 Name: ____________________

Collaborators: ____________________

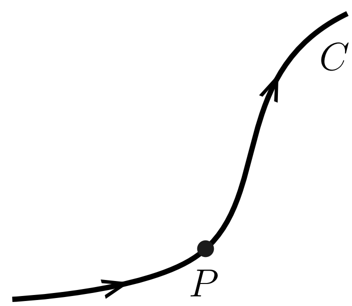
Instructions: Unless otherwise indicated, you may use Maple to assist with computations on this assignment. Upload your Maple file to the Moodle dropbox along with the scanned pdf of your work. You may not use solution manuals / websites (e.g. Chegg). AI tools like ChatGPT are also disallowed.

1. Suppose a particle traverses a curve C at constant speed of 5 m/s, where the shape of C is given by $\vec{c}(u) = \left\langle \ln\left(\frac{u}{3}\right),\ \sin\left(\frac{\pi u^2}{9}\right),\ \cos\left(\frac{\pi u}{3}\right) \right\rangle$ meters on $1 \leq u \leq 4$.

   (a) Find the particle's acceleration vector at $(0, 0, -1)$. Round to three decimal places.

   (b) Find the point on C where the magnitude of the particle's acceleration is as large as possible. Round to three decimal places.

2. A particle traverses the curve C shown in the figure, in the direction indicated. At point P, the curvature of C is $\frac{1}{12}$ m^{-1}. Moreover, at P the particle is traveling at a speed of 6 m/s and is slowing down at a rate of 9 m/s^2.



(a) Find the components of the particle's acceleration, a_T and a_N.

(b) At point P, on the figure, carefully sketch and clearly label $\vec{a}_T$, $\vec{a}_N$, and the particle's full acceleration vector. Draw these vectors with correct relative magnitudes (e.g. if $|\vec{a}_T|$ is supposed to be half of $|\vec{a}_N|$, then make sure your sketch looks that way). Briefly explain your logic.

3. On a planet where gravity imparts a constant downward acceleration of 25 feet/second^2, the position of a rocket (with distances in feet and time in seconds) is given by

$$\vec{r}(t) = \left\langle t^2,\ t,\ \frac{1}{4}t^4 \right\rangle.$$

The z-coordinate represents the rocket's altitude. At $t = 10$, the rocket's steering and propulsion systems are turned off, after which its flight is influenced by gravity only. (Ignore air resistance.)

(a) Find the position function of the rocket for its projectile motion, i.e. the part of its flight after the propulsion is turned off.

(b) The rocket eventually impacts a mountain modeled by $z = 10000 - 5 \cdot \sqrt{(x-3000)^2 + y^2}$. (All coordinates in feet.) At what altitude does the rocket hit the mountain? Round to three decimal places.

4. Express the natural domain D of $f(x, y) = \ln(x^2 - 2x + 4y^2 - 3)$ in set notation, then carefully sketch and shade D. Be very clear about which points are in D and which are not.

5. Let $f(x, y) = 4 - \sqrt{x^2 + y^2}$.

   (a) On the same set of axes, sketch and clearly label the level sets $\{(x, y) : f(x, y) = k\}$ for $k = 0, 1, 2, 3, 4$.

   (b) Use the level sets to sketch the graph of f.

   (c) Is f differentiable at all (x, y)? Explain why or why not.