PHYS 512 - Problem Set 2 Solutions Explanation

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1 Question 1

Consider the charge, dq, of a small area, $da = R^2 sin\theta d\theta d\phi$, on the surface of an infinitesimally thin spherical shell with radius R and charge density σ.

$$dq = \sigma da = \sigma R^2 sin\theta d\phi. \tag{1}$$

Since we have a hollow shell, $z < R$ is the easy case: $E = 0$. Take $z > R$. Define the distance between the middle of da and z to be s. Then we draw the following relationship:

$$s^2 = R^2 + z^2 - 2Rzcos\theta. \tag{2}$$

We recall that the E-field from a continuous charge distribution with uniform surface charge density can be found with

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma}{|s|^3} \vec{s} da' \tag{3}$$

We note that $z - Rcos(\theta) = \sqrt{R^2 + z^2 - 2Rzcos\theta}$. Substituting all this in and evaluating $phi \in (0, 2\pi)$, we get:

$$\vec{E}(\vec{r}) = \frac{1}{4\pi\epsilon_0} \int \frac{\sigma R^2 sin\theta d\theta d\phi (z - Rcos\theta)}{(R^2 + z^2 - 2Rzcos\theta)^{3/2}} da'$$
$$\vec{E}(\vec{r}) = \frac{2\pi\sigma R^2}{4\pi\epsilon_0} \int \frac{sin\theta (z - Rcos\theta)}{(R^2 + z^2 - 2Rzcos\theta)^{3/2}} d\theta$$

$$\vec{E}(\vec{r}) = \frac{\sigma R^2}{\pi\epsilon_0} \int \frac{sin\theta (z - Rcos\theta)}{(R^2 + z^2 - 2Rzcos\theta)^{3/2}} d\theta \tag{4}$$

Equation 4 is the integral we will integrate with the code in ps2_problem1.py. The follow plot of the magnitude of E field as a function of position (with $R = 5\sigma = 1, \epsilon = 1$) was found using the code:

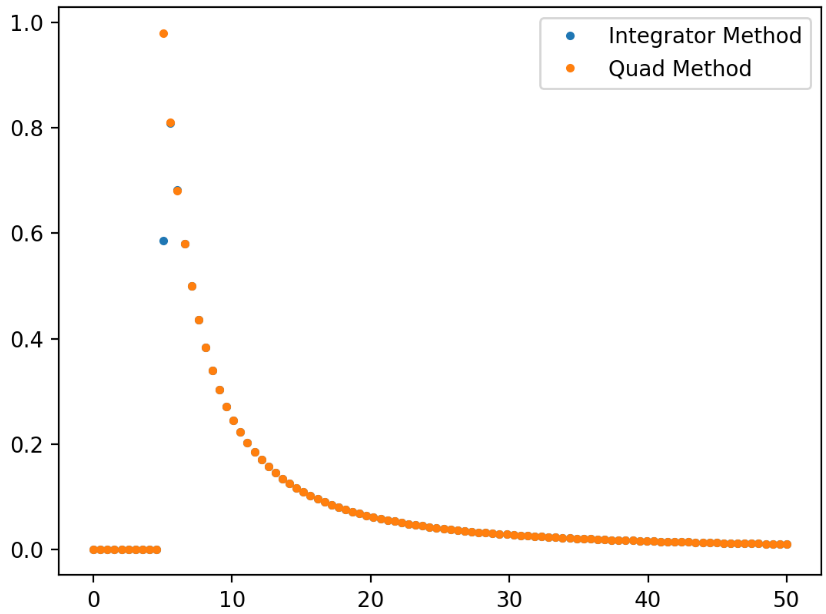

Figure 1: E field as a function of z position using the two integrator methods. $R = 5\sigma = 1, \epsilon = 1$.

Notice the weird point near $z = R$ with the integrator method. This is a singularity resulting from getting 0 for denominator of the integrand at that point. We observe that quad does not have this issue. Since the we are numerically integrating, the singularity will cause issues near $z = R$.

2 Question 2

In class, the function was called for five points each time it was called. Since these function use the same definition for error, if we test two functions with the same function, domain and tolerance, and say, there were n recursive calls. In the class example, each time the recursive function is called, the function is evaluated at 5 points. So in total, the function would have be called 5^n times! With the recursive function method, we save the function value at the ends and pass them when we cut to a smaller region. For each recursive call, two function calls are used. So for this function, the function would be called $2n$ times total. This is much smaller if we want smaller errors!

3 Question 3

The following figure shows the comparison between using np.log(x) and the mylog2 function that was implemented. The error (mylog2(x) - np.(x)) is within the 10^{-8} t0 10^{-6} range.

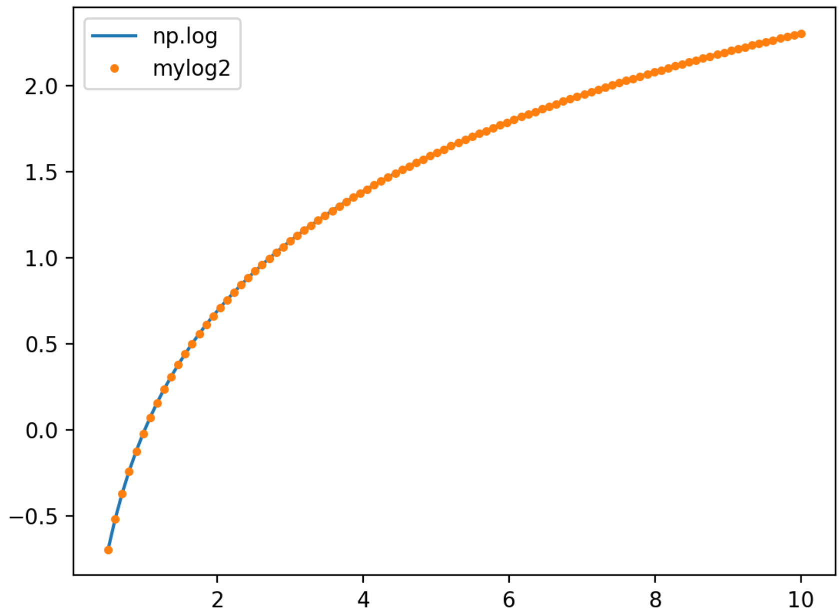

Figure 2: Comparison of calculating ln(x) using np.log(x) and the mylog2(x) function.