Ukulelor Formulation

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The statement of the use case is on Mip Wise's website: www.mipwise.com/use-cases/ukulelor.

This formulation is not generic in the sense that it uses the given data explicitly. For a data agnostic model, see ukulelor_pro.ipynb.

Input Data Model

Decision Variables

- x_i: The number of ukuleles shipped to retailer i, for all $i = R1, R2, \cdots, R7$.
- z_i: Equals 1 if Ted ships to retailer i, 0 otherwise, for all $i = R1, R2, \cdots, R7$.

Notice that x_i are integer variables while z_i are binary variables (they can only take 0-1 values). We can't define x_i as continuous variables since shiping 50.3 ukuleles, for instance, wouldn't make sense.

To simplify notation, let's write $x_1, x_2, \cdots, x_7$ instead of $x_{R1}, x_{R2}, \cdots, x_{R7}$ when writing constraints and the objective function. Similarly, let's write $z_1, z_2, \cdots, z_7$ instead of $z_{R1}, z_{R2}, \cdots, z_{R7}$.

Constraints

- C1) Production capacity:

$$x_1 + x_2 + x_3 + x_4 + x_5 + x_6 + x_7 \leq 650.$$

  This constraint is saying that the total shipped quantity cannot exceed 650 units.

- C2) If ship to retailer i, then ship at least 50:

$$50 z_i \leq x_i, \quad \forall i.$$

  These inequalities are saying that, if $z_i = 1$, meaning that Ted ships at least one unit to retailer i, then x_i must be at least 50, meaning that Ted must ship at least 50 units to that retailer.

- C3) If no shipping to retailer i, then $x_i = 0$:

$$x_i \leq d_i \cdot z_i, \quad \forall i.$$

  Here, d_i represents the demand of retailer i. These inequalities are sort of the complement of the previous ones: if $z_i = 0$, meaning that Ted decided not to ship to retailer i, then x_i must be zero, meaning that Ted must not ship any unit to that retailer.

Objective

The objective is to maximize total profit, which is total revenue minus penalty.

The total revenue is obtained by simply multiplying the number of units shipped to each retailer by the wholesale price paid by that retailer:

$$\text{revenue} = 47x_1 + 65x_2 + 70x_3 + 68x_4 + 46x_5 + 78x_6 + 55x_7.$$

For the total penalty we need to multiply the individual penalty (20 times the wholesale price) that Ted would pay to retailer i by $1 - z_i$. So we get:

$$\text{penalty} = 940(1 - z_1) + 1300(1 - z_2) + 1400(1 - z_3) + 1360(1 - z_4) + 920(1 - z_5) + 1560(1 - z_6) + 1100(1 - z_7).$$

Then, if $z_1 = 0$, meaning that Ted does not ship to retailer $R1$, then the quantity in the brackets is 1 and, therefore, the penalty of $20 \cdot 47 = 940$ gets counted. On the flip side, if $z_1 = 1$, meaning that Ted does ship to retailer $R1$, then the quantity in the brackets is 0 and, therefore, no penalty gets counted.

Using summation, the final objective becomes:

$$\max \sum_i p_i \cdot x_i - \sum_i 20 \cdot p_i \cdot (1 - z_i).$$

Here, p_i represents the wholesale price paid by retailer i.

Final Formulation

Putting everything together, we obtain:

$$
\begin{array}{ll}
\max & \sum_i p_i \cdot x_i - \sum_i 20 \cdot p_i \cdot (1 - z_i) \\
\text{s.t.} & \sum_i x_i \leq 650, \\
& 50 \cdot z_i \leq x_i, \quad \forall i, \\
& x_i \leq d_i \cdot z_i, \quad \forall i, \\
& x_i \geq 0, z_i \in \{0, 1\} \quad \forall i.
\end{array}
$$