PHYS 512 - Problem Set 7 Solutions

Minya Bai (260856843)

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1 Question 1

If we were to plot out the randomly generated numbers, using an interactive plot at certain angles we observe multiple lines which we can takes to be planes that the data points reside one. Similarly, we can observe this by plotting $ax+by$ against z, for a and b are real numbers. I tested for multiple values of a and b and I found that we again can observe very distinct lines of points. This can be observed on the left plots in Figure 1.

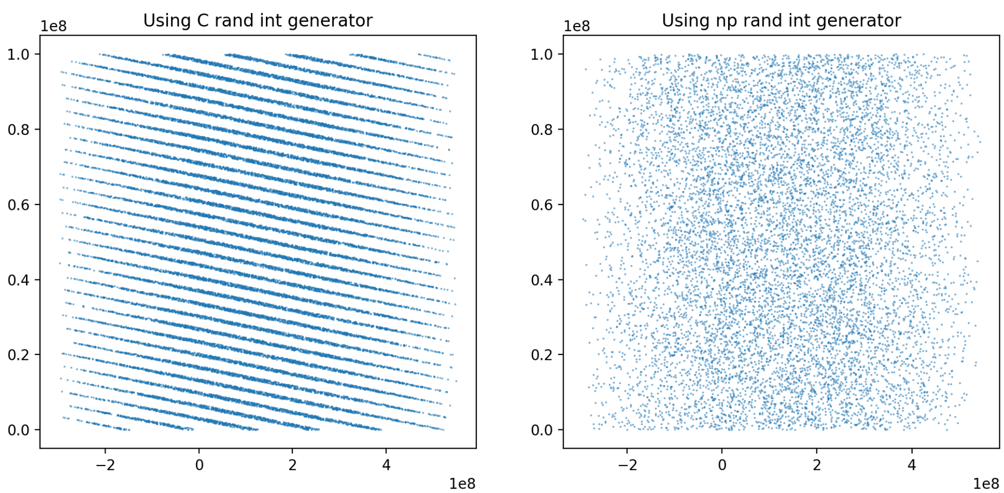

Figure 1: *(left)* Points generated by the C random number generator for $a = 5.5$, and $b = -3$. *(right)* Points generated by the python (numpy) random generator for $a = 5.5$ and $b = -3$.

2 Question 2

We have the option of choosing between a Gaussian, Lorentzian and a power series. These functions are defined as follows:

$$g(x) = e^{-\frac{x^2}{2\sigma^2}} \tag{1}$$

$$l(x) = \frac{1}{1+x^2} \tag{2}$$

$$p(x) = (x+a)^b + c \tag{3}$$

where σ is the standard deviation of the gaussian function, and a, b, c are the horizontal shift, the order, and the vertical shift of the power series. Before we get into calculating the functions for the deviates, we can first plot out these functions again the exponential function we are trying to calculate the deviates for.

From the graph above we can see that the Gaussian intersects the function, so it would not be a suitable function for us to use. When the power series has a vertical shift set to 0, we see that it deviates from the function a lot more near x=0. This will cause the function to blow up. So we need a c to ensure this doesn't happen. The Lorentzian looks good, and it is always above the exponential.

Now, in order to calculate the deviates, we need to take the integral of these PDF and take the inverse. I was unable to take the inverse of the CDF of the power series, so I took this to mean the Lorentzian was the ideal choice.

Calculating for the CDF and the inverse of CDF we get:

$$CDF = \int \frac{1}{1+x^2} dx = arctan(x) \tag{4}$$

$$CDF^{-1} = tan(y) \tag{5}$$

for $y \in (-\frac{\pi}{2}, \frac{\pi}{2})$.

Now, using the rejection method, we want to calculate the exponential deviates. The following histogram was obtained using the Lorentzian to calcalate the deviates.

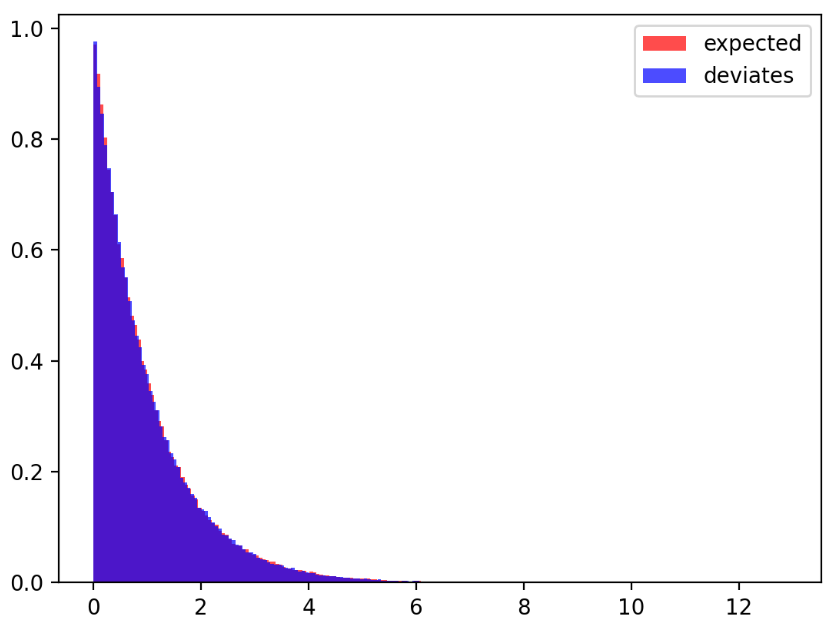

Figure 2: Histogram of exponential deviates using rejection method.

The max acceptance is 0.9999986977784492, but the actual accepted fraction was found to be 0.6358491549132068.

3 Question 3

This time, we want to repeat question 2, but use a ratio-of-uniform generator instead. To do this, we define a space (u, v) by generating a list of random numbers between 0 and 1 for both. Next, we take the ratio of v/u, and we want the region to be bounded by $0 \leq u \leq [p(v/u)]^{0.5}$, where $p(x)$ is our exponential function. The following histogram was obtained using this method:

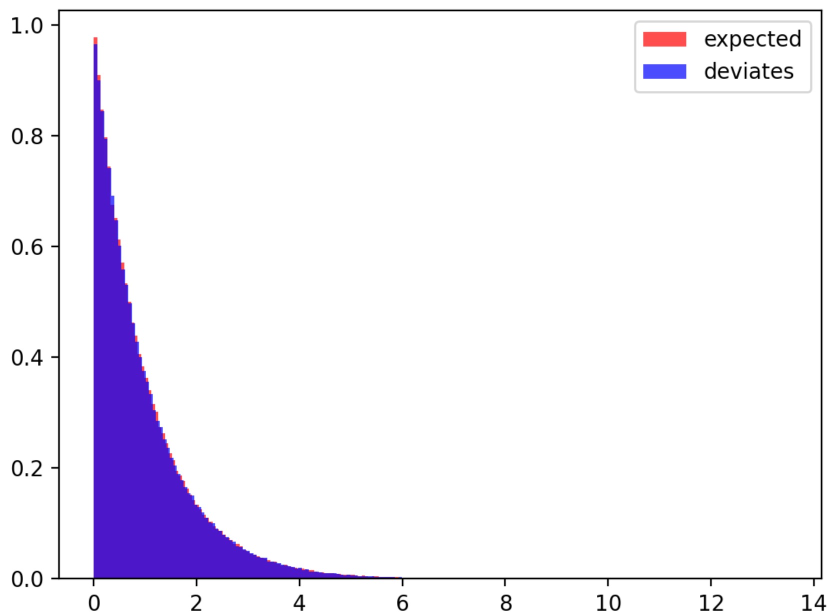

Figure 3: Histogram of the exponential deviates using ratio-to-uniform method.

The acceptance fraction is 0.500238, which is lower than what was calculated in Question 2.