Exercise: Finding the area under the curve using scissors and a scientific weight

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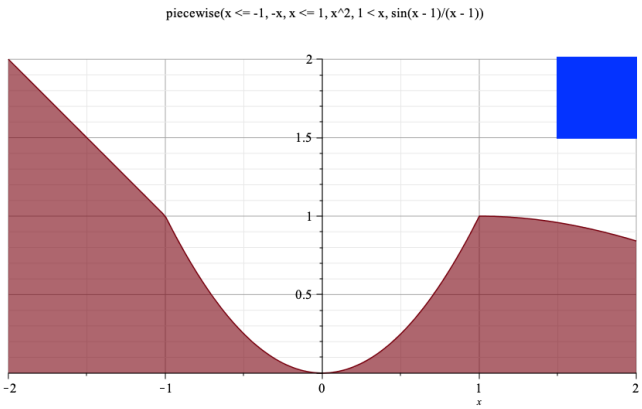
In this exercise the students are asked to determine the definite integral of different functions using only scissors and a scientific weight (the weight must be fairly precise since we need to weight small pieces of paper).

What you need:

- Some functions printed on paper (see appendix for examples). **Important! The axis must be scaled equally!**
- Some scissors
- A least one weight with good precision (minimum 0.1 g accuracy)

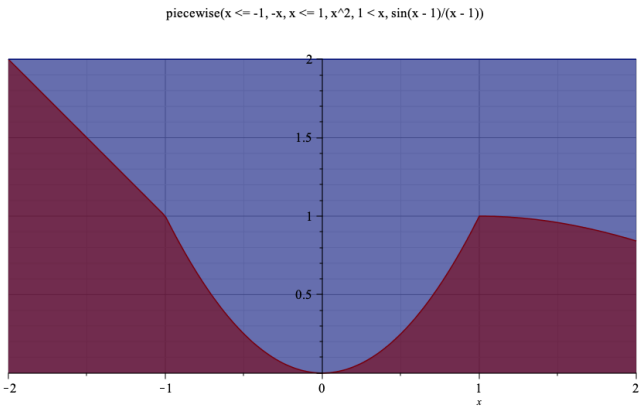
Solution(s): There exist multiple ways of solving this problem. An idea could be to look at how many "reference squares" there is in the area

$$\int_a^b f(x)\,\mathrm{d}x = \frac{\text{Weight-of-area}}{\text{Weight-of-reference-square}} \cdot \text{Area-of-reference-square} = \frac{\square}{\square} \cdot \text{Area}\,(\square) \tag{0.1}$$



Another way of solving this problem is to look at the fraction of the grid that the area consumes.

$$\int_a^b f(x)\,\mathrm{d}x = \frac{\text{Weight-of-area}}{\text{Weight-of-full-grid}} \cdot \text{Area-of-full-grid} = \frac{\square}{\square + \square} \cdot \text{Area}\,(\square + \square) \tag{0.2}$$



A fun note about this exercise is that they do not need to know what function generated the plot in order to compute the integral. This can be used to start a discussion with the class about "black-box" functions (and the integral of them). This highlights why numerical integration is needed for some applications.

Appendix A

Some examples of functions

$$f_1(x) = \sin(x) \tag{A.1}$$

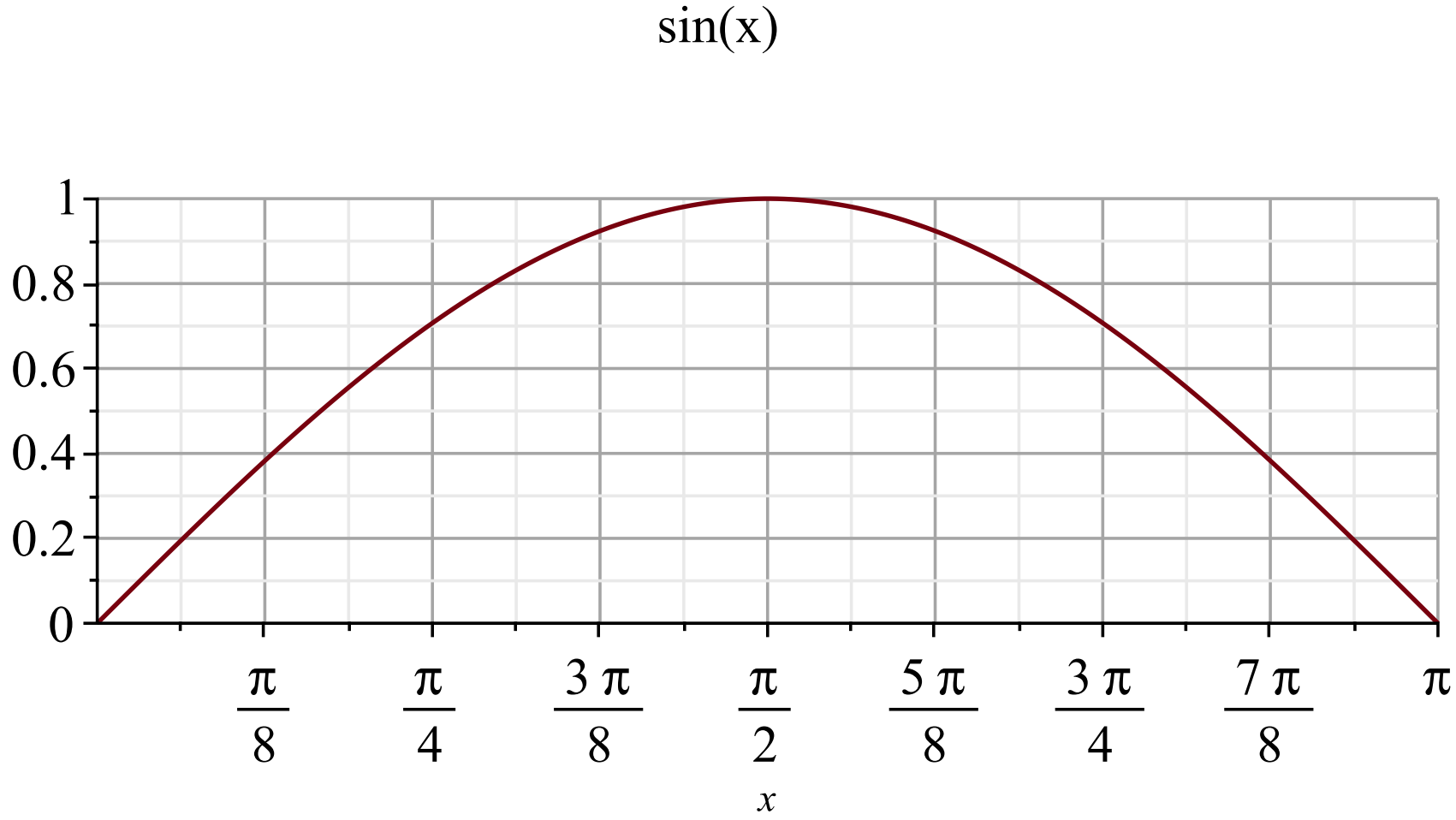
$$f_2(x) = \frac{\sin(x)^2}{2} \tag{A.2}$$

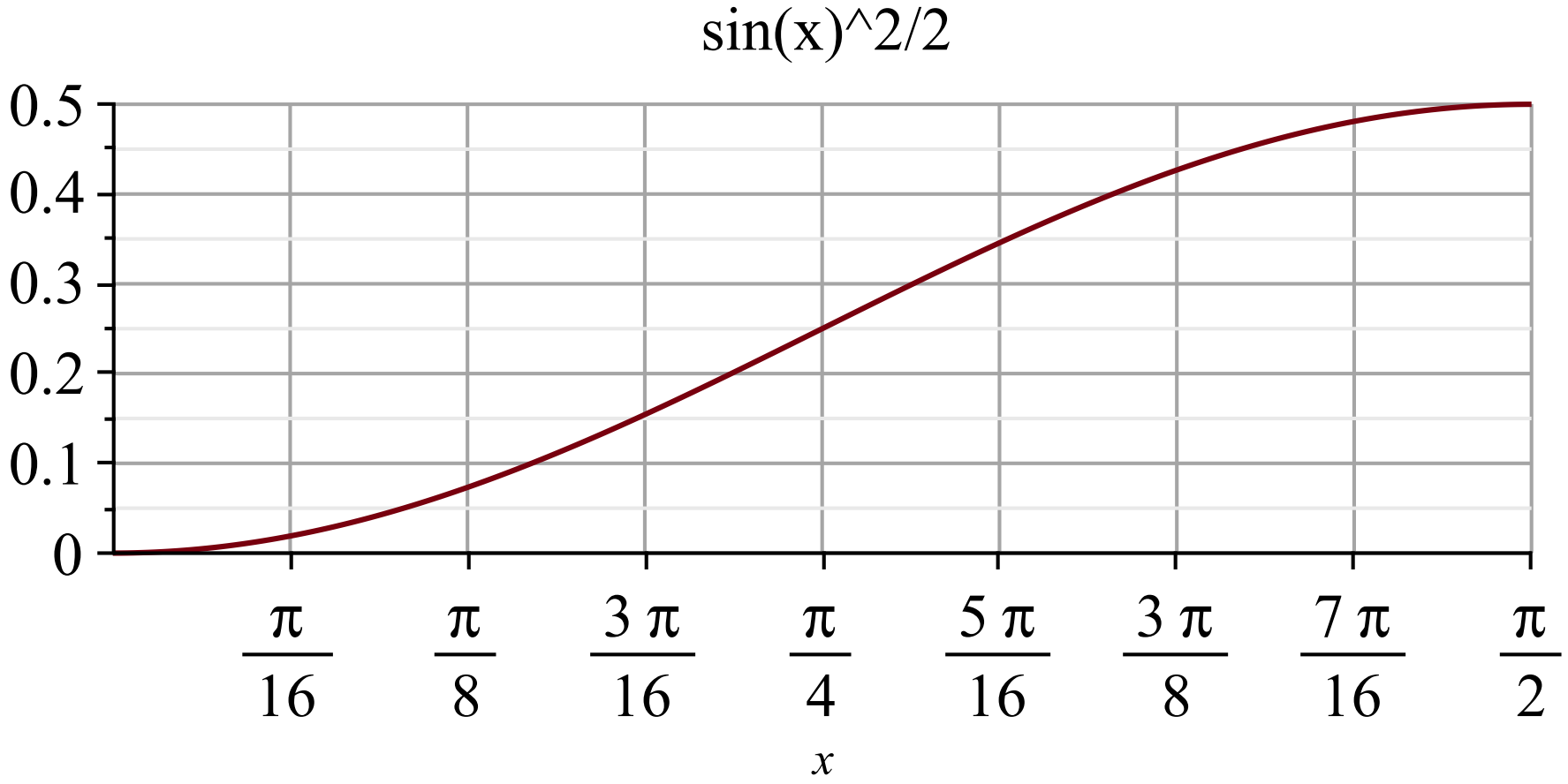
$$f_3(x) = x^2 \tag{A.3}$$

$$f_4(x) = \begin{cases} x^2, & x < 0 \\ \sin(x), & x \geq 0 \end{cases} \tag{A.4}$$

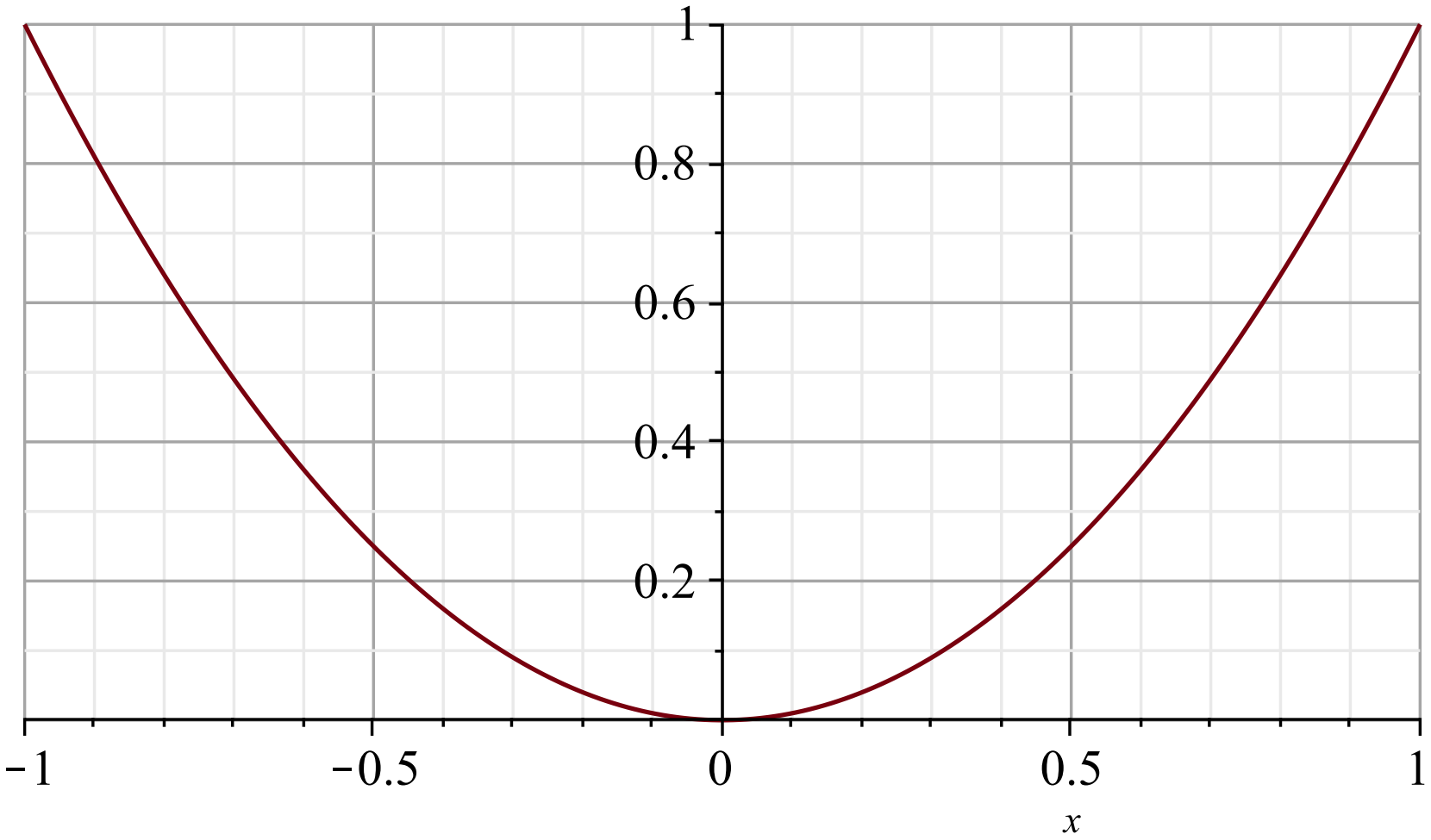
$$f_5(x) = \begin{cases} -x, & x < -1 \\ x^2, & -1 \leq x \leq 1 \\ \frac{\sin(x-1)}{x-1}, & x > 1 \end{cases} \tag{A.5}$$

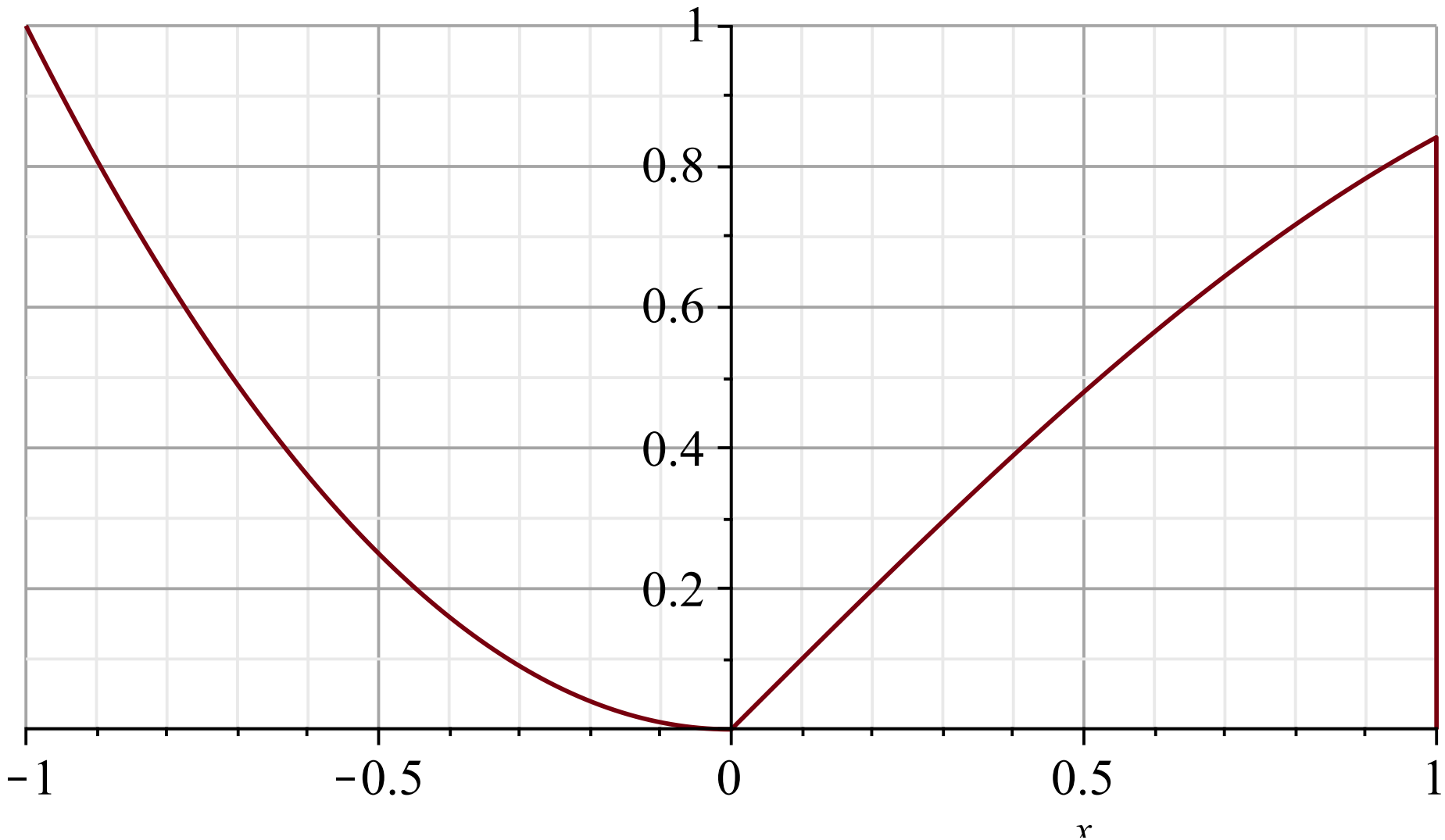
Each of the above functions is plotted in `Maple` (see next 5 pages). If you want to make your own functions then i stress again: **The axis must be scaled equally!**

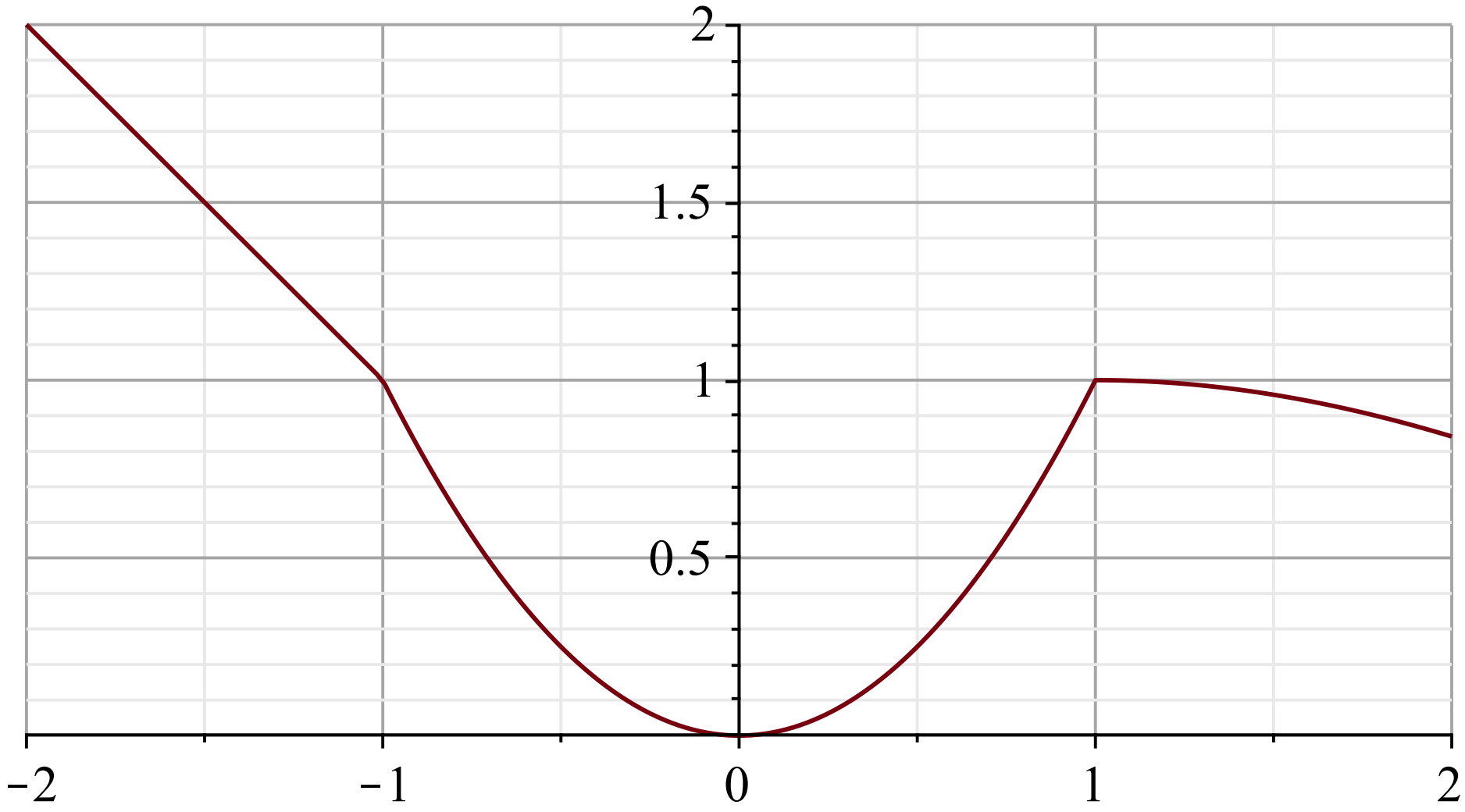

sin(x)
1
0.8
0.6
0.4
0.2
0
π/8
π/4
3π/8
π/2
5π/8
3π/4
7π/8
π
x


sin(x)^2/2
0.5
0.4
0.3
0.2
0.1
0
π/16
π/8
3π/16
π/4
5π/16
3π/8
7π/16
π/2
x

x^2




1
0.8
0.6
0.4
0.2
−1
−0.5
0
0.5
1
x


2
1.5
1
0.5
−2
−1
0
1
2