Supplementary material for the letter “Reduced Nodal Admittance Matrix Method for Probabilistic GIC Analysis in Power Grids”

In the main text of the letter, the GICs in power grids are calculated based on sparse matrix factorization. In this supplementary material, we compared the efficiency of GIC computation based on sparse and full matrix factorization, as shown in Table 2. The best results of each method are marked in red. The results show that, except for large-scale power grids (although the efficiency remains very similar in relative terms), the BAM method outperforms the NAM method if full matrix factorization is used.

Table 2 Calculation time (s) of four methods using sparse and full matrix factorization (using **1-sec** geoelectric field data, and the number of time instants is 3,600 × 24 = 86,400)*

Power grid test cases	Number of Monte Carlo samples	Calculation time (s) using **sparse** matrix factorization				Calculation time (s) using **full** matrix factorization			
		LP	NAM	BAM	RNAM	LP	NAM	BAM	RNAM
EPRI-21	1000	29.2	11.5	20.9	8.5	4.9	6.3	4.6	4.4
IEEE 118-GMD	1000	397	164	253	99	111	133	78	72
Sanhua UHV-EHV	1000	478	210	353	141	150	172	116	106
ACTIVSg2000	2000	14620	6329	11012	4269	15429	16152	6435	6298

* MATLAB provides two built-in functions `lu` and `decomposition` for LU factorization, and two built-in functions `chol` and `decomposition` for Cholesky factorization. The calculation time here refers to the best result of different functions for matrix factorization in MATLAB R2020b software.

In addition, we analyzed the influence of the different numbers of time instants on the calculation time of the four methods. Taking the largest power grid test case as an example, we use the 1-min geoelectric field data as input, and the calculation time of the four methods is shown in Table 3. The best results of each method are marked in red. It shows that the ranking of computational efficiency of the BAM and NAM methods may also vary under inputs with different number of time instants. And the RNAM method has the fastest computational speed in both sparse and full matrix factorization simulation configurations.

Table 3 Calculation time (s) of different matrix factorization methods for power grid test case ACTIVSg2000 (using **1-min** geoelectric field data, and the number of time instants is 60 × 24 = 1440)

Simulation configurations	Calculation time (s)			
	LP	NAM	BAM	RNAM
Sparse matrix factorization using built-in function “`decomposition`”	368	110	548	74
Sparse matrix factorization using built-in function “`lu`”/“`chol`”	254	246	193	158
Full matrix factorization using built-in function “`decomposition`”	604	478	181	152
Full matrix factorization using built-in function “`lu`”/“`chol`”	646	471	4203	150

We also compared the storage of the design matrices of the four methods under the simulation configuration of sparse and full design matrices. As shown in Table 4, the storage required for full matrix inversion is much larger than that for sparse matrix inversion, especially when the size of design matrix is large.

Table 4 Comparison of storage requirement of full and sparse design matrix

Power Grid Test Cases	Storage of full design matrix (kB)				Storage of sparse design matrix (kB)				Ratio of storage of full matrix to storage of sparse matrix			
	LP	NAM	BAM	RNAM	LP	NAM	BAM	RNAM	LP	NAM	BAM	RNAM
EPRI-21	2.26	2.26	0.95	0.95	1.36	1.09	1.34	0.89	1.7	2.1	0.7	1.1
IEEE 118-GMD	395.5	395.5	108.8	108.8	17.7	14.2	16.3	10.2	22	28	7	11
Sanhua UHV-EHV	608.1	608.1	270.3	270.3	19.8	15.5	16.4	11.9	31	39	16	23
ACTIVSg2000	6.1E+04	6.1E+04	1.9E+04	1.9E+04	177.1	148.7	157.7	120.2	346	412	119	156

Moreover, the condition numbers of the design matrices of the four methods are compared in Table 5, which may affect the numerical stability of the solution. It can be seen that the proposed RNAM method has the smallest condition number.

Table 5 Condition numbers of the design matrices of the four methods

Power grid test cases	Condition number of the design matrix			
	LP	NAM	BAM	RNAM
EPRI-21	7.81E+12	421.89	324.86	276.99
IEEE118GMD	3.83E+12	652.27	2544.03	84.34
Sanhua UHV-EHV	2.92E+12	184.41	92.23	91.71
ACTIVSg2000	1.90E+13	3.46E+03	4.10E+04	1.42E+03