NUMERICAL SIMULATIONS APPLIED TO ENGINEERING
First Exam, October 2024

Name and Family Name: . . . .

Mark the correct answer in the table below. The following grading policy will be adopted: *Correct answer: +1 point; Wrong answer: -0.5 points.*

	1	2
True		
False		

1) All the coefficients of the homogeneous transformation matrix change because they depend on the transformation.

2) Euler's method is one of the most accurate schemes used to approach ODEs.

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Problem 1 (5 points): A satellite is tracked and different points of its trajectory are being recorded.

a) (2 Points) Interpolate the following points with a Lagrange polynomial:

(-1.0, 2.2)
(0.0, 1.2)
(1.0, 0.8)
(2.0, 1.0)

b) (1 Point) What is the actual order of the polynomial?

c) (2 Points) Evaluate the polynomial at points x = 1.5 and 2.5. Do you think you can trust the values obtained for x = 1.5? And for x = 2.5?
Justify your answers.

Problem 2 (5 points): A missile is launched from the Earth surface with an initial velocity v_0. The missile follows a straight trajectory, perpendicular to Earth's surface.

a) (3 Points) Calculate the distance of the missile to the surface, r, as a function of time by solving Newton's second law:

$$\frac{d^2r}{dt^2} = -G\frac{M_{\text{Earth}}}{r^2}$$

where $M_{\text{Earth}} = 5.97\,10^{24}$ kg is the mass of the Earth. Assume the following boundary conditions: $r(t=0) = R_{\text{Earth}}$, and $v(t=0) = v_0$, with $R_{\text{Earth}} = 6400$ km being the radius of the Earth. The value of the gravitational constant is $G = 6.67\,10^{-20}$ km^3 kg^{-1} s^{-2}. Integrate the corresponding ODE using an Euler second-order centered scheme for the interval $0 \leq t(\text{hours}) \leq 3$, considering three different cases with $v_0 = 9$ km/s, $v_0 = 10$ km/s and $v_0 = 11$ km/s.

b) (2 Points) Compare the results with the analytical solution (obtained from the conservation of energy):

$$\frac{1}{r} = \frac{1}{GM_{\text{Earth}}} \left[\frac{GM_{\text{Earth}}}{R_{\text{Earth}}} + 0.5(v^2 - v_0^2) \right]$$