## MODELING A HIGH-MASS TURN-DOWN IN THE STELLAR INITIAL MASS FUNCTION

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## **ABSTRACT**

Statistical sampling from the stellar initial mass function (IMF) for all star-forming regions in the Galaxy would lead to the prediction of  $\sim 1000~M_{\odot}$  stars unless there is a rapid turn-down in the IMF beyond several hundred solar masses. Such a turn-down is not necessary for dense clusters because the number of stars sampled is always too small. Although no upper mass limits to star formation have ever been observed, a theory for the IMF should be able to explain the lack of  $\sim 1000~M_{\odot}$  stars in normal galaxy disks. Here we explore several mechanisms for an upper mass cutoff, including an exponential decline of the star formation probability after a turbulent crossing time. The results are in good agreement with the observed IMF over the entire stellar mass range, and they give a gradual turn-down compared to the Salpeter function above  $\sim 100~M_{\odot}$  for the normal thermal Jeans mass,  $M_{\rm J}$ . However, they cannot give both the observed power-law IMF out to the high-mass sampling limit in dense clusters and the observed lack of supermassive stars in whole galaxy disks. The exponential decline is too slow for this. Either there is a sharp upper mass cutoff in the IMF, perhaps from self-limitation, or the IMF is different for dense clusters than for the majority of star formation that occurs at lower density. In the latter case, dense clusters would have to form an overabundance of massive stars relative to the average IMF in a galaxy. Evidence for a difference in the cluster and field IMFs supports this picture, but systematic effects could mimic this evidence even with a universal IMF. Within the framework of the sampling model, the upper mass turn-down should shift toward higher mass when  $M_1$  shifts upward, as might be the case in some starburst galaxies, and shift toward lower mass when  $M_J$  is lower, as might be the case in ultracold or high-pressure regions. Supermassive stars may therefore be possible in starburst galaxies, while in low surface brightness regions, where ultracold gas might exist at normal pressures, or in galactic cluster cooling flows, where cold gas could have extremely high pressures, a high fraction of the star formation could end up as brown dwarfs.

Subject headings: stars: formation — stars: luminosity function, mass function

## 1. INTRODUCTION

A recent model for the stellar initial mass function (IMF), in which stellar masses are randomly sampled down to the thermal Jeans mass from hierarchically structured, prestellar clouds (Elmegreen 1997b, 1999a, 2000a; hereafter Papers I, II, and III), is extended here to consider the simultaneous evolution of small and large turbulent cloud structures, introducing an additional competition for mass that was not present in the earlier work. As a result, the slope of the power law comes closer to the Salpeter IMF  $(M^{-1.3}d \log M \text{ changes to } M^{-1.35}d \log M)$ , and there is an interesting turn-down in the IMF at high mass that might have important consequences. Such a turn-down has not yet been observed in real clusters because the masses of the largest stars generally scale with the total cluster mass, as expected for random sampling (Elmegreen 1983, 1997b; Schroeder & Comins 1988; Massey & Hunter 1998; Selman et al. 1999), but statistical considerations (§ 2) suggest that there should be a turn-down in the IMF where this scaling stops, somewhere in the range from 100 to 1000  $M_{\odot}$ . Otherwise, the largest star in a galaxy such as ours would be several thousand solar masses, and it would be even larger in starbursts.

There have been several previous suggestions that stars have a maximum mass (Larson 1982; Khersonsky 1997). Larson & Starrfield (1971), Kahn (1974), and Wolfire & Cassinelli (1986, 1987) suggested that radiation pressure on dust grains halts the accretion onto a massive star, thereby

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limiting the mass. However, there is no obvious dependence between maximum stellar mass in a cluster and metallicity (Freedman 1985; Massey, Johnson, & DeGioia-Eastwood 1995a), so radiation pressure effects are probably not important. In addition, a large part of the stellar accretion may be optically thick through an accretion disk (e.g., Nakano 1989; Nakano, Hasegawa, & Norman 1995; Jijina & Adams 1996) or protostar coalescence (see § 5), and in that case, radiation pressure effects would not be expected.

There seems to be no fundamental limit to the stellar mass at the time of birth from the Eddington limit. The most massive stars have luminosities that increase roughly proportional to mass (Massey 1998), so they are probably near their Eddington limits on the main sequence and consequently shed a lot of mass after they form. Such rapid mass loss is observed around the most massive stars (Figer et al. 1998; Morse et al. 1998; Grosdidier et al. 1998). However, there would seem to be nothing to prevent the rapid accumulation of optically thick gas during star formation, especially if it comes in through a dense disk or by some prestellar coalescence. As long as the accretion rate during star formation exceeds the maximum wind loss rate, which is about the stellar density at one optical depth on the surface multiplied by the surface area and the escape velocity, and as long as the total accretion time is less than the nuclear burning time in the core, which is about  $2 \times 10^6$  yr for a massive star, then a star with an arbitrarily large mass can form.

A more important question is whether other things happen first in a massive clump that makes most of the gas unavailable for accretion to a single star. Here we examine the possibility that star formation in the smaller, denser, and faster evolving subclumps is likely to happen first, and that competition for gas and cloud dispersal, rather than self-destruction, are what limit the *initial* mass of the largest star that can form. This solution is obvious in the extreme case: whole molecular clouds do not form single supermassive stars because smaller stars form first and either take the gas themselves or disperse it away from the cloud. The same competition for mass should arise on smaller scales as well, preventing the formation of  $\sim 1000~M_{\odot}$  stars, for example, in  $10^4~M_{\odot}$  cores, by the earlier formation of 1–100  $M_{\odot}$  stars in the same gas.

Some level of competition between low-mass star formation in small clumps and more massive star formation in the larger clumps that contain them was already present in the models of Papers I-III. It had the effect of steepening the mass function from  $M^{-1.15}d \log M$  for pure root-density-weighted sampling to  $\sim M^{-1.3}d \log M$  after numerical simulations of this process. However, there was no constraint on timing for this type of mass competition, as there is in the present models. This means that the Salpeter function would extend to arbitrarily high masses if a large clump can wait forever to form its star. However, such infinite waiting times are unrealistic, because the gas inside a large clump should continuously move and make new small clumps, forming additional small stars, and because the winds and radiation from stars that have already formed in the small clumps should significantly alter the conditions for future star formation in the large clump. If the gas that was originally available for the large star is systematically channeled into small stars first, or if this gas is removed from the large-scale clump altogether, then a large star will not form there.

We believe that this is a realistic situation for turbulent clouds and attempt to model it here within the context of the random sampling model for the IMF. We do this by including an exponentially decreasing function,  $e^{-\omega t(M)/t(M)}$ , for the probability of forming a star on a certain scale; t(M) is the crossing time on the scale that forms a star of mass M,  $M_J$  is the mass at the lower limit to the power law in the IMF, considered here to be the thermal Jeans mass, and  $\omega$  is a constant of order unity. With this function, large clumps with long crossing times have a decreased chance of forming a massive star.

In the discussion that follows, we first present evidence that there should be a turn-down in the IMF for normal star formation regions at masses larger than  $\sim 100~M_{\odot}$ . We then present a model for the origin of this turn-down that is consistent with the prevailing picture of interstellar clouds, i.e., one in which prestellar clouds have scale-free spatial structures and dynamical evolution on all scales.

## 2. A STATISTICAL LIMIT FOR THE UPPER STELLAR MASS

For a universal IMF that is randomly sampled in a particular region over time, the largest stellar mass that is expected to be present is given by the expression  $\int_{M_{\text{max}}}^{\infty} n(M)dM = 1$ , while the total cluster mass in the power-law part of the IMF is

$$M_{\text{cluster}} = \int_{M_{\text{min}}}^{\infty} Mn(M)dM = \frac{xM_{\text{max}}^{x} M_{\text{min}}^{(1-x)}}{x-1},$$
 (1)

given a power law slope -1-x in the expression  $n(M)dM \propto M^{-1-x}dM$ . For the Salpeter slope of x=1.35 and a lower limit to the power law part of 0.3  $M_{\odot}$ , this integral is

$$M_{\rm cluster} \sim 3 \times 10^3 \left(\frac{M_{\rm max}}{100 \ M_{\odot}}\right)^{1.35} M_{\odot} \ .$$
 (2)

This equation does not apply to clusters that are old enough to have lost their most massive members via supernovae; then the maximum current stellar mass will be lower than the initial value. It also does not apply to the low-mass flattened part of the IMF. It applies only to the stars in the power-law part of the IMF that are less massive than the most massive surviving member of the contemporary cluster. In fact, young star clusters always have masses that are just large enough to account for their largest stars. That is, there are usually only a few stars, or perhaps just one, with a mass close to the maximum stellar mass in the cluster. Violation of equation (2) would require a significant number of stars in the highest mass bin of the IMF, with a sudden lack of stars any more massive than this. Without such a violation, there is no evidence for the existence of an absolute largest stellar mass.

The situation changes when all of the current star formation in a galaxy is considered as the statistical ensemble. According to our model, stellar masses are randomly chosen from the same IMF regardless of where they form, so the IMF from each star-forming region and the summed IMF from all star-forming regions in a galaxy are the same. This is in agreement with observations, which suggest that the IMF from integrated light in a galaxy (e.g., Bresolin & Kennicutt 1997) is the same as the IMF in individual clusters (see review in Elmegreen 1999b). When such random sampling is considered in the context of a pervasive and interconnected interstellar fractal structure, then  $M_{\text{cluster}}$  in equation (2) can be very large. Indeed, the entire ensemble of molecular clouds, along with their internal dense substructures and peripheral atomic structures, is apparently a continuum of forms extending over a wide range of cloud and intercloud media (Elmegreen & Falgarone 1996; Elmegreen 1997a). For example, the interstellar media in the Small Magellanic Cloud (Stanimirovic et al. 1999) and M81 group galaxies (Westpfahl et al. 1999) look like continuous power-law structures with no characteristic scales in either the clouds or the spaces between them.

What this continuity means is that the IMF could really be sampling from a total gas mass that is the summed mass over many neighboring star-forming clouds, instead of just the mass in any one giant molecular cloud core. Neighboring clouds are just parts of the larger structure in the overall interstellar hierarchy. This summed mass might be more like  $10^6~M_{\odot}$  of gas, corresponding to  $\sim 10^{4.5}~M_{\odot}$  of stars, considering typical efficiencies. Then  $M_{\rm max}$  in the above expression would be  $\sim 600~M_{\odot}$ . The summed mass for samiltonian to the summed mass for same same samiltonian to the summed mass for same same same same same pling could even be the whole molecular interstellar medium. If the main-sequence lifetime of a massive star is 2 Myr (Massey 1998), then typical galactic star formation rates of 5  $M_{\odot}$  per year will have  $10^7 M_{\odot}$  of young mainsequence stars at any one time. If these stars are considered as the statistical sample for the IMF, then the maximum stellar mass would be 40,000  $M_{\odot}$  for an extrapolated Salpeter function. Obviously, the IMF must turn down somewhere above  $\sim 100~M_{\odot}$  if interstellar structures on

kiloparsec scales provide an ensemble for sampling the IMF.

These observations suggest that the power-law IMF drops more rapidly than the Salpeter slope above several hundred solar masses, giving, in effect, two characteristic masses to the IMF, one at each end. These two characteristic masses may have different origins and scale differently with cloud properties, or they could be related and scale together. The model proposed next is in the latter category. Other models are given in § 4.

# 3. A HIGH-MASS TURN-DOWN IN THE RANDOM SAMPLING MODEL FOR THE IMF

## 3.1. A Timing Limitation Converted into a Mass Range

The first stars that are likely to form in a cloud are those collapsing from the densest pieces, which evolve the quickest. These are the lowest mass pieces, according to the turbulence scaling laws discovered by Larson (1981), and in the random sampling model they give the smallest stars. An important aspect of interstellar cloud structure is that these low-mass clumps are themselves clumped together into larger cloud fragments, which in turn are clumped further into even larger pieces (see reviews in Scalo 1985; Elmegreen & Efremov 2000). Such hierarchical structure gives the fractal aspect of interstellar clouds that has been discussed extensively in the literature. Fractal structure has been found at the edges of atomic (Vogelaar & Wakker 1994), molecular (Dickman, Horvath, & Margulis 1990; Falgarone, Phillips, & Walker 1991), and dust (Beech 1987; Bazell & Désert 1988; Scalo 1990) clouds, in the intensity distributions across clouds (Stutzki et al. 1998), and in the clump-to-clump and cloud-to-cloud distribution of masses, considering a hierarchy of structures that extends up to galactic scales (Elmegreen & Falgarone 1996).

In the random-sampling model, stars can form from any level in this hierarchy of structures as long as the gas can be made significantly self-gravitating by any of a variety of physical processes. In fact, only a small range of hierarchical levels is necessary to give the observed factor of  $\sim 100$  for stellar mass in the power-law part of the IMF. This mass factor corresponds to a size factor of  $\sim 7$  for prestellar clumps, considering the scaling between mass M and size Sthat comes from the fractal dimension  $D \sim 2.3$ :  $M \propto S^{2.3}$ (Elmegreen & Falgarone 1996; Heithausen et al. 1998). Thus, all stars in the power-law part of the IMF start within a limited range of scales in the interstellar fractal, from the smallest gravitationally unstable piece at several tenths of a solar mass up to fragments only  $\sim 7$  times larger in size. The whole interstellar fractal probably spans a factor of 10<sup>5</sup> or more in size, although it does not need to be continuous everywhere. This limited range for all star formation, out of the enormous range of available cloud structures, emphasizes the need for a physical limitation to the mass of the most massive star, since there is a physical limitation to the lowest mass star.

Here we consider what happens if small stars form so much faster than the evolution time of the larger clump that contains them that the cloudy gas around the smallest stars has time to re-establish a fresh hierarchy of structures before the larger clump gets a chance to form its larger star. This new structure might be driven by a variety of processes, including small-star winds and continued turbulence decay. Whatever the cause of mixing, the large-scale

gaseous structure that was available at first for the formation of a large star will get redistributed into smaller pieces and end up making only smaller stars. Thus, the large star will have a reduced chance of forming. In terms of actual timescales, this constraint corresponds to a formation time of the largest star that must be within a factor of only a few times the formation time of the smallest stars in any one branch of the hierarchical tree of gas structures. This factor follows from the relatively short timescale for turbulence to redistribute and reform gaseous structures inside each existing scale, and from the relatively short time for young stellar winds to stir and distort the surrounding gas.

Considering the turbulent scaling relationships for molecular clouds, this constraint on timing can be translated into a constraint on mass. The turbulence correlations show that the total line width or velocity dispersion in a clump,  $\Delta v$ , scales with the clump size, S, to about the 0.4 or 0.5 power (Larson 1981; for a compilation of clump studies, see Efremov & Elmegreen 1998). This means the crossing time  $t_{\rm crossing} \propto S/\Delta v \propto S^{\alpha}$  for  $\alpha \sim 0.5-0.6$ . The same clumps also have a relation between mass M and size, which is  $M \propto S^{\kappa}$  for  $\kappa \sim 2-3$ . In individual surveys,  $\kappa \geq 3$  (e.g., Rosette:  $\kappa = 3.20 \pm 0.43$ , Williams, de Geus, & Blitz 1994; Ophiuchus:  $3.67 \pm 0.71$ , Loren 1989; Maddalena-Thaddeus cloud:  $3.12 \pm 0.23$ , Williams et al. 1994; M17:  $3.18 \pm 0.56$ , Stutzki & Güsten 1990), while for the whole inner Galaxy,  $\kappa \sim 2.38 \pm 0.09$  from Solomon et al. (1987), and for all clouds taken together,  $\kappa \sim 2.3$  (Elmegreen & Falgarone 1996). This latter value has been interpreted as the fractal dimension by Pfenniger & Combes (1994), Larson (1994), Elmegreen & Falgarone (1996), and Heithausen et al. (1998). In Larson (1981),  $\kappa \sim 2$ , which follows for virialized clouds with  $\alpha \sim 0.5$ . From these  $\alpha$  and  $\kappa$ , the crossing time scales with mass as

$$t_{\rm crossing} \propto M^{\alpha/\kappa} \quad \text{for } \alpha/\kappa \sim 0.17-0.3 \ .$$
 (3)

In the examples below, we consider  $\alpha/\kappa=0.2$  and 0.3. For  $\alpha/\kappa=0.2$ , a constraint on the ratio of crossing times between the smallest star-forming scale and the largest corresponds to a range in stellar mass equal to this timing ratio to the power  $\kappa/\alpha\sim5$ . Thus, a factor of  $\sim100$  in stellar mass corresponds to a factor of only  $\sim2.5$  in turbulent crossing time. Regardless of the theory of star formation, most of the power-law range of the IMF is created in regions with dynamical and turbulent timescales that are within a factor of  $\sim2.5$  of the timescale at the minimum gravitationally unstable mass. This extremely tight schedule for the relative formation times of intermediate- and high-mass stars suggests that turbulence might be involved, since turbulent fluids mix and destroy structures on similar timescales.

## 3.2. A Stochastic IMF Model with a Timing Limitation

To illustrate the effects of timing constraints on the IMF, we modified the previous models of the IMF that were based on root-density—weighted, random samples of mass in hierarchically structured clouds (Papers I–III). We used the formalism of Paper III to get the flattening at low mass, which assumes that there is a separate and uniform mass distribution for the formation of stars inside each clump.

We simulate the dynamical competition for gas mass in evolving turbulent clouds by introducing the additional probability

$$P(M) = e^{-\omega t(M)/t(M_{\rm J})} \sim e^{-\omega(M/M_{\rm J})^{\alpha/\kappa}}$$
(4)

for the formation of a star of mass M, where t(M) is the

dynamical timescale for regions of mass M, and  $\omega$  is a dimensionless parameter of order unity.

There are several ways to view this exponential in physical terms. If low-mass stars mix up the gaseous structures in their immediate neighborhoods, triggering other low-mass stars directly or in some other way preventing the gas from coming together into a single massive star, then the timescale for this mixing is on the order of a few crossing times at the low-mass end. This time includes the star formation time for the low-mass stars, as well as the mixing time itself, assuming that the star formation time is relatively quick in terms of the local dynamical time (Elmegreen 2000b). If turbulence alone mixes the gas as part of the continued decay of turbulent energy by shocking, viscosity, and magnetic diffusion, then the time during which the massive structures are free to evolve into massive stars is limited again to a few crossing times (Stone, Ostriker, & Gammie 1998; MacLow et al. 1998). Thus, there is a window of opportunity to form a star of mass M in gas that also forms other stars, and this window has a characteristic timescale equal to the crossing time on the lowest mass scale,  $M_{\rm J}$ , multiplied by some factor of order unity,  $\omega^{-1}$ .

From a mathematical point of view, the exponential is the Poisson probability that no significant mass redistribution inside scale M has occurred, given a mean number of redistribution times equal to  $\omega t(M)/t(M_J)$ . Recall that the Poisson probability for N events given an expected number of events  $\lambda$  is  $\lambda^N e^{-\lambda}/N!$ , so the case of no events, N=0, has probability  $e^{-\lambda}$ . Here, the waiting time for redistribution is taken to equal  $\omega^{-1}$  times the crossing time at the lower mass end, which is the Jeans mass. If  $\omega=1$ , then the gas structure at scale  $M_J$  completely redistributes itself in one crossing time on that scale.

The thermal Jeans mass in equation (4) is only one of several interpretations for the low-mass limit of star formation. Another is that pre-main-sequence winds, possibly induced by deuterium burning, set the lower limit to what can accrete onto a star (Nakano, Hasegawa, & Norman 1995; Adams & Fatuzzo 1996). For the wind model, the lower mass objects are assumed to continue accreting until they reach at least this limit. The nature of the lower mass limit is not important for the present paper. Perhaps there is a combination of effects, including both the requirement for clump self-gravity (the  $M_J$  limit) and the self-limitation by winds. In any case, we use the notation  $M_J$  to represent the lower limit to the power-law part of the IMF, where it turns over to a somewhat flattened distribution at lower mass.

In the algorithm for determining the IMF, the probability P in equation (4) is applied after the clump mass is chosen from the hierarchical tree. The whole algorithm runs as follows. A hierarchical tree of masses inside other masses is set up initially using a random number generator, with H = 10 hierarchical levels and an average of N = 3.2 subclumps per clump, distributed as a Poisson variable in the interval from 1 to 5 (this choice of N is discussed below). These masses are then chosen sequentially, using more random numbers, with a weighting for clump choice that scales with the square root of the local density in the "cloud." Density is related to mass using the fractal dimension D = 2.3. This sequential choice gives a realistic aspect of competition for mass, because any clump that is removed to make a star cannot be used again later to make a different star at a higher level. The square root of density enters because that is the relative rate for most of the dynamical

processes that make stars. After this one mass is chosen, we generate another random number, R, uniformly distributed between 0 and 1, and keep that mass for the final IMF if P(M) > R, where P is given by equation (4) with  $\omega = 1$ ,  $\alpha/\kappa = 0.2$ , and  $M_J = 2$  in program units. This simulates the second aspect of competition for mass, namely, the requirement that large regions not be given much time to turn their gas into massive stars, considering that the smaller regions inside of them should evolve more quickly. To get the actual star mass, we take a random fraction of this chosen clump mass. This random fraction is sampled from a probability distribution function that is flat in logarithmic mass intervals, as discussed in Paper III; it gives a realistic flat part to the IMF at low mass without affecting the high-mass power law

There are five parameters in the model:  $M_{\rm J}$ ,  $\mathcal{R}$ ,  $\omega$ ,  $\alpha/\kappa$ , and D. The mass,  $M_{\rm J}$ , is the basic scale that determines whether or not stars will form in a cloud; in the model, it is the lower limit to the power-law part of the IMF and so is directly observable. The dimensionless parameter  $\mathcal{R}$  is the ratio of the largest to the smallest mass for stars that form in any one clump; it should be the same as the mass range for the flat part of the IMF and is therefore also observable. The dimensionless quantity  $\omega$  is the rate of significant mixing at the scale  $M_J$  multiplied by the dynamical time on that scale; a high mixing rate makes it difficult to form massive stars, so  $\omega$  affects where the high-mass turn-down occurs. The ratio  $\alpha/\kappa$  affects the conversion of mixing time to mass; this depends on the scaling properties of the turbulent fluid as discussed in § 3.1 and is therefore observable, although inaccurately known.

The fractal dimension D enters only into the conversion of mass to density (and therefore the relative sampling rate). The power-law slope in the model does not depend much on any of these parameters; it depends mostly on D, but only by the amount 0.5-1.5/D, which has a relatively small range for D between 2 and 3.

The other parameters affect the parts of the IMF that are beyond the power law, at low and high masses. We have remarked previously how the slope of the IMF should not vary much from region to region because it depends on the geometric properties of the cloud, such as D. The endpoints of the power-law part of the IMF should depend on the details and physical parameters of star formation, however, and that is the case for the dependence of the present models on the parameters  $M_1$ ,  $\mathcal{R}$ ,  $\omega$ , and  $\alpha/\kappa$ .

Figure 1 shows the results of two numerical simulations in randomly generated hierarchical clouds having H=10 levels with an average of 3.2 subclumps per level. The bottom panel shows the IMF and the top panel shows the IMF multiplied by M, giving the relative mass distribution rather than the relative number distribution. In both panels, the dotted line displays the new result, including the exponential probability to simulate dynamical competition from equation (4), and the solid line is without this exponential. The stellar masses in the dimensionless units used by the computer program are given along the top axes of each panel. In these units,  $M_{\rm J}=2$ . The masses in physical units are given along the bottom axes. For physical units, we choose  $M_{\rm J}=0.3~M_{\odot}$  from the definition of the thermal Jeans mass (cf. Papers I–III).

The P=0 case for the solid line in Figure 1 was run in order to normalize the desired result against computer limitations. The main limitation is the computer memory (2 GB per processor), which would cause a turn-down at high

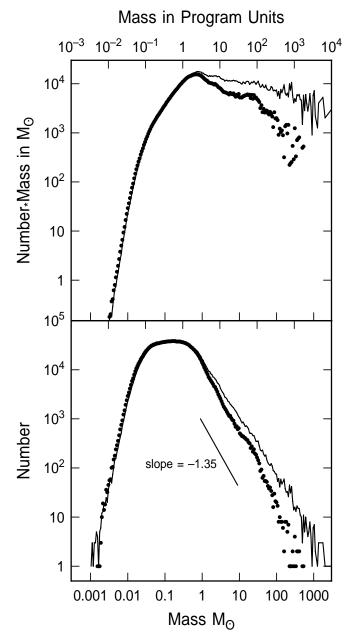


FIG. 1.—Bottom: IMF simulation for two cases: solid line is for a model with no timing constraint in the choice of clumps for star formation, while the dotted line is with the timing constraint, assuming  $\omega=1$  and a power in eq. (3) equal to 0.2. Both models contain  $2\times 10^6$  stars. Top: Product of the mass and the IMF.

mass if  $N^H$  is small because of an inability of the hierarchical tree in the model to span the observed stellar mass range (see Paper I). For these runs,  $N^H$  is sufficiently high that there is no artificial turn-down. The normalization run also indicates how much noise should be present in the result. To minimize noise, we ran this model for a large number of randomly generated trees until the number of final stars was  $\sim 2 \times 10^6$ . This is comparable to the total number of stars in the Milky Way that are younger than  $\sim 0.2$  Myr, if the birthrate is  $\sim 10$  stars per year. The stellar masses are binned in intervals of 0.05 in the logarithm (base 10), and the numbers on the ordinate of the lower panel are the numbers of model stars actually chosen for these bins. Because this number is low for the massive stars, the noise is relatively large there. Note that we could not have limited

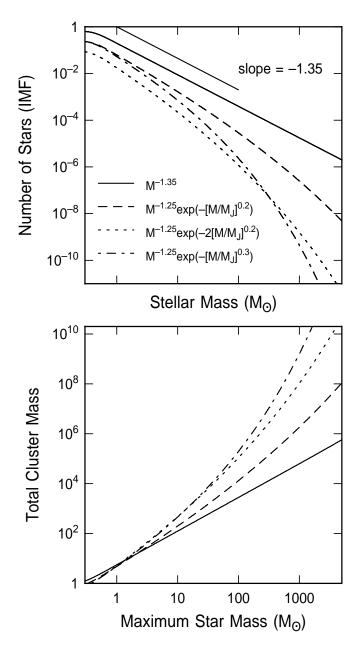


FIG. 2.—IMF models vs. stellar mass (top), and total cluster masses versus maximum stellar mass (bottom).

this simulation to high-mass stars to get better accuracy there; a competition for mass among all of the levels in the hierarchy is necessary to get the IMF.

The expected mass range for the calculation can be determined from the details of the algorithm. The smallest masses in the model are the building blocks for all of the clumps, and they occupy the range from 0.01 to 1 program units, which are the units indicated at the top of the figure. These smallest masses are distributed as  $M^{-1}d\log M$  from 0.1 to 1 to simulate a continuing hierarchical structure below  $M_{\rm J}$  (see Paper I). Thus, the average mass of the smallest unit is 0.2558 program units. The choice of star mass from a given clump mass introduces another factor. This comes from a distribution that is uniform in  $\log \epsilon$  for  $\epsilon$  equal to the star/clump mass fraction. For a total range of  $\epsilon = \Re = 30$ , as assumed here, the average value of  $\epsilon = 0.2842$ . The average number of subclumps is chosen to be 3.8, but computer memory limitations force us to allow

variations in the actual number of subclumps per clump that only range from 1 to 5. The average of a Poisson variable in the interval from 1 to 5 with a mean of 3.8 equals 3.2, which is the effective average number of subclumps per clump in the simulations. With these three numbers in runs with nine hierarchical sublevels (the tenth level is the whole cloud), the model IMF is expected to begin to drop significantly because of computer limitations at a mass equal to  $M_J$  times  $0.2558 \times 0.2842 \times 3.2^9 = 2560$ . This is larger than the maximum masses for the lines in Figure 1 (in terms of  $M_J$ ), so there are no significant drops in the IMF from memory limitations.

A comparison between the two lines in Figure 1 illustrates the main point of this paper: the timing constraint steepens the IMF a little for the whole power law range, actually making the slope closer the Salpeter value, and it produces a noticeable (factor of 10) dip in the IMF below the P=0 case at masses larger than  $\sim 1000 M_{\rm J}$  (= 300  $M_{\odot}$  in the figure).

For a realistic physical model, the mass at the high-mass turn-down should depend on the average number of crossing times for the redistribution of mass inside each particular mass scale. We have assumed in the above calculations that only one crossing time ( $\omega=1$ ) is enough to completely mix up a clump and render it improbable to convert a high fraction of its mass into a single star. This may be too long a time. If it takes  $\frac{1}{2}$  crossing time for mixing to have this effect, then we should have written  $P(M)=e^{-2(M/M_3)^{a/\kappa}}$  in equation (4). This has the effect of decreasing the mass at the upper mass turn-down, which depends sensitively on this factor in the exponent. We consider this change for Figure 2, discussed below.

## 4. UPPER STELLAR MASS CUTOFFS

The high-mass turn-down in the model IMF shown by Figure 1 is enough to account for the lack of supermassive stars in most star-forming regions, and it agrees well with the observation of a Salpeter slope out to 100–130  $M_{\odot}$  in the R136 cluster (Massey et al. 1995a; Selman et al. 1999). It still predicts too large a value for the maximum stellar mass in a whole galaxy disk, however.

Figure 2 shows IMFs and the expected maximum initial stellar masses versus the total stellar masses for four cases: (1) a pure power-law Salpeter function above 0.3  $M_{\odot}$ , having the form  $M^{-1.35}d\log M$ ; (2) an IMF of the form  $M^{-1.25}e^{-\omega(M/0.3\,M_{\odot})^{0.2}}d\log M$  for  $\omega=1$  that matches the Salpeter IMF at low mass and has the proposed drop-off at high mass from timing limitations, as in the model of § 3.2; (3) the same as case 2 but with  $\omega=2$ ; and (4) the same as case 2 but with a steeper power law in the exponent  $(\alpha/\kappa=0.3)$ . The pure Salpeter function reproduces equation (4), and the modified functions indicate how much more stellar mass must be sampled to get a massive star when there is a timing constraint.

Figure 2 suggests that if  $\omega$  and  $\alpha/\kappa$  are large enough to avoid stars more massive than  $\sim 300~M_{\odot}$  in a whole galaxy disk with  $10^7~M_{\odot}$  of young stars, then the IMF slope out to  $130~M_{\odot}$  is too steep to explain the observations of R136. This result is fundamental to IMF theory and not particularly dependent on the present models: There is a problem getting both the Salpeter function out to  $\sim 130~M_{\odot}$  in dense clusters (e.g., Selman et al. 1999) and at the same time not getting any  $\sim 300~M_{\odot}$  stars at all in a whole galaxy. Some type of IMF cutoff at high mass must begin just beyond the

observed mass for the most massive stars, and it must be steep to avoid forming overly massive stars in whole galaxies

We can see six ways around this problem. First, supermassive stars really could exist, particularly in other galaxies, but not be recognized or resolved yet. We discuss in § 6 how their presence in starburst regions could support the IMF model.

Second, there could be a self-limitation of newborn stellar mass from winds or radiation pressure beyond the mass of the largest observed star. Self-limitation of stellar mass is reasonable, but there are no direct observations of it in the pre-main-sequence phase, and theoretical considerations for how it might come about are beyond the scope of this paper.

Third, supermassive stars could form with the relative abundance predicted by the modified IMF models, but then erode so quickly during the first few hundred thousand years of the main sequence that they do not come out of their primordial cloud cores at their full initial masses. In this case, they would be observed primarily as embedded ultraluminous infrared sources and not be seen otherwise.

Fourth, there could be a limit to the mass of a cloud in which coherent star formation samples the complete IMF. The upper mass limit for a cloud that is necessary to avoid clusters much larger than  $10^{5.5}~M_{\odot}$  (at which point the maximum stellar mass begins to exceed the maximum observed mass for the  $\omega \sim 1$ ,  $\alpha/\kappa = 0.2$  models) is in fact the upper mass limit that is really observed for clouds, namely  $\sim 10^7 M_{\odot}$  for the giant spiral arm complexes (Elmegreen & Elmegreen 1983, 1987; Rand 1993, 1995), considering an efficiency of  $\sim 3\%$ . Since most stars form in these complexes in any case, we only need to assume that each one contains an independent hierarchical tree of cloud structure and that IMF sampling does not span the distance between them. The same would be true for the giant cloud complex in the LMC that formed the 30 Dor region, perhaps with a slightly higher efficiency.

In a small galaxy such as the LMC, this sampling limit can explain the maximum stellar mass without recourse to timing or other limitations during the star formation process. In massive galaxies, however, the analogous argument becomes more questionable. If there are many  $10^7$  $M_{\odot}$  clouds in a galaxy, then the combined mass available for star formation can be much larger than  $10^7 M_{\odot}$ , and in this combined mass, purely random star formation would sample the IMF out to prohibitively large stellar masses. To get around this problem, we would have to assume that several independent  $10^7 M_{\odot}$  clouds do not sample the IMF in the same way as one extended cloud (or spiral arm) with the same total mass. However, if we make this assumption, we have lost one important advantage of the randomsampling model, i.e., that star formation on the subparsec scale does not have to know about gas on the kiloparsec scale. If  $10^7 M_{\odot}$  is really a limit to the horizon of random sampling, then a  $4 \times 10^3~M_{\odot}$  clump inside such a cloud, which might try to make a  $10^3~M_{\odot}$  star all at once if it were in a larger cloud, must instead know that there are no other  $10^7 M_{\odot}$  clouds immediately nearby that would increase the total gas mass available for sampling. We view such environmental knowledge as unreasonable. If star formation processes are really local, then the mass of the peripheral cloud cannot affect the IMF except to influence the total number of stars that are formed.

Fifth, there could be a bias in the sampling of star mass as a function of cloud mass that is not from the formation of stars, which was considered to be unlikely in the previous paragraph, but from the destruction of clouds. It is possible that stars form randomly everywhere with no local knowledge of the overall cloud mass around them, but that star formation stops in a cloud of a certain mass once a star of a certain mass forms. Since there are more low-mass clouds than high-mass clouds, an increasing function of maximum star mass versus cloud mass at the time of destruction will steepen the IMF (Paper II). This is because low-mass clouds are more easily disrupted than high-mass clouds, and as a result are able to form stars only up to a certain low mass compared to the stars that are able to form in high-mass clouds. With such a bias, the larger number of low-mass clouds ensures that there will be more star formation episodes with a small maximum mass than with a large maximum mass. We discussed previously (Paper II) how this process could explain the apparently steep IMF in the extreme field regions of the LMC and Galaxy (Massey et al. 1995b), but concluded that the similarity between the cluster IMF and the whole-galaxy IMF limited this steepening effect to only a small fraction of the total mass. The onset of rapid and thorough cloud destruction above  $\sim 10^5~M_{\odot}$  of gas could in principle steepen the IMF for stars greater than 100  $M_{\odot}$  without any of the timing considerations modeled in the previous section.

A sixth possibility is that there are different IMFs in different regions, with shallow slopes in dense clusters to satisfy the constraint from R136, and steep slopes in other regions to give a low maximum mass. The constraint on these parameters from R136 is for an extremely dense, self-gravitating core. If most star formation is not in such cores but in lower density regions, then  $\omega$  and  $\alpha/\kappa$  could be higher in general, and supermassive stars could be avoided outside these cores.

In summary, the modified IMF with a timing constraint discussed in the previous section explains the observed IMF and the lack of  $\sim\!300~M_\odot$  stars in normal star formation regions and clusters, which is something the pure Salpeter function cannot do. However, no single model of the type discussed here can explain the simultaneous appearance of a Salpeter IMF out to  $\sim\!130~M_\odot$  in R136 and a lack of  $\sim\!300~M_\odot$  stars in a whole-galaxy sample. Either supermassive stars limit their own mass or destroy their own clouds excessively compared to low-mass stars, or an excess of massive stars forms in dense clusters. The next section considers this last possibility in more detail.

## 5. AN EXCESS OF HIGH-MASS STARS IN CLUSTERS?

One possible difference in the IMF for high- and low-density environments was mentioned in the previous section as a way to explain the relatively shallow IMFs out to  $100-130~M_{\odot}$  in dense clusters such as R136 and at the same time explain the relatively steep drop-off beyond this mass that is imposed by the lack of  $\sim 300~M_{\odot}$  stars in whole galaxies. One thing that would do this is protostar coalescence or enhanced protostar accretion in dense clusters but not elsewhere. Then any tendency for the galaxy-wide IMF to decrease at high mass from timing constraints or other causes can be compensated for by a slight overabundance of  $50-130~M_{\odot}$  stars in dense clusters. At the moment, there is no direct observation of protostar coalescence, and there are observations of massive stars in

the 30 Dor region that are not in the dense R136 core, so perhaps this possibility is unreasonable. Nevertheless, we consider the implications here briefly.

The observation of massive stars in extremely dense environments has often led to the idea that some of the largest stars may form by coalescence (Zinnecker 1986; Larson 1990; Price & Podsiadlowski 1995; Bonnell, Bate, & Zinnecker 1998; Stahler, Palla, & Ho 2000) or enhanced accretion (Larson 1978, 1982; Zinnecker 1982; Bonnell et al. 1997). Coalescence of pre-main-sequence stars requires extreme densities, even higher than what is observed in the densest clusters, but coalescence after accretional drag (Bonnell et al. 1998) or coalescence of protostars with extended disks (McDonald & Clarke 1995) may be possible. We showed in Paper II that even protostellar coalescence is likely to operate at a rate that scales with the square root of the local density, so the same basic model of the IMF should apply if stars routinely coalesce. The question is whether high-mass stars form more by coalescence than low-mass stars, because if they do, then the self-similarity assumed by the model would not be appropriate.

There is considerable evidence that the IMF is steeper in regions of lower density, but it is unclear how much of this evidence is free from selection and systematic effects. For example, local field-star IMFs, corrected for past birthrates and vertical Galaxy drift, typically give x in the range of 1.5 to 2, instead of  $\sim 1.35$  (Miller & Scalo 1979; Garmany, Conti, & Chiosi 1982; Humphreys & McElroy 1984; Scalo 1986; Blaha & Humphreys 1989; Basu & Rana 1992; Kroupa, Tout, & Gilmore 1993; Parker et al. 1998). The same steep IMF has been derived by Brown (1998) for local OB associations using *Hipparcos* positions for membership. LMC clusters in regions of low young-star density (J. K. Hill et al. 1994; R. S. Hill et al. 1995) and unclustered embedded stars in Orion (Ali & DePoy 1995) also have  $x \sim 1.5-2$ . An extreme example is the steep high-mass IMF  $(x \sim 4)$  in the field regions of the LMC and solar neighborhood (Massey et al. 1995b).

These relatively steep IMFs contrast the shallow IMFs often found in dense clusters and in normal cluster cores (Sagar et al. 1986; Sagar & Richtler 1991; Hunter et al. 1996a, 1996b, 1997; Selman et al. 1999; see review in Massey 1998, and Fig. 5 in Scalo 1998).

The problem with these measurements is that systematic effects are expected that would give such a density dependence even if the IMF at birth is the same on average. Mass segregation could make the IMFs in cluster cores shallower than at the edges, and differential drift between the shortlived, high-mass stars and the long-lived, low-mass stars that are leaving OB associations can overpopulate the field with low-mass stars. Unknown star formation histories in OB associations can also produce uncertainties in the upper mass IMF when corrected for evolved stars that are no longer present. There has been little modeling to quantify these effects, except for mass segregation, and that seems too slow to produce the observed flatness in the youngest cluster cores (e.g., Subramaniam, Sagar, & Bhatt 1993; Hillenbrand & Hartmann 1998; Fischer et al. 1998; Bonnell & Davies 1998). If this is the case, then massive stars would be born preferentially in the cores, perhaps as a result of the coalescence or accretion mechanisms discussed above.

Unfortunately, no one knows whether the IMFs at the faint edges of clusters are steep enough to compensate for the flat IMFs in the cores, giving the same IMF on average

as in nonclustered regions. Nor are the dynamics of a cluster during star formation known well enough to rule out mass segregation. For example, Giersz & Heggie (1996) found that mass segregation is virtually complete in only one core collapse time. This implies that if magnetic pressure is comparable to turbulent pressure in a cloud core, and so the initial motions of the stars are subvirial, leading to a significant and rapid collapse of stellar mass to the cloud core, then by the time we see a tight cluster in the core, the segregation of high and low stellar mass is finished.

The observation of dense clustering around high-mass stars has also been used to suggest that massive stars require coalescence or excessive accretion (Testi, Palla, & Natta 1999). However, the rarity of massive stars compared to low-mass stars implies that any region of star formation will generally form many low-mass stars along with the few high-mass stars. The correlation between the mass of the largest star and the density of the cluster, found in their survey (Fig. 7), is also a correlation between maximum star mass and cluster mass, considering that the cluster radius is about constant, as shown in their Figure 2. Then the maximum star mass should correlate with cluster density because of equation (1). For example, in Figure 7 of Testi et al. (1999), the cluster density varies from  $\sim 10^{3.3}$  pc<sup>-3</sup> for an O5 star to  $\sim 10^2$  pc<sup>-3</sup> for an A0 star. The masses of these stars vary from 40  $M_{\odot}$  at O5 to 4  $M_{\odot}$  at A0 (Mihalas & Binney 1981). Then the ratio of cluster densities,  $10^{1.3}$ which is also the ratio of cluster masses for a constant cluster radius, is equal to the ratio of stellar masses, 10, to a power equal to the Salpeter slope, x = 1.35, as predicted by equation (1). Thus, the trend between cluster density and maximum stellar mass found by Testi et al. (1999) could be the same as equation (1). A similar point was made by Bonnell & Clarke (1999).

The Trapezium cluster is a good place to check the coalescence model. It is extremely dense, containing  $\sim 5000$  stars pc $^{-3}$  (Prosser et al. 1994) or more (McCaughrean & Stauffer 1994), and it has several massive stars with close companions. According to Hillenbrand (1997), Hillenbrand et al. (1998), and Weigelt et al. (1999), the Trapezium primary stars  $\Theta^1 A$ , B, C, and D have masses of 20, 7, 45, and 17  $M_{\odot}$ , respectively, and they have 2, 4, 0, and 1 close, lower mass companions.

The number of massive stars expected in the Trapezium cluster can be determined from an IMF extrapolation of the low-mass star count. Palla & Stahler (1999) fit the IMF for stars below 10  $M_{\odot}$  to the recent average IMF derived by Scalo (1998), which has slopes of -0.2 between  $0.1~M_{\odot}$  and  $1~M_{\odot}$ , and -1.3 between  $1~M_{\odot}$  and  $10~M_{\odot}$ , and -1.3 between  $10~M_{\odot}$  and  $100~M_{\odot}$ , considering logarithmic intervals of mass. For a total number of Trapezium stars equal to 258 in the mass range from  $10^{-0.4}~M_{\odot}$  to  $10^{0.8}~M_{\odot}$ , the resulting IMFs for these mass ranges are  $380M^{-0.2}$ ,  $380M^{-1.7}$ , and  $150M^{-1.3}$ , respectively. If we fit the 258 stars to a single slope of -1.3, then the result is  $100M^{-1.3}$  for all masses.

The integral of these IMFs over mass from  $30\,M_\odot$  to  $100\,M_\odot$  is 1.1 for the Scalo (1998) fit and 0.7 for the constant slope of -1.3. This implies that the presence of a single 45  $M_\odot$  star in the Trapezium cluster is not unusual. Similarly, the expected number of stars between 10 and  $20\,M_\odot$  is 3.4 for the Scalo fit and 2.2 for the constant slope. These compare well with the observed number of 2. Thus, the massive stars in the Trapezium cluster were just as likely to form there as anywhere else with a normal IMF, including a

much more dispersed cluster or association having the same total mass. There is no need to postulate that the 45  $M_{\odot}$  star or any of the other massive stars required the companions to be there or that they formed by coalescence.

In summary, the IMF slope at intermediate to high mass is about the same in clusters and associations spanning a factor of ~200 in density (Hunter et al. 1997; Massey & Hunter 1998; Luhman & Rieke 1998). This suggests that theories of the IMF should not rely on processes that require high densities, including coalescence, protostar interactions, or enhanced accretion, to give a slope in the range of  $x \sim 1-1.5$ . The lowest density regions of star formation have no protostellar coalescence, yet their intermediate- to high-mass IMFs look about the same as in dense clusters, to within the statistical noise, and aside from the expected effects of mass segregation, differential dispersal, and post-main-sequence evolution. While this apparent IMF uniformity would seem to make the theory of the IMF simpler, it is a problem when it comes to understanding the upper mass limit. Why is the maximum stellar mass not larger in extended regions of star formation than it is in the most massive dense clusters, where  $\sim 130 \, M_{\odot}$  stars are observed among only  $\sim 10^4 M_{\odot}$  of other stars? The upper mass cutoff that is necessary seems to be sharper than what is likely to arise from self-similar turbulence and timing constraints. Perhaps self-limitation of maximum stellar mass is the only alternative.

## 6. VARIATIONS IN $M_1$

If the proposed top-heavy IMF in starburst regions (Rieke et al. 1980) is the result of a higher minimum mass,  $M_{\rm I}$  (Paper I), then, without self-limitation, there should be unusually massive stars in these regions as well, according to the IMF model presented here. Stars with initial masses of  $\sim 10^3~M_{\odot}$  at their Eddington limits might be possible, forming from the larger clumps in normal (but warm) clouds. The presence of such stars would have important consequences, such as a higher luminosity-to-mass ratio for a stellar population, a higher supernova or gamma-ray burst rate per unit gas mass, and stronger winds and galactic outflows per unit gas mass, as well as unusual abundance ratios from nuclear processing. The supermassive stars themselves might look odd too, having unusual stellar spectra indicating rapid mass loss, or possibly appearing primarily in the infrared (Hoyle, Solomon, & Woolf 1973). The post-main-sequence stars could include the proposed "warmers" (Terlevich & Melnick 1985) or an unusual abundance of W-R stars compared to other stars, as in W-R galaxies (e.g., Ohyama, Taniguchi, & Terlevich 1997). Any observation of an IMF that is shifted entirely to larger masses, from the lower mass turnover to an upper mass turndown, would support the IMF model developed here.

The consequences of a systematic shift toward lower stellar masses resulting from a smaller value of  $M_{\rm J}$  would also be important. This may allow brown dwarfs to form without normal stars at ultralow temperatures (Elmegreen 1999c) or extremely high pressures (Fabian 1994).

#### 7. SUMMARY

Turbulent and young-star mixing of star-forming gas can limit the masses of the largest stars by continuously forming smaller scale structures and smaller stars inside all of the larger regions. A model for this mixing uses a time constraint for star formation at every mass M that decreases exponentially with the turbulent crossing time at that mass, normalized to the crossing time at the lowest significant mass, which is taken to be  $M_J$ . The results are presented in the form of a random-sampling simulation in Figure 1 and with the analytical form in Figure 2.

The simulations suggest that the timing constraint makes the slope of the IMF slightly steeper by eliminating a small fraction of the intermediate-mass stars compared to the low-mass stars, but the greatest effect is for the high-mass stars, which are significantly reduced compared to the Salpeter function. The result is a Salpeter IMF starting at a lower mass limit of several  $M_{\rm J}$  and continuing to higher masses, with a gradual turn-down in the upper mass range. At masses of about  $10^3 M_{\rm J}$ , the turn-down amounts to a decrease by a factor of  $\sim 10$ .

These results explain the lack of young stars larger than

several hundred solar masses in giant star-forming regions or spiral-arm pieces where random sampling from the Salpeter function alone would predict several such stars. The results do not explain how dense clusters such as R136 can have a Salpeter function out to  $\sim\!130~M_{\odot}$  while a whole galaxy does not have stars greater than 300  $M_{\odot}$ . That is, the required falloff at masses greater than 130  $M_{\odot}$  is too steep for the assumed timing constraint to explain. There are several ways out of this problem, discussed in § 4, including an excess of high-mass stars in dense clusters, where the Salpeter IMF is commonly measured, and self-limitation of the maximum star mass.

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