$x(t) = e^{-t/2}$; $w_0 = \frac{2\pi}{T_0}$ $\quad T_0 = \pi$

$w_0 = \frac{2\pi}{\pi} = w_0 = 2$

$$a_0 = \frac{1}{T_0}\int_{T_0} x(t)\,dt \rightsquigarrow a_0 = \frac{1}{\pi}\int_0^{\pi} e^{-t/2}\,dt = -\frac{2}{\pi} e^{-t/2}\Big|_0^{\pi} = -\frac{2}{\pi}\left(e^{-\pi/2} - 1\right)$$

$$\boxed{a_0 \approx 0{,}504279524}$$

$$a_n = \frac{2}{T_0}\int_{T_0} x(t)\cos(n w_0 t)\,dt \rightsquigarrow n \in \mathbb{N}$$

$$\boxed{a_n = \frac{2}{\pi}\int_0^{\pi} e^{-t/2}\cos(2nt)\,dt}$$

$$\int_0^{\pi} e^{-t/2}\cos(2nt)\,dt$$

1º $u = e^{-t/2}$ $\qquad v = \int \cos(2nt)\,dt = \frac{1}{2n}\operatorname{sen}(2nt)$

$du = -\frac{1}{2} e^{-t/2}\,dt$

$$= \frac{e^{-t/2}\operatorname{sen}(2nt)}{2n}\Big|_0^{\pi} + \frac{1}{4n}\int_0^{\pi} e^{-t/2}\operatorname{sen}(2nt)\,dt$$

2º $u = e^{-t/2}$; $v = \int \operatorname{sen}(2nt)\,dt = -\frac{1}{2n}\cos(2nt)$

$d = -\frac{1}{2} e^{-t/2}\,dt$

$$= \frac{e^{-t/2}\operatorname{sen}(2nt)}{2n}\Big|_0^{\pi} - \frac{e^{t/2}\cos(2nt)}{8n^2}\Big|_0^{\pi} - \frac{1}{16n^2}\int_0^{\pi} e^{-t/2}\cos(2nt)\,dt$$

$$\left(1 + \frac{1}{16n^2}\right)\int_0^{\pi} e^{-t/2}\cos(2nt)\,dt = \frac{e^{-t/2}\operatorname{sen}(2nt)}{2n}\Big|_0^{\pi} - \frac{e^{t/2}\cos(2nt)}{8n^2}\Big|_0^{\pi}$$

$$\left(\frac{16n^2+1}{16n^2}\right)\int_0^{\pi} e^{-\frac{t}{2}}\cos(2nt)\,dt = \frac{e^{-\frac{t}{2}}\overset{0}{\cancel{\operatorname{sen}(2nt)}}}{2n}\Bigg|_0^{\pi} - \frac{e^{-\frac{t}{2}}\cos(2nt)}{8n^2}\Bigg|_0^{\pi}$$

Obs.: $\operatorname{sen}(2n\pi) = 0 \;\; \forall \; n \in \mathbb{N}$ $\qquad \operatorname{sen}(0) = 0$

$\cos(2n\pi) = 1 \;\; \forall \; n \in \mathbb{N}$ $\qquad \cos(0) = 0$

$$= \left(\frac{\cancel{16n^2}}{1+16n^2}\right)\cdot\left(-\frac{e^{-\frac{t}{2}}\cos(2nt)}{\cancel{8n^2}}\right)\Bigg|_0^{\pi} = \left(\frac{2}{1+16n^2}\right)\left(-e^{-\frac{t}{2}}\cos(2nt)\right)\Big|_0^{\pi}$$

$$= \left(\frac{2}{1+16n^2}\right)\cdot\left(-e^{-\frac{\pi}{2}}\overset{1}{\cancel{\cos(2n\pi)}} + e^{0}\overset{1}{\cancel{\cos(0)}}\right) = \left(\frac{2}{1+16n^2}\right)\left(1-e^{-\frac{\pi}{2}}\right)$$

$$a_n = \frac{2}{\pi}\int_0^{\pi} e^{-\frac{t}{2}}\cos(2nt)\,dt = \underbrace{\frac{2\cdot\left(1-e^{-\frac{\pi}{2}}\right)}{\pi}}_{a_0}\left(\frac{2}{1+16n^2}\right)$$

$$\boxed{a_n = a_0\cdot\left(\frac{2}{1+16n^2}\right)}$$

$$b_n = \frac{2}{T_0}\int_{T_0} x(t)\operatorname{sen}(n\omega_0 t)\,dt$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^{-\frac{t}{2}}\operatorname{sen}(2nt)\,dt$$

$$\boxed{\int_0^{\pi} e^{-\frac{t}{2}}\operatorname{sen}(2nt)\,dt}$$

1ª $\quad u = e^{-\frac{t}{2}} \quad ; \quad v = \int \operatorname{sen}(2nt)\,dt$

$du = -\frac{1}{2}e^{-\frac{t}{2}}\,dt \qquad v = -\frac{1}{2n}\cos(2nt)$

$$= -\frac{e^{-\frac{t}{2}}\cos(2nt)}{2n}\Bigg|_0^{\pi} - \frac{1}{4n}\int_0^{\pi} e^{-\frac{t}{2}}\cos(2nt)\,dt$$

(2º)

$$u = e^{-\frac{t}{2}} \qquad ; \qquad v = \int \cos(2nt)\,dt$$

$$du = -\frac{1}{2} e^{-\frac{t}{2}}\,dt \qquad v = \frac{1}{2n}\operatorname{sen}(2nt)$$

$$-\frac{e^{-\frac{t}{2}}\cos(2nt)}{2n}\Bigg|_0^{\pi} - \frac{e^{-\frac{t}{2}}\operatorname{sen}(2nt)}{8n^2}\Bigg|_0^{\pi} - \frac{1}{16n^2}\int_0^{\pi} e^{-\frac{t}{2}}\operatorname{sen}(2nt)\,dt$$

$$\left(1+\frac{1}{16n^2}\right)\int_0^{\pi} e^{-\frac{t}{2}}\operatorname{sen}(2nt)\,dt = -\frac{e^{-\frac{t}{2}}\cos(2nt)}{2n}\Bigg|_0^{\pi} - \frac{e^{-\frac{t}{2}}\operatorname{sen}(2nt)}{8n^2}\Bigg|_0^{\pi}$$

$$\left(\frac{16n^2+1}{16n^2}\right)\int_0^{\pi} e^{-\frac{t}{2}}\operatorname{sen}(2nt)\,dt = -\frac{e^{-\frac{t}{2}}\cos(2nt)}{2n}\Bigg|_0^{\pi} - \cancel{\frac{e^{-\frac{t}{2}}\overset{0}{\operatorname{sen}(2nt)}}{8n^2}}\Bigg|_0^{\pi}$$

$$= \left(\frac{16n^{\cancel{2}}}{1+16n^2}\right)\left(\frac{-e^{-\frac{t}{2}}\cos(2nt)}{\cancel{2n}}\right)\Bigg|_0^{\pi} \Rightarrow \left(\frac{8n}{1+16n^2}\right)\left(-e^{-\frac{t}{2}}\cos(2nt)\right)\Big|_0^{\pi}$$

$$= \left(\frac{8n}{1+16n^2}\right)\left(-e^{-\frac{\pi}{2}}\overset{1}{\cancel{\cos(2n\pi)}} + \overset{1}{\cancel{e^{0}\cos(0)}}\right) \Rightarrow \left(\frac{8n}{1+16n^2}\right)\left(1-e^{-\frac{\pi}{2}}\right)$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} e^{-\frac{t}{2}}\operatorname{sen}(2nt)\,dt \Rightarrow \underbrace{\frac{2}{\pi}\left(1-e^{-\frac{\pi}{2}}\right)}_{a_0}\cdot\left(\frac{8n}{1+16n^2}\right)$$

$$\boxed{b_n = a_0\cdot\left(\frac{8n}{1+16n^2}\right)}$$

$$C_n = \sqrt{a_n^2 + b_n^2} \quad ; \quad C_0 = a_0$$

$$= \sqrt{\left(\frac{2a_0}{1+16n^2}\right)^2 + \left(\frac{8na_0}{1+16n^2}\right)^2}$$

$$\sqrt{\frac{(2a_0)^2+(8na_0)^2}{(1+16n^2)^2}} \Rightarrow \sqrt{\left(\frac{1}{\cancel{1+16n^2}}\right)^2(2a_0)^2(\cancel{1+16n^2})}$$

$$= \sqrt{\frac{(2a_0)^2}{1+16n^2}} = \frac{2a_0}{\sqrt{1+16n^2}} \; ; \quad C_n = \frac{2a_0}{\sqrt{1+16n^2}}$$

$$\theta = tg^{-1}\left(\frac{-b_n}{a_n}\right) = -tg^{-1}\left(\frac{b_n}{a_n}\right) = -tg^{-1}\left[\left(\frac{8n\cancel{a_0}}{\cancel{1+16n^2}}\right)\cdot\left(\frac{\cancel{1+16n^2}}{2\cancel{a_0}}\right)\right]$$

$$\boxed{\theta = -tg^{-1}(4n)}$$