30/11/2018

End Semester Examination

Winter-2018

Summer-2022

1. Solve the partial differential equation

$$(x^2-yz)p+(y^2-zx)q=z^2-xy$$

→ Lagranges linear form $Pp+Qq=R$

Here $P=x^2-yz$, $Q=y^2-zx$, $R=z^2-xy$

Lagranges Auxiliary equation are $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$

$$\therefore \frac{dx}{x^2-yz}=\frac{dy}{y^2-zx}=\frac{dz}{z^2-xy}$$

$$\frac{dx-dy}{(x^2-yz)-(y^2-zx)}=\frac{dy-dz}{(y^2-zx)-(z^2-xy)}=\frac{xdx+ydy+zdz}{x^3+y^3+z^3-3xyz}=\frac{dx+dy+dz}{x^2+y^2+z^2-xy-yz-zx}$$

1st two ratio.

$$\frac{d(x-y)}{(x-y)(x+y+z)}=\frac{d(y-z)}{(y-z)(x+y+z)}$$

$$\frac{d(x-y)}{(x-y)}-\frac{d(y-z)}{(y-z)}=0$$

$$\log(x-y)-\log(y-z)=\log c_1$$

$$\boxed{\frac{x-y}{y-z}=c_1}$$

3rd & last ratio.

$$\frac{xdx+ydy+zdz}{(x+y+z)(x^2+y^2+z^2-xy-yz-zx)}=\frac{dx+dy+dz}{x^2+y^2+z^2-xy-yz-zx}$$

$$\frac{xdx+ydy+zdz}{x+y+z}=d(x+y+z)$$

$$xdx+ydy+zdz-(x+y+z)d(x+y+z)=0$$ after integration

$$\frac{x^2}{2}+\frac{y^2}{2}+\frac{z^2}{2}-\frac{(x+y+z)^2}{2}=c_2$$

$$\boxed{xy+yz+zx=c_2}$$

$\therefore$ General solution is $\phi(c_1,c_2)=0$

$$\boxed{\phi\left(\frac{x-y}{y-z},\ xy+yz+zx\right)=0}$$

May 2019

(2) Solve: $xz(z^2+xy)p - yz(z^2+xy)q = x^4$

$\longrightarrow$ Lagranges linear form $Pp+Qq = R$

Here $P = xz(z^2+xy)$, $Q = -yz(z^2+xy)$, $R = x^4$

Lagranges auxiliary eqn are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{xz(z^2+xy)} = \frac{dy}{-yz(z^2+xy)} = \frac{dz}{x^4}$$

1st two ratio

$$\frac{dx}{xz(z^2+xy)} = \frac{dy}{-yz(z^2+xy)}$$

$\frac{dx}{x} = -\frac{dy}{y}$ Integrate

$\int\frac{dx}{x} = -\int\frac{dy}{y} + \log c_1$

$\log x + \log y = \log c_1$

$\log xy = \log c_1$

$\boxed{xy = c_1}$

1st & 3rd ratio.

$$\frac{dx}{xz(z^2+xy)} = \frac{dz}{x^4}$$

$\frac{x^4\,dx}{x} = z(z^2+xy)\,dz$

$x^3dx - z^3dx - zc_1dz = 0$ Integrate

$\int x^3dx - \int z^3dx - c_1\int z\,dz = c_2$

$\frac{x^4}{4} - \frac{z^4}{4} - c_1\frac{z^2}{2} = c_2$

$x^4 - z^4 - 2c_1z^2 = c_2$

$\boxed{x^4 - z^4 - 2xyz^2 = c_2}$

$\therefore$ General solution is $\phi(c_1, c_2) = 0$

$$\boxed{\phi(xy,\ x^4 - z^4 - 2xyz^2) = 0}$$

Page No.:
Date:

winter-2019

Solve: $pz - qz = z^2 + (x+y)^2$

$\rightarrow$ Lagranges linear form $Pp + Qq = R$

Here $P = z$, $Q = -z$, $R = z^2 + (x+y)^2$

Lagranges auxiliary eq^n are

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{z} = \frac{dy}{-z} = \frac{dz}{z^2 + (x+y)^2}$$

1st two ratio.

$$\frac{dx}{z} = \frac{dy}{-z}$$

$$dx = -dy$$

$dx + dy = 0$. Integrate

$$\int dx + \int dy = c_1$$

$$\boxed{x + y = c_1}$$

1st & 3rd ratio

$$\frac{dx}{z} = \frac{dz}{z^2 + (x+y)^2}$$

$$dx = \frac{z\,dz}{z^2 + c_1^2} \quad \text{Integrate.}$$

$$\int dx - \frac{1}{2}\int \frac{2z\,dz}{z^2 + c_1^2} = c_2$$

$$x - \frac{1}{2}\log|z^2 + c_1^2| = c_2$$

$$x - \frac{1}{2}\log|z^2 + (x+y)^2| = c_2$$

$\therefore$ General solution is

$$\phi(c_1, c_2) = 0$$

$$\boxed{\phi\left(x+y,\ x - \frac{1}{2}\log|z^2 + (x+y)^2|\right) = 0}$$

may 2019

From the partial differential equation by eliminating arbitrary function f from $f(x+y+z, x^2+y^2+z^2)=0$

→ Let $u = x+y+z$ $v = x^2+y^2+z^2$

$\frac{\partial u}{\partial x} = 1$ $\frac{\partial v}{\partial x} = 2x$, $\frac{\partial v}{\partial y} = 2y$, $\frac{\partial v}{\partial z} = 2z$

$\frac{\partial u}{\partial y} = 1$

$\frac{\partial u}{\partial z} = 1$

$\therefore f(u,v) = 0$

Diff partially w.r.t. x

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial x}\right) = 0$$

$$\frac{\partial f}{\partial u}(1+p) + \frac{\partial f}{\partial v}(2x+2zp) = 0$$

$$\frac{\partial f}{\partial u}(1+p) = -\frac{\partial f}{\partial v}(2x+2zp)$$

$$\frac{\frac{\partial f}{\partial u}}{\frac{\partial f}{\partial v}} = \frac{-(2x+2zp)}{(1+p)} \quad \text{---} \ (1)$$

Diff partially w.r.t. y.

$$\frac{\partial f}{\partial u}\left(\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\frac{\partial z}{\partial y}\right) + \frac{\partial f}{\partial v}\left(\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial y}\right) = 0$$

$$\frac{\partial f}{\partial u}(1+q) + \frac{\partial f}{\partial v}(2y+2zq) = 0$$

$$\frac{\partial f}{\partial u}(1+q) = -\frac{\partial f}{\partial v}(2y+2zq)$$

$$\frac{\frac{\partial f}{\partial u}}{\frac{\partial f}{\partial v}} = \frac{-(2y+2zq)}{(1+q)} \quad \text{---} \ (2)$$

$\therefore$ From Eqn 1 & 2, we have

$$\frac{2y+2zq}{1+q} = \frac{2x+2zp}{1+p}$$

$$(1+p)(2y+2zq) = (1+q)(2x+2zp)$$

$$2y+2zq+2py+\cancel{2zpq} = 2x+2zp+2qx+\cancel{2zpq}$$

$$y+zq+py = x+zp+qx$$

$$py-zp+zq-qx = x-y$$

$$\boxed{(y-z)p+(z-x)q = x-y}$$

which is a partial differential equation of first order.

winter-2019

From the partial differential equation by eliminating arbitrary function f from $f(x^2+y^2+z^2,\ 3x+5y+7z)=0$.

→

Let $u = x^2+y^2+z^2$ $\qquad v = 3x+5y+7z$

$\frac{\partial u}{\partial x} = 2x,\ \frac{\partial u}{\partial y} = 2y,\ \frac{\partial u}{\partial z} = 2z \qquad \frac{\partial v}{\partial x} = 3,\ \frac{\partial v}{\partial y} = 5,\ \frac{\partial v}{\partial z} = 7$

$\therefore\ f(u,v) = 0$.

Diff partially w.r.t. x.

$$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}\right] + \frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial x}\right] = 0$$

$$\frac{\partial f}{\partial u}\left[2x+2zp\right] + \frac{\partial f}{\partial v}\left[3+7p\right] = 0$$

$$\frac{\partial f}{\partial u}(2x+2zp) = -\frac{\partial f}{\partial v}(3+7p)$$

$$\frac{\frac{\partial f}{\partial u}}{\frac{\partial f}{\partial v}} = -\frac{(3+7p)}{(2x+2zp)} \quad \text{—— } ①$$

Diff partially w.r.t. y.

$$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y}\right] + \frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\cdot\frac{\partial z}{\partial y}\right] = 0$$

$$\frac{\partial f}{\partial u}(2y+2zq) + \frac{\partial f}{\partial v}(5+7q) = 0$$

$$\frac{\partial f}{\partial u}(2y+2zq) = -\frac{\partial f}{\partial v}(5+7q)$$

$$\frac{\frac{\partial f}{\partial u}}{\frac{\partial f}{\partial v}} = -\frac{(5+7q)}{(2y+2zq)} \quad \text{—— } ②$$

$\therefore$ From eqn ① & ②.

$$\frac{5+7q}{2y+2zq} = \frac{3+7p}{2x+2zp}$$

$$(5+7q)(2x+2zp) = (3+7p)(2y+2zq)$$

$$10x+10zp+14xq+\cancel{14zpq} = 6y+6zq+14yp+\cancel{14zpq}$$

$$5x+5zp+7xq = 3y+3zq+7yp$$

$$5zp-7yp+7xq-3zq = 3y-5x$$

$$7yp-5zp+3zq-7xq = 5x-3y$$

$$\boxed{(7y-5z)p+(3z-7x)q = 5x-3y}$$

summer-2022

From the partial differential equation by eliminating arbitrary function f from $f(xy+z^2, x+y+z)=0$.

→ Let $u = xy+z^2$ $\qquad$ $v = x+y+z$

$\frac{\partial u}{\partial x} = y, \ \frac{\partial u}{\partial y} = x, \ \frac{\partial u}{\partial z} = 2z$ $\qquad$ $\frac{\partial v}{\partial x} = 1, \ \frac{\partial v}{\partial y} = 1, \ \frac{\partial v}{\partial z} = 1$

$\therefore \ f(u,v)=0$

Diff partially w.r.t. x

$$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial x} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial x}\right] + \frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial x} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial x}\right] = 0$$

$$\frac{\partial f}{\partial u}(y+2zp) + \frac{\partial f}{\partial v}(1+p) = 0$$

$$\frac{\partial f}{\partial u}(y+2zp) = -\frac{\partial f}{\partial v}(1+p)$$

$$\frac{\frac{\partial f}{\partial u}}{\frac{\partial f}{\partial v}} = \frac{-(1+p)}{(y+2zp)} \quad \text{—— (1)}$$

Diff partially w.r.t. y.

$$\frac{\partial f}{\partial u}\left[\frac{\partial u}{\partial y} + \frac{\partial u}{\partial z}\cdot\frac{\partial z}{\partial y}\right] + \frac{\partial f}{\partial v}\left[\frac{\partial v}{\partial y} + \frac{\partial v}{\partial z}\frac{\partial z}{\partial y}\right] = 0$$

$$\frac{\partial f}{\partial v}(x+2zq) + \frac{\partial f}{\partial v}(1+q) = 0$$

$$\frac{\partial f}{\partial u}(x+2zq) = -\frac{\partial f}{\partial v}(1+q)$$

$$\frac{\frac{\partial f}{\partial u}}{\frac{\partial f}{\partial v}} = \frac{-(1+q)}{(x+2zq)} \quad \text{—— (2)}$$

$\therefore$ From Eqn (1) + (2)

$$\frac{(1+p)}{(y+2zp)} = \frac{(1+q)}{(x+2zq)}$$

$$(1+p)(x+2zq) = (1+q)(y+2zp)$$

$$x+2zq+xp+\cancel{2zpq} = y+2zp+yq+\cancel{2zpq}$$

$$xp-2zp+2zq-yq = y-x$$

$$\boxed{(x-2z)p+(2z-y)q = y-x}$$

Winter 2018

May-2019

Find the temperture in bar of length 2 units whose ends are kept at zero temperature and lateral surface insulated if initial temperature is $\sin\left(\frac{\pi x}{2}\right) + 3\sin\left(\frac{5\pi x}{2}\right)$

→ One-dimensional heat equation is $\frac{\partial u}{\partial t} = c^2 \frac{\partial^2 u}{\partial x^2}$ ——— (1)

The soln. of eqn (1) consistent with physical nature of problem is given by

$$u = c_1 e^{-m^2c^2t}\,(c_2\cos mx + c_3 \sin mx) \quad \text{——— (2)}$$

where $u(x,t) = 0$ at $x = 0$ ——— (3)

$u(x,t) = 0$ at $x = 2$ ——— (4)

$u(x,t) = \sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2}$ at $x = 0$ ——— (5)

Using Conditions (3) in (2) we get $0 = c_1 e^{-m^2c^2t}(c_2) \Rightarrow \boxed{c_2 = 0}$

From (2) $u = c_1 c_3 e^{-m^2c^2t} \sin mx$ ——— (6)

Using Conditions (4) in (2) we get $0 = c_1 c_3 e^{-m^2c^2t} \sin 2m$

$\Rightarrow \sin 2m = 0 = \sin n\pi \Rightarrow 2m = n\pi \Rightarrow \boxed{m = \frac{n\pi}{2}}$

From (6), we have

$$u = c_1 c_3 e^{-\left(\frac{n\pi}{2}\right)^2 c^2 t} \sin\frac{n\pi x}{2} \quad \text{——— (7)}$$

∴ The general soln is

$$u = \sum_{n=1}^{\infty} b_n e^{-\left(\frac{n\pi}{2}\right)^2 c^2 t} \sin\frac{n\pi x}{2} \quad \text{——— (8)}$$

Using (5) in (8), we have

$$\sin\frac{\pi x}{2} + 3\sin\frac{5\pi x}{2} = \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{2}$$

$$= b_1 \sin\frac{\pi x}{2} + b_2 \sin\frac{2\pi x}{2} + b_3 \sin\frac{3\pi x}{2} + b_4 \sin\frac{4\pi x}{2} + b_5 \sin\frac{5\pi x}{2} + \cdots$$

$\Rightarrow b_1 = 1,\ b_5 = 3,\ b_2 = b_3 = b_4 = b_6 = b_7 = b_8 = \cdots = 0$

∴ From (8), we have

$$u = b_1 e^{-\frac{\pi^2}{4}c^2 t} \sin\frac{\pi x}{2} + b_5 e^{-\left(\frac{5\pi}{2}\right)^2 c^2 t} \sin\frac{5\pi x}{2}$$

$$\boxed{u = e^{-\frac{\pi^2}{4}c^2 t} \sin\frac{\pi x}{2} + 3 e^{-\left(\frac{5\pi}{2}\right)^2 c^2 t} \sin\frac{5\pi x}{2}}$$

Which is the requried temperature.

winter 2018 2019

Use method of separation of variables to solve the equation.

$\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$, given that $u(x,0) = 6e^{-3x}$.

$\rightarrow$ The given PDE is $\frac{\partial u}{\partial x} = 2\frac{\partial u}{\partial t} + u$ —— ①

Let $u = X(x)T(t)$ —— ② $\boxed{u = XT}$

where $X(x)$ is function of x alone and $T(t)$ is fun of t alone.

From ② $\frac{\partial u}{\partial x} = \frac{dX}{dx}T$, $\frac{\partial u}{\partial t} = X\frac{dT}{dt}$

put in eqn ①, we have

$$\frac{dX}{dx}T = 2X\frac{dT}{dt} + XT$$

$$\frac{1}{X}\frac{dX}{dx} = \frac{2}{T}\frac{dT}{dt} + 1 = -k^2 \text{ (say)}$$

$$\frac{1}{X}\frac{dX}{dx} = -k^2, \quad \frac{2}{T}\frac{dT}{dt} + 1 = -k^2$$

$$\frac{dX}{dx} = -k^2 X,$$

$$\frac{dX}{X} = -k^2 dx$$

$$\int\frac{dX}{X} = -k^2\int dx + \log c_1$$

$$\log X = -k^2 x + \log c_1$$

$\log e = 1$

$$\log X = -k^2 x \log e + \log c_1$$

$$= \log e^{-k^2 x} + \log c_1$$

$$\log X = \log c_1 e^{-k^2 x}$$

$$\boxed{X = c_1 e^{-k^2 x}}$$

$$\frac{2}{T}\frac{dT}{dt} = -(k^2+1)$$

$$\frac{dT}{T} = -\frac{(k^2+1)}{2}dt.$$

$$\int\frac{dT}{T} = -\frac{(k^2+1)}{2}\int dt + \log c_2$$

$$\log T = -\frac{(k^2+1)}{2}t + \log c_2$$

$$= -\frac{(k^2+1)}{2}t\log e + \log c_2$$

$$= \log e^{-\frac{(k^2+1)}{2}t} + \log c_2$$

$$\log T = \log c_2 e^{-\frac{(k^2+1)}{2}t}$$

$$\boxed{T = c_2 e^{-\frac{(k^2+1)}{2}t}}$$

$$\therefore u = XT = (c_1 e^{-k^2 x})(c_2 e^{-\frac{(k^2+1)}{2}t}) = c_1c_2 e^{-k^2 x} e^{-\frac{(k^2+1)}{2}t} \quad \text{——③}$$

Given $u(x,0) = 6e^{-3x}$ at $t = 0$.

$$6e^{-3x} = c_1c_2 e^{-k^2 x} \Rightarrow \boxed{c_1c_2 = 6}, \boxed{k^2 = 3}$$

From ③ $u = 6e^{-k^2 x} e^{-\frac{(k^2+1)}{2}t} = 6e^{-3x}e^{-2t} = 6e^{-3x-2t}$.

$$\boxed{u = 6e^{-(3x+2t)}}$$

which is the required solution.

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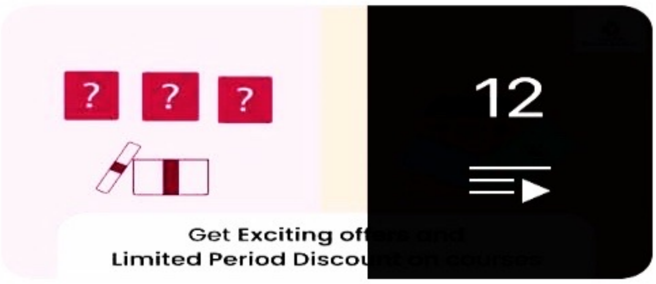

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