Ukulelor Formulation

Author(s): Aster Santana
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The statement of the use case is on Mip Wise's website: www.mipwise.com/use-cases/ukulelor.

For a beginner-friendly formulation, see ukulelor.ipynb.

Input Data Model

Indices

- I: Collection of retailers.

Parameters

- pu: Production upper bound (units), i.e., production capacity of the factory.
- sl: Shipment lower bound (units), i.e., minimum shipment quantity to avoid penalty.
- pn: Penalty (num. of units) paid for each order that has less than S ukuleles shipped.
- p_i: Unit price at which ukuleles are sold to retailer i.
- d_i: Demand (units) of ukuleles from retailer i.

Decision Variables

- x_i: The number of ukuleles shipped to retailer i.
- z_i: Equals 1 if Ted ships to retailer i, 0 otherwise.

Constraints

- C1) Production capacity:

$$\sum_i x_i \leq pu.$$

- C2) If ship to retailer i, then ship at least 50:

$$sl \cdot z_i \leq x_i, \quad \forall i.$$

- C3) If no shipping to retailer i, then $x_i = 0$:

$$x_i \leq d_i \cdot z_i, \quad \forall i.$$

Objective

The objective is to maximize total profit, which is total revenue minus penalty.

$\text{revenue} = \sum_i p_i \cdot x_i.$

$\text{penalty} = \sum_i pn \cdot p_i \cdot (1 - z_i).$

$$\min \ \text{revenue} - \text{penalty}.$$

Final Formulation

Putting everything together, we obtain:

$$
\begin{aligned}
\min \quad & \textstyle\sum_i p_i \cdot x_i - \sum_i pn \cdot p_i \cdot (1 - z_i) \\
\text{s.t.} \quad & \textstyle\sum_i x_i \leq pu, \\
& sl \cdot z_i \leq x_i, \quad \forall i, \\
& x_i \leq d_i \cdot z_i, \quad \forall i, \\
& x_i \geq 0, z_i \in \{0, 1\} \quad \forall i.
\end{aligned}
\tag{1}
$$