Voltwise Formulation

Input Data Model

Indices

- I - Collection of tasks.

Parameters

- r_i - Regular completion time (days) for task i.
- c_i - Execution cost (dollar/day) of task i.
- ce_i - Expedition cost (dollar/day) of task i.
- eu_i - Expedition upper bound (day) of task i.
- p - List of (i, j)-pairs where task i must be completed before task j starts.
- U - Maximum time to complete the project.

Decision Variables

- x_i - Starting time of task i.
- y_i - Total time taken to complete task i.

Constraints

- C1) Task j can only start after Task i is complete:

$$x_i + y_i \leq x_j, \quad \forall i, j \in p.$$

- C2) Maximum time to complete task i:

$$y_i \leq r_i, \quad \forall i.$$

- C3) Minimum time to complete task i:

$$r_i - eu_i \leq y_i$$

- C4) Maximum time to complete the project:

$$x_i + y_i \leq U, \quad \forall i.$$

Objective

The objective is to minimize total cost, which is composed by regular cost plus extra cost due to completion delay.

$\text{regular\_cost} = \sum_i c_i y_i.$

$\text{expedition\_cost} = \sum_i ce_i (r_i - y_i).$

$$\min \ \text{regular\_cost} + \text{expedition\_cost}.$$

Final formulation

$$
\begin{aligned}
\min \quad & \textstyle\sum_i c_i y_i + \sum_i ce_i (r_i - y_i) \\
\text{s.t.} \quad & x_i + y_i \le x_j, \quad \forall i, j \in p, \\
& y_i \le r_i, \quad \forall i, \\
& r_i - eu_i \le y_i, \quad \forall i, \\
& x_i + y_i \le U, \quad \forall i, \\
& x_i, y_i, z_i \ge 0, \quad \forall i.
\end{aligned} \tag{1}
$$