(a) $P(\mu, \tau^2, \mu_0 \mid y) \propto P(y \mid \mu, \tau^2, \cancel{\mu_0})$

$$\propto P(y \mid \mu, \tau^2)\, P(\mu \mid \mu_0)\, P(\mu_0)\, P(\tau^2)$$

$$\propto P(y \mid \mu, \tau^2) \prod_{j=1}^{J=5} P(\mu_j \mid \mu_0)\, P(\mu_0)\, P(\tau^2)$$

$$\propto \prod_{j=1}^{J=5} \prod_{i=1}^{n_j=100} N(\mu_j, \tau^2) \times \prod_{j=1}^{J=5} N(\mu_0, 25^2)$$

$$\times\, N(0, 50^2) \times \text{Inv-}\chi^2(1, 0.5^2)$$

(b) $P(\mu_j \mid y, \cancel{\mu_0}, \sigma^2) \propto P(\mu_j \mid y_{\cdot j}, \mu_0, \sigma^2)$

$$\propto P(y_{\cdot j} \mid \mu_j, \sigma^2)\, P(\mu_j \mid \mu_0)$$

$$\propto \prod_{i=1}^{100} N(y_{ij} \mid \mu_j, \sigma^2) \times N(\mu_0, 25^2)$$

$\Updownarrow$ same, different notation

$$\propto \prod_{i=1}^{100} N(\mu_j, \sigma^2) \times N(\mu_0, 25^2)$$

$$\prod_{i=1}^{100} e^{-(\frac{1}{2\sigma^2}(y_{ij}-\mu_j)^2} \times e^{(-\frac{1}{2\tau_0^2}(\mu_j-\mu_0)^2)}$$

$\Downarrow$

$$e^{(-\frac{1}{2}(\frac{1}{\tau_0^2}(\mu_j-\mu_0)^2+\frac{1}{\sigma^2}\sum_{i=1}^{n=100}(y_{ij}-\mu_j)^2)}$$

$\Uparrow$ estimating a normal mean with known variance and n observation.

$\Downarrow$ base on the result of textbook P42.

$$P(\mu_j \mid \sigma^2, y_1, \dots y_n) = N(\mu_j \mid \mu_n, \tau_n^2)$$

where $\mu_n = \dfrac{\frac{1}{\tau_0^2}\mu_0 + \frac{n_j \bar{y}_{\cdot j}}{\sigma^2}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}$ and

$$\tau_n^2 = \frac{1}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}$$

(c) $P(\sigma^2 \mid y, \mu, \mu_0) \propto P(\sigma^2, y, \mu, \mu_0)$

$$\propto P(y \mid \sigma^2, \mu, \cancel{\mu_0})\, P(\sigma^2)$$

$$\propto \prod_{j=1}^{J} \prod_{i=1}^{n_j} N(y_{ij} \mid \mu_j, \sigma^2) \times \text{Inv-}\chi^2(1, 0.5^2)$$

normal distribution with known mean and unknown variance.

book P43

we have

$$\sigma^2 \mid y \sim \text{Inv-}\chi^2\left(\nu_0 + n,\ \frac{\nu_0 \sigma_0^2 + n v}{\nu_0 + n}\right)$$

(d) $P(\mu_0 \mid \mu, y, \sigma^2) = P(\mu_0 \mid \mu) \propto P(\mu \mid \mu_0) P(\mu_0)$

$$P(\mu \mid \mu_0) = \prod_{j=1}^{J=5} N(\mu_0, 25^2) \; N(0, 50^2)$$

$$\propto e^{\{-\frac{1}{2\times 25^2} \sum_{j=1}^{5} (\mu_j - \mu_0)^2\}} \times e^{-\frac{1}{2\times 50^2}(\mu_0 - 0)^2}$$

$$\propto e^{\{-\frac{1}{2\tau_0^2} \sum_{j=1}^{5} (\mu_j - \bar{\mu})^2 + n(\bar{\mu} - \mu_0) - \frac{1}{2\times 50^2}(\mu_0^2)\}}$$

$$\propto e^{(-\frac{1}{2\tau_0^2}((n-1)^2 s^2 + n(\bar{\mu} - \mu_0) - \frac{1}{2\times 50^2}(\mu_0^2)))} = N(\mu_0 \mid \mu_n, \tau_n^2)$$

$\boxed{\text{here } n=5}$

known σ^2 & n data. so use result at P42 at text book

$$\mu_n = \frac{\frac{1}{\tau_0^2}\mu_0 + \frac{n}{\sigma^2}\bar{\mu}}{\frac{1}{\tau_0^2} + \frac{n}{\sigma^2}}$$

and

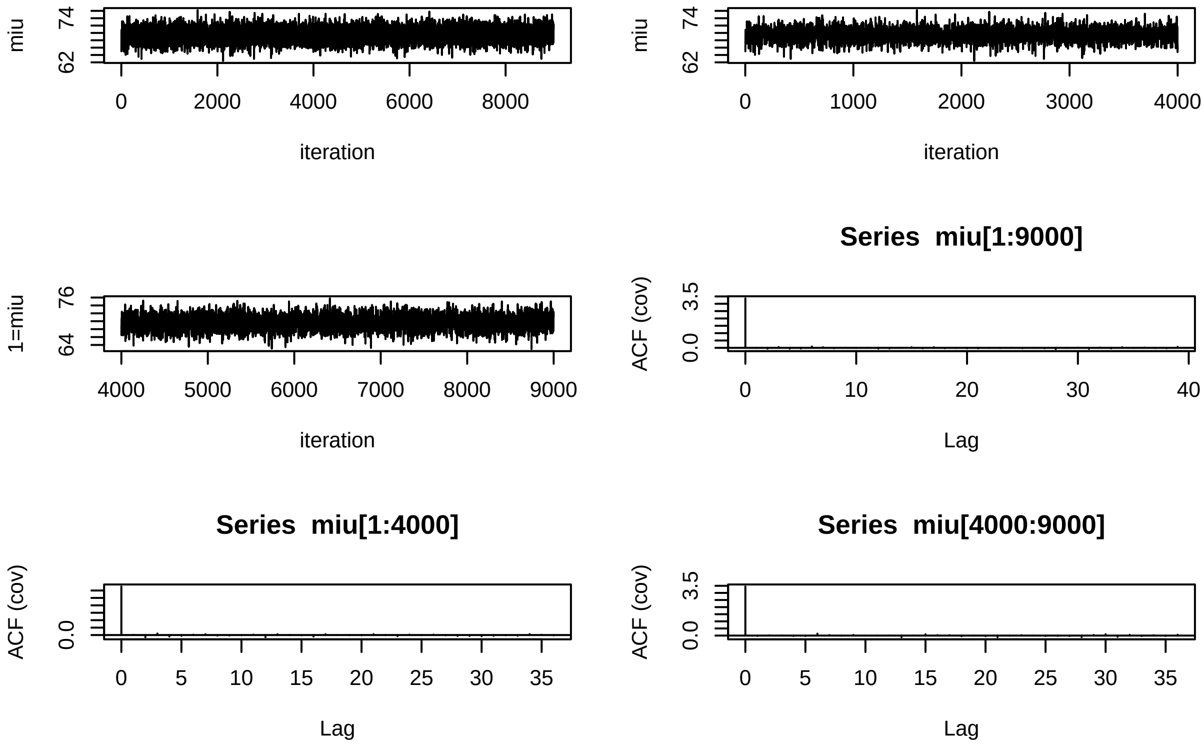
$$\frac{1}{\tau_n^2} = \frac{1}{\tau_0^2} + \frac{n}{\sigma^2}$$

$n=5$

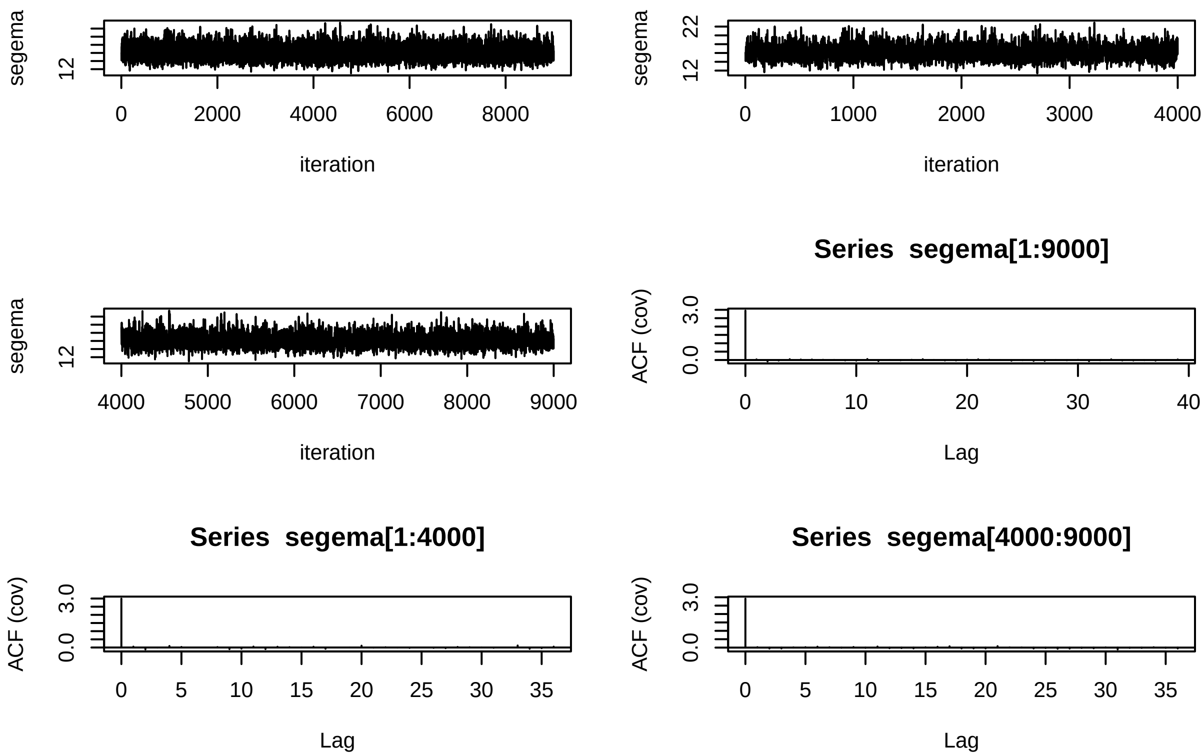
Assignment5

```r
##Q3

##(1) At iteration t, the Gibbs sampler draws Thetaj^t~p(thetaj|theta-j^(t-1),y),
##    for j=1,...,d, where theta-j^(t-1) represents all the components of theta,
##    expect for thetaj, at their current value:
##    Theta-j^(t-1)=(theta1^(t),..., thetaj-1^(t),thetaj+1^(t-1),...,thetad^(t-1))

## Based on the lecture note, we know that miu|,y ~
## N(((miu0/Tao^2)+(n*y-bar/segema^2))/ (1/Tao^2)+(n/segema^2)), 1/(1/tao^2+n/segema^2)))

##  segema^2| miu,y ~ Inv-X^2 (v+n, ((v*segema^2+nv)/(v+n))), v=1/n sum(yi-miu^t)^2

##  Initializing (miu^(0),segema^2(0))=(65,16)

##  For each iteration/update,
##  miu^(t) ~ miu| segema^(t-1),y
##  segema^2(t) ~segema^2| miu^t, y

## and we have data y1 = 70, y2=75 and y3=72.

## Code for sampling
library("invgamma") ## I will use rinvchisq in this package to sample
##                     from inverse chi-square distribution
y1<-70
y2<-75
y3<-72
ybar<-mean(c(70,75,72))
ysum<-sum(c(70,75,72))
rho<-0.8
N_iter<-9000


miu<-rep(0,N_iter)
segema<-rep(0,N_iter)
X<-rep(0,N_iter)
miu[1]<-65
segema[1]<-16

for(t in 2:N_iter){
   miu1<-((65/9+ysum/segema[t-1])/(1/9+3/segema[t-1]))
   segema1<-(1/(1/9+3/segema[t-1]))
   miu[t]<-rnorm(1,miu1,sqrt(segema1))

   VV<-function(x,x0){
        return(sum((x-x0)^2))}
```

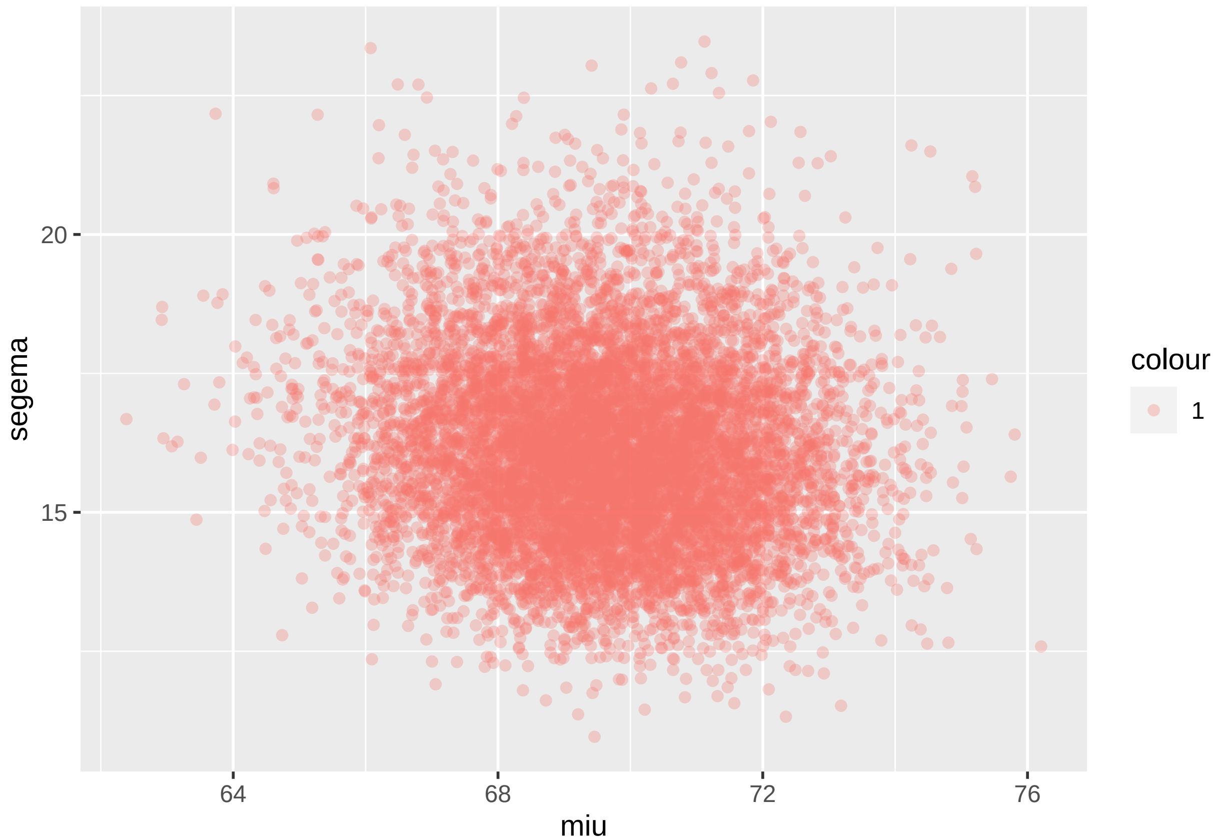
```r
  V<-VV(c(70,75,72),miu[t])

  X<-rchisq(1,178)
  segema2<-(175*16+V)/(178)
  segema[t]<-((178*segema2)/X)


}

### Yes, the chain mixing well, there is no visual evidence of high autocorrelation.
# make a "trace plot" of the draws of theta

## Yes,the Markov Chain appear to converged after 4000 burn-in samples.(because of the
## low correlation)
## I will used first 4000 as burn-in data.
## For the trace plot at 4000:9000 iterations, there is no clear upward/downward trend
## so we conclude the chain has converged.
## The convergence is better for segema than for miu.

## With the zoom in of both trace and afc plot of miu and segema

## We could see the correlation at 4000:9000 iterations and at 1:4000 iterations
## all have very low autocorrelation and less correlation at 4000:9000 iterations.


## There is no visual evidence of high autocorrelation.But somehow bit correlated
## The autocorrelation drop down with the iterations go on.

par(mfrow=c(3,2))
plot(x=c(1:9000),y=miu[1:9000],type="l",xlab="iteration",ylab="miu")
plot(x=c(1:4000),y=miu[1:4000],type="l",xlab="iteration",ylab="miu")
plot(x=c(4000:9000),y=miu[4000:9000],type="l",xlab="iteration",ylab="1=miu")
acf(miu[1:9000], type = "covariance")
acf(miu[1:4000], type = "covariance")
acf(miu[4000:9000], type = "covariance")
```




```r
par(mfrow=c(3,2))
plot(x=c(1:9000),y=segema[1:9000],type="l",xlab="iteration",ylab="segema")
plot(x=c(1:4000),y=segema[1:4000],type="l",xlab="iteration",ylab="segema")
plot(x=c(4000:9000),y=segema[4000:9000],type="l",xlab="iteration",ylab="segema")
acf(segema[1:9000], type = "covariance")
acf(segema[1:4000], type = "covariance")
acf(segema[4000:9000], type = "covariance")
```




```r
### (3) two-demensional plot for miu and segema.
dataa<-as.data.frame(cbind(miu,segema))
library("ggplot2")

colnames(dataa)[1] <- "miu"
colnames(dataa)[2] <- "segema"
ggplot()+geom_point(data =dataa ,aes(miu,segema, color = '1'), alpha = 0.3)
```



segema
20
15
64
68
72
76
miu
colour
1