 6.66860392e-03 |

Table 1: Fit parameters using LM method.

## 3 Question 3

Run mcmc we get following graphs for each parameter that shows the values converges and taking the average we get the following values

We observe that the params do indeed converge with the following plots. Note that for the first coloumn, we observe that at later times the data resembles white noise.

| Parameter | Value |
|---|---|
| $H_0$ | 68.4513939992969 ± 1.6904967965474136 |
| $\Omega_b h^2$ | 0.022382400740447225 ± 0.0002315449158901239 2 |
| $\Omega_C h^2$ | 0.11730356764479602 ± 0.0036462682055554887 |
| $\tau$ | 0.08046363574700982 ± 0.03209770920008682 |
| $A_s$ | 2.1991199629160267e-09 ± 1.3258536145474286e-10 |
| $n_s$ | 0.9737398016898084 ± 0.007889759555378947 |

Table 2: Mean fit parameters using mcmc.

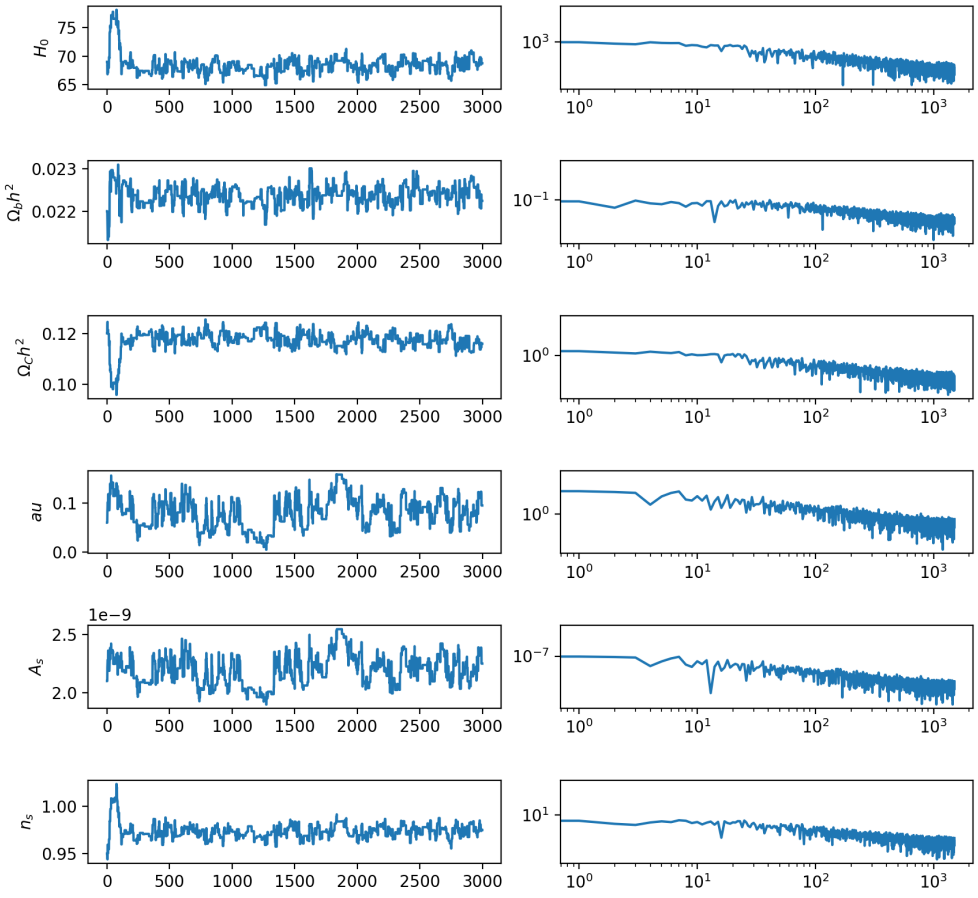


Figure 3: Parameters as a function of steps from the mcmc chain.

We can calculate the value for $\Omega_\lambda$ using the following equation and calculate the standard deviation to determine the uncertainty,

$$\Omega_\lambda = 1 - \Omega_b - \Omega_C. \tag{1}$$

From the mcmc I calculate

$$\Omega_\lambda = 0.701882006747265 \pm 0.07715242608374544 \tag{2}$$

# 4 Question 4

When running the mcmc this time, we want to re-estimate the covariance matrix using the results from Question 3. To do this, we take the values for $\tau$ that we got from 3 and calculate it for a gaussian centered at $\tau = 0.054$ with a standard deviation of 0.0074. We will use this as our weights to re-estimate the covariance matrix. Running a mcmc gives the following fit parameters:

| Parameter | Value |
|---|---|
| $H_0$ | 68.56681388022216 ± 1.775466992934356 |
| $\Omega_b h^2$ | 0.022390764358972925 ± 0.0002407029377142806 3 |
| $\Omega_C h^2$ | 0.11718628722121541 ± 0.003782683444287137 |
| $\tau$ | 0.08966962961413484 ± 0.028602254464678078 |
| $A_s$ | 2.2383332655077826e-09 ± 1.1884055508404824e-10 |
| $n_s$ | 0.9743665216746519 ± 0.008871981321501164 |

Table 3: Mean fit parameters while constraining the values of tau

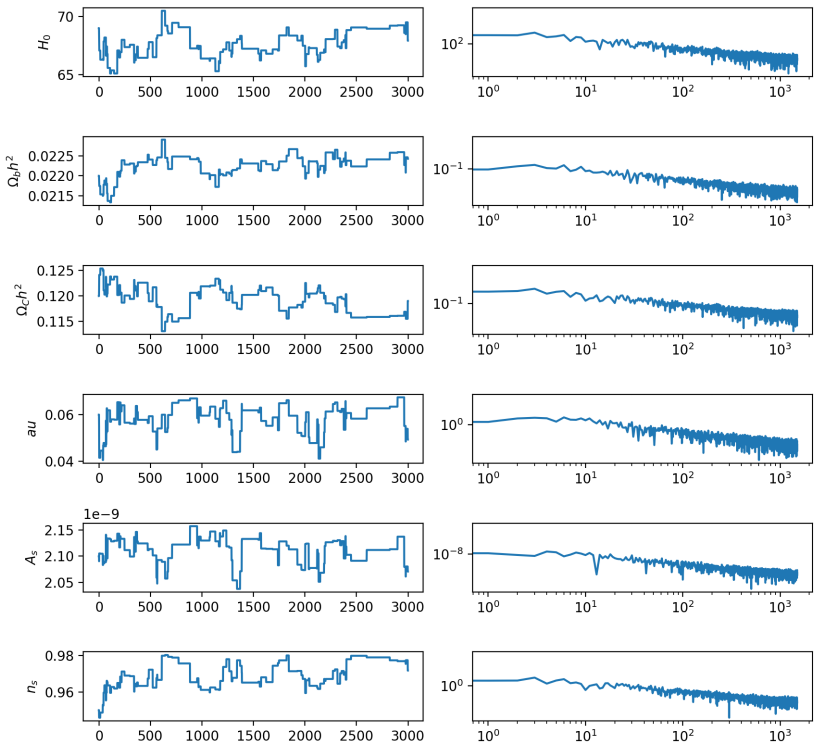


Figure 4: Parameters as a function of step size while constraining the values of tau

We can observe that these results are off by the factor from the importance sampling such that the params from Question 4 can be reproduced using answers from 3 and the importance sampling.