Image Operations

Mathematical operations
Linear Operations

- Let H be an operation which is applied to image f(x,y) to produce the image g(x,y); that is g(x,y)=**H[**f(x,y)**]**

- H is said to be a linear operator if
$H\left[a_i f_i(x, y) + a_j f_j(x, y)\right]$

$$= a_i H[f_i(x, y)] + a_j H\left[f_j(x, y)\right]$$
$$= a_i g_i(x, y) + a_j g_j(x, y)$$

Mathematical operations
Nonlinear Operations

- Any operation that fails to satisfy the equation of the previous slide; example: min and max operations
- For example consider the following two images each with four pixels

- $f_1 = \begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}, f_2 = \begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}, a_1 = 1, a_2 = -1.$
- Then;
- $max\left\{(1)\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix} + (-1)\begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}\right\} = max\left\{\begin{bmatrix} -6 & -3 \\ -2 & -4 \end{bmatrix}\right\} = -2$
- But;
- $(1)max\left\{\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}\right\} + (-1)max\left\{\begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}\right\} = 3 + (-1)7 = -4$

Arithmetic Operations

- Most branches of image processing

- Arithmetic operations are carried out between **coresponding pixel pairs**
  - **Pixel by pixel,** computation-intensive!
    - How to perform it?

- This involves images with the **same size**.

- The arithmetic operations are +, -, *, /
  - Ex: $s(x,y)= f(x,y) * g(x,y)$

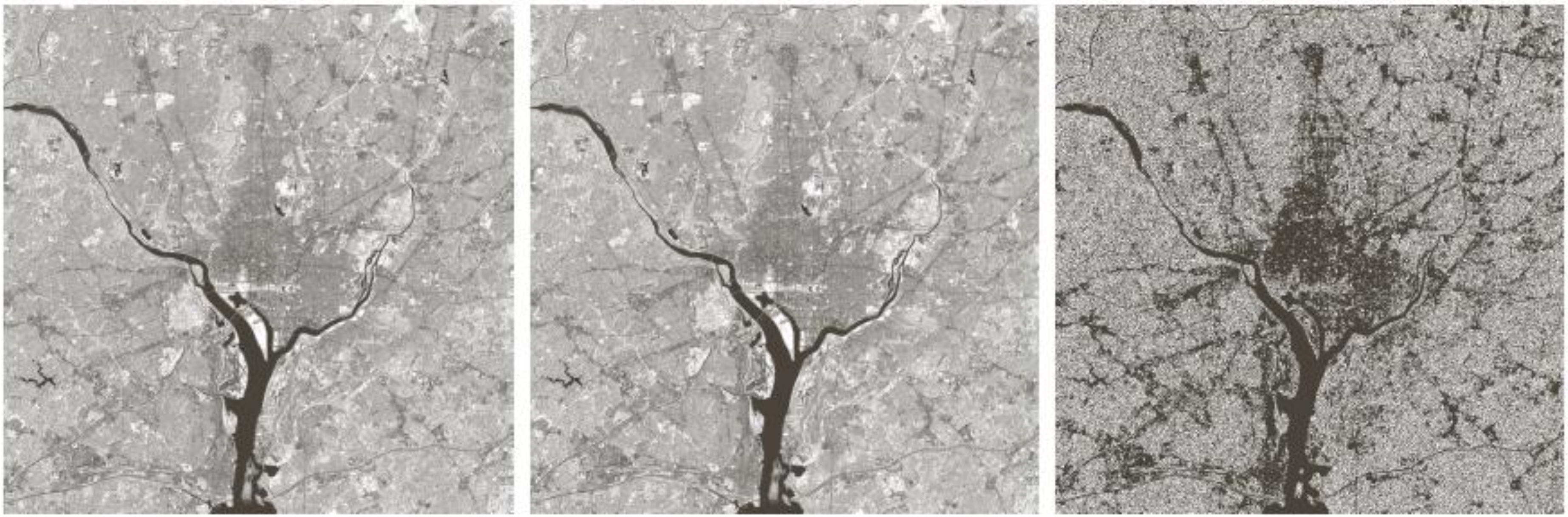
Images are from the Digital Image Processing textbook, see slide 7

Arithmetic Operators

- Addition:
  – Image averaging to reduce the noise.
- Subtraction:
  – Basic tool in medical imaging and motion detection
- Multiplication:
  – To correct gray-level shading result from:
    - non-uniformities in illumination
    - or in the sensor used to acquire the image
- Division

Image Subtraction

- Finding the differences between images



a b c

FIGURE 2.27 (a) Infrared image of the Washington, D.C. area. (b) Image obtained by setting to zero the least significant bit of every pixel in (a). (c) Difference of the two images, scaled to the range [0, 255] for clarity.

Image Multiplicative (Masks)

- To focus on some regions

FIGURE 2.30 (a) Digital dental X-ray image. (b) ROI mask for isolating teeth with fillings (white corresponds to 1 and black corresponds to 0). (c) Product of (a) and (b).

Image Averaging

- Consider a noisy image modeled as:

$$g(x,y) = f(x,y) + \eta(x,y)$$

Where $f(x,y)$ is the original image, and $\eta(x,y)$ is an uncorrelated zero-mean noise process

- Objective: to reduce the noise content by averaging a set of noisy images

Image Averaging (cont.)

- Define an image formed by averaging K different noisy images:

$$\overline{g}(x,y)=\frac{1}{K}\sum_{i=1}^{K}g_i(x,y)$$

- It follows that:

$$E\{\overline{g}(x,y)\}=f(x,y)$$

= expected value of g (output after averaging)
=~ original image f(x,y)

Image Averaging (cont.)

- Note: the images $g_i(x,y)$ (noisy images) must be registered (aligned) in order to avoid the introduction of blurring and other artifacts in the output image.

Important Notice

- In arithmatic operations, values may exceed the upper and lower limits
  - In gray levels the upper level is 255 and the lower is 0
- Some software simply set the values exceeding the upper limit to the upper limit and the ones below the lower limit to the lower limit
  - In gray levels, any value more than 255 will be set to 255 and any one below 0 will be set to zero.
- This approach is not accurate.

Important Notice (cont.)

- Another approch which guarantees that the full range is scaled between the allowed limits is as follows
  - Suppose the produced image after processing is given by f
  - First find $f_m = f - min(f)$, this produces an image with min value 0
  - Then perform the operation $f_s = K[f_m/max(f_m)]$, this will create an image that its values range from 0 to K. In gray level K would be set to 255.
  - E.g. Please perform addition operation on the following two images and scale the output to the gray levels 0-7

Image 1

0	1
2	7

Image 2

1	3
4	6

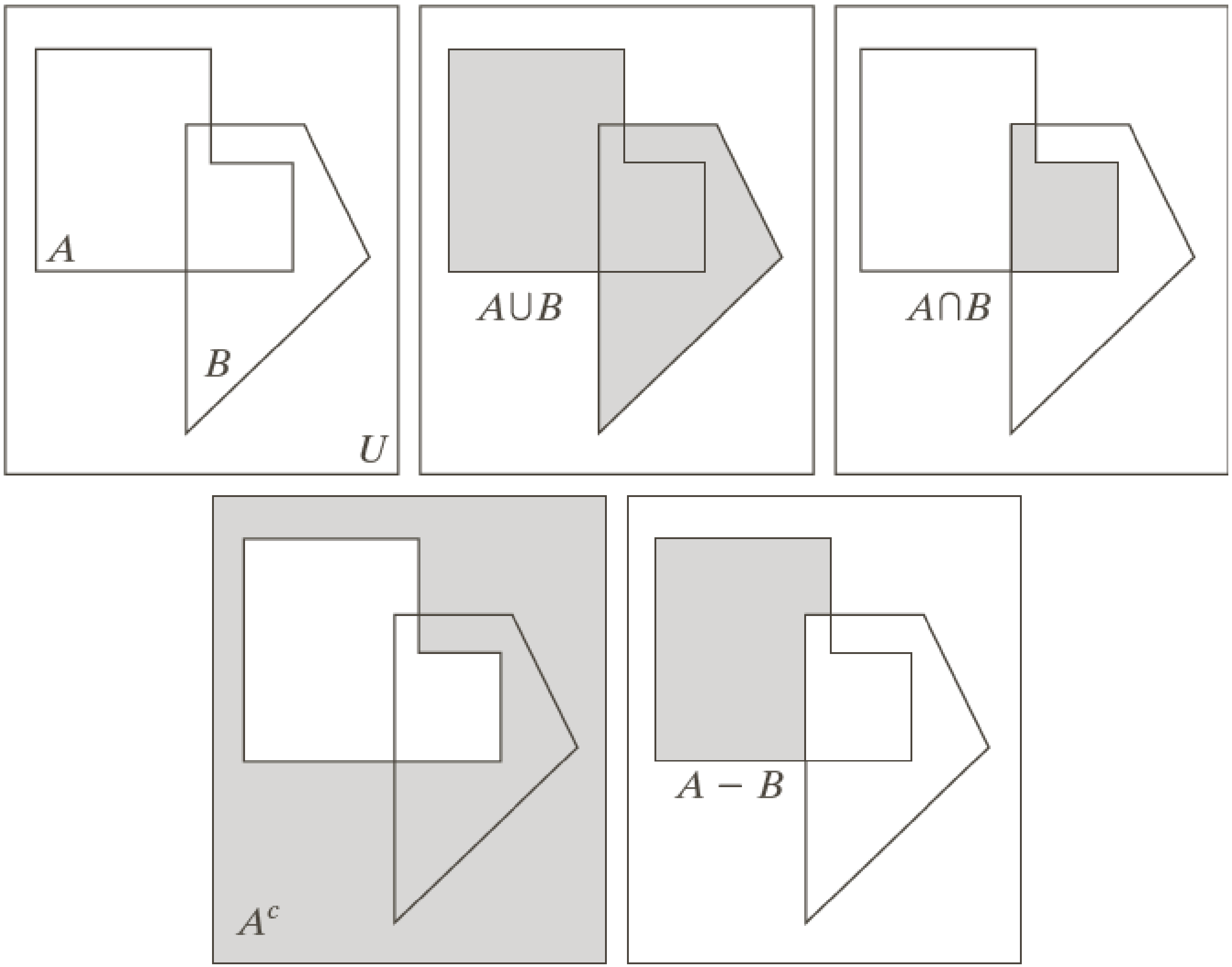
Final image

0	2
3	7

Set Operations

- Let A be a set of pixels (intensities) then
- The complement of a set A is the set $A^c=\{w|w \notin A\}$
- The difference between two sets A and B is $A-B=\{w|w \in A, w \notin B\}=A \cap B^c$
- When dealing with gray level pixels, union and intersection operations are usually defined as max and min or corresponding pixel pairs, respectively.
- The complement is defined as the pairwise difference between a constant and the intensity of every pixel in an image.

Example of set operations (binary images)

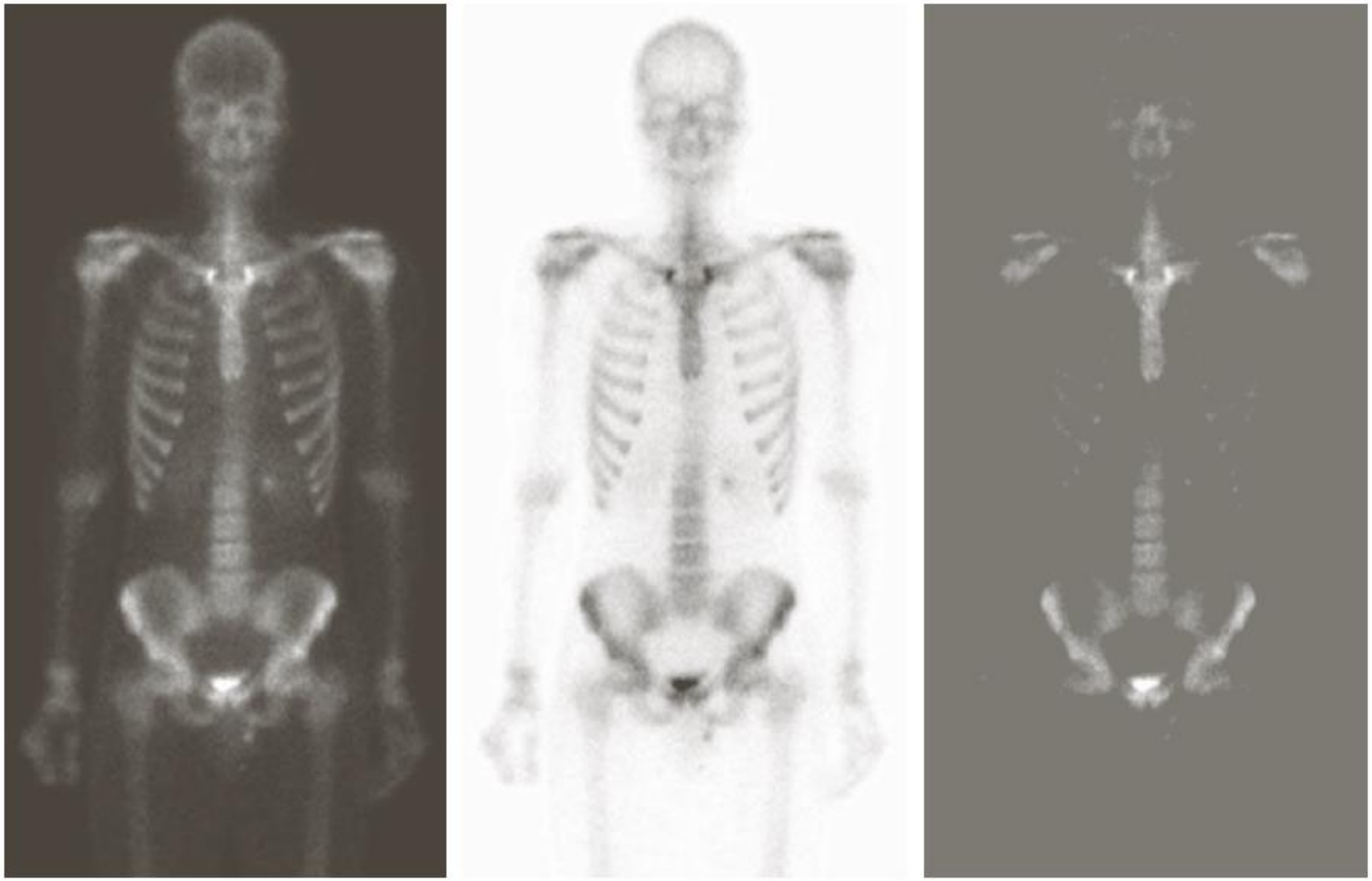


a b c
d e

FIGURE 2.31
(a) Two sets of coordinates, A and B, in 2-D space. (b) The union of A and B. (c) The intersection of A and B. (d) The complement of A. (e) The difference between A and B. In (b)–(e) the shaded areas represent the member of the set operation indicated.

Images are from the Digital Image Processing textbook, see slide 7

Example, set operations of gray level images



a b c

FIGURE 2.32 Set operations involving gray-scale images. (a) Original image. (b) Image negative obtained using set complementation. (c) The union of (a) and a constant image. (Original image courtesy of G.E. Medical Systems.)

Logical operations

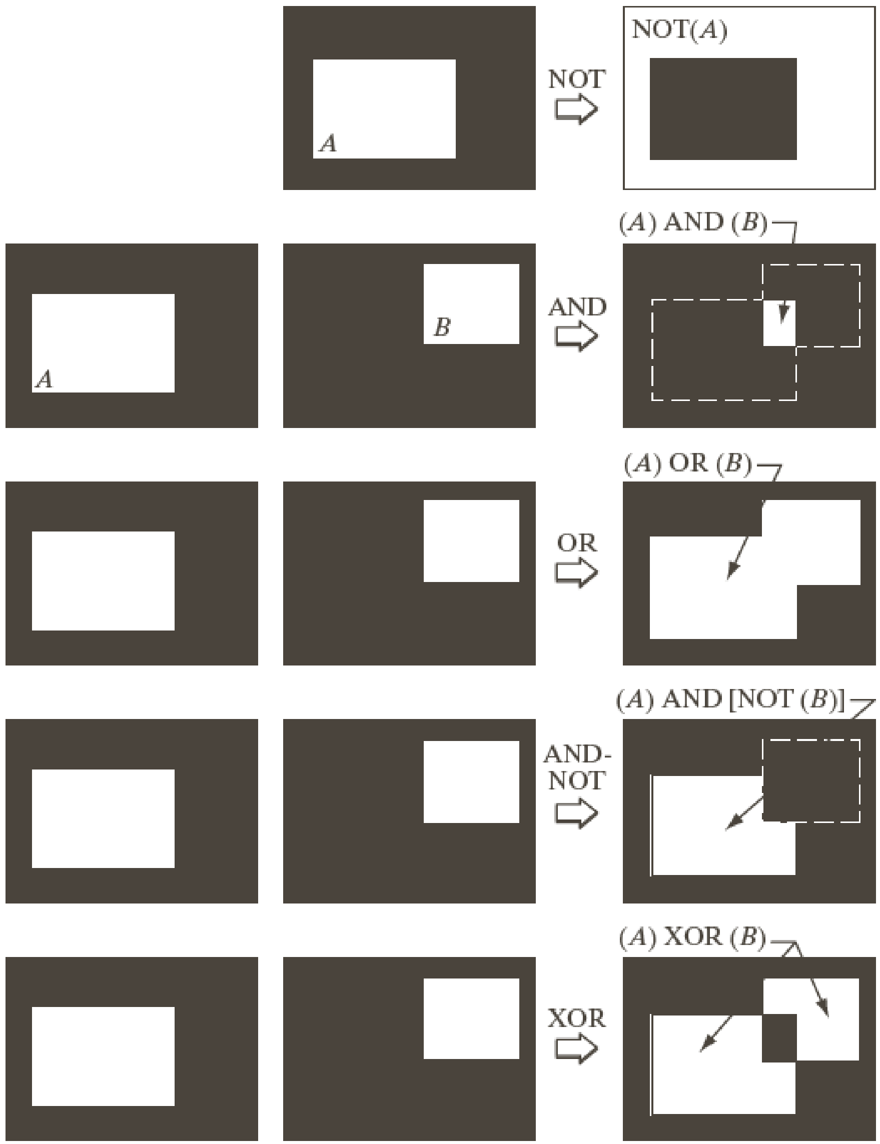


FIGURE 2.33 Illustration of logical operations involving foreground (white) pixels. Black represents binary 0s and white binary 1s. The dashed lines are shown for reference only. They are not part of the result.

Logical operations (cont.)

- logic operations apply only to binary images.
- Arithmetic operations apply to multivalued pixels.
- Logic operations used for tasks such as masking, feature detection, and shape analysis.

Spatial Operations

- Can be classified into:
  - (1) Single-pixel operations
  - (2) Neighborhood operations
    - Mask operations

Single-pixel operations

- Alter a pixel based on its intensity only.
- Can be expressed as $s=T(z)$, where
  - z is the intensity of a pixel in the original image
  - s is the mapped intensity in the transformed image,
  - T is the transformation function which depends on a single pixel intensity.

- Example: Producing the negative image.

Neighborhood Operations

- The **pixel *s*** in the processed image, is produced by processing **the corresspponding pixel *z*** in the original unporcessed image and its **neighbor pixels set** Z_1**-**Z_8

	Z_1	Z_2	Z_3	
	Z_4	z	Z_5	
	Z_6	Z_7	Z_8	

Images are from the Digital Image Processing textbook, see slide 7

Neighborhood Operations (cont.)

- e.g. average filter
  - Replace the gray value of the pixel with the average gray values of the same pixel and its neighbor pixels within a mask of 3x3
  - $s = \frac{1}{9}(z+Z_1+Z_2+ \ldots +Z_7+Z_8)$

	Z_1	Z_2	Z_3	
	Z_4	z	Z_5	
	Z_6	Z_7	Z_8	

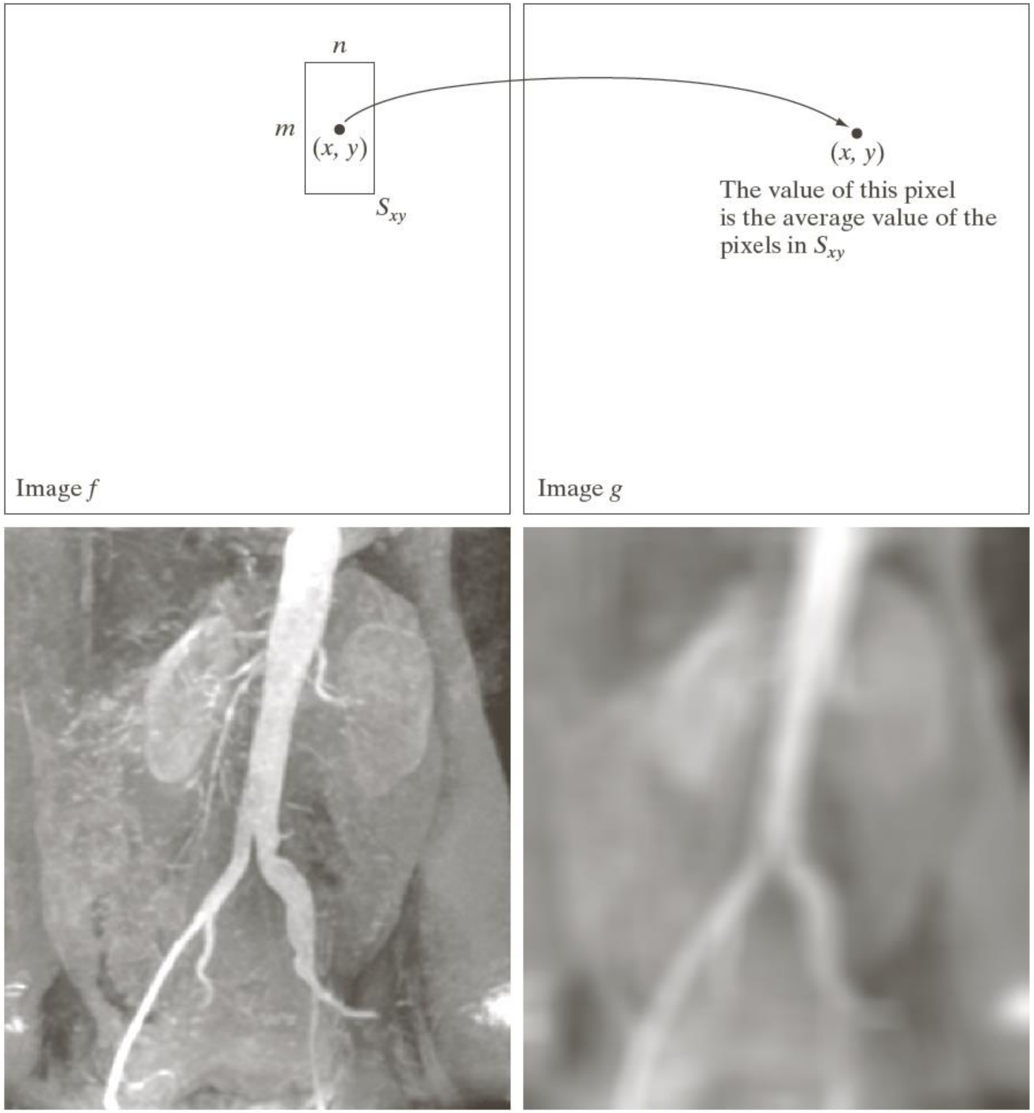
Mask Operator

- $s = \frac{1}{9}(z+Z_1+Z_2+ \ \ldots \ +Z_7+Z_8)$

$$= \frac{1}{9}z + \frac{1}{9}Z_1 + \ldots. + \frac{1}{9}Z_8 = w_1 z + w_2 Z_1 + \ldots. + w_9 Z_8$$

w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

Avarage example



a b
c d

FIGURE 2.35 Local averaging using neighborhood processing. The procedure is illustrated in (a) and (b) for a rectangular neighborhood. (c) The aortic angiogram discussed in Section 1.3.2. (d) The result of using Eq. (2.6-21) with $m = n = 41$. The images are of size 790×686 pixels.

Principle Objective of Enhancement

- Process an image so that the result will be **more suitable** than the original image
  - for a **specific application**.
- Techniques are **problem oriented**.
- A method which is quite useful for enhancing an image may not necessarily be the best approach for enhancing another images
- **No general theory** on image enhancement exists.

Spatial vs. Frequency Domain

- Spatial Domain (image plane):
  - Techniques are based on **direct manipulation of pixels** in an image.
    - Gray level transformations.
    - Histogram processing.
    - Arithmetic/Logic operations.
    - Filtration techniques.

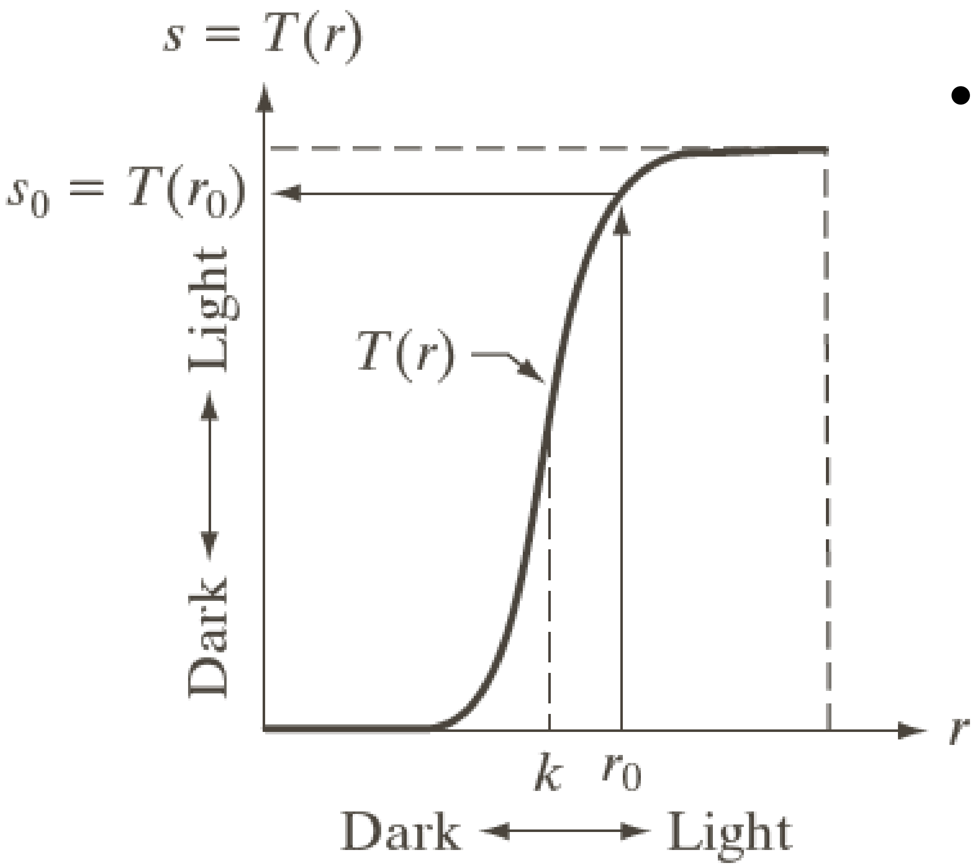
- Frequency Domain :
  - Techniques are based on modifying the Fourier transform of an image

Good images

- For **human visual**
  - The visual evaluation of image quality is a highly **subjective** process.
  - It is **hard to standardize** the definition of a good image.
- For **machine perception**
  - The evaluation task is easier.
  - A good image is one which gives **the best machine recognition results**.

- A certain amount of **trial and error** usually is required before a particular image enhancement approach is selected.

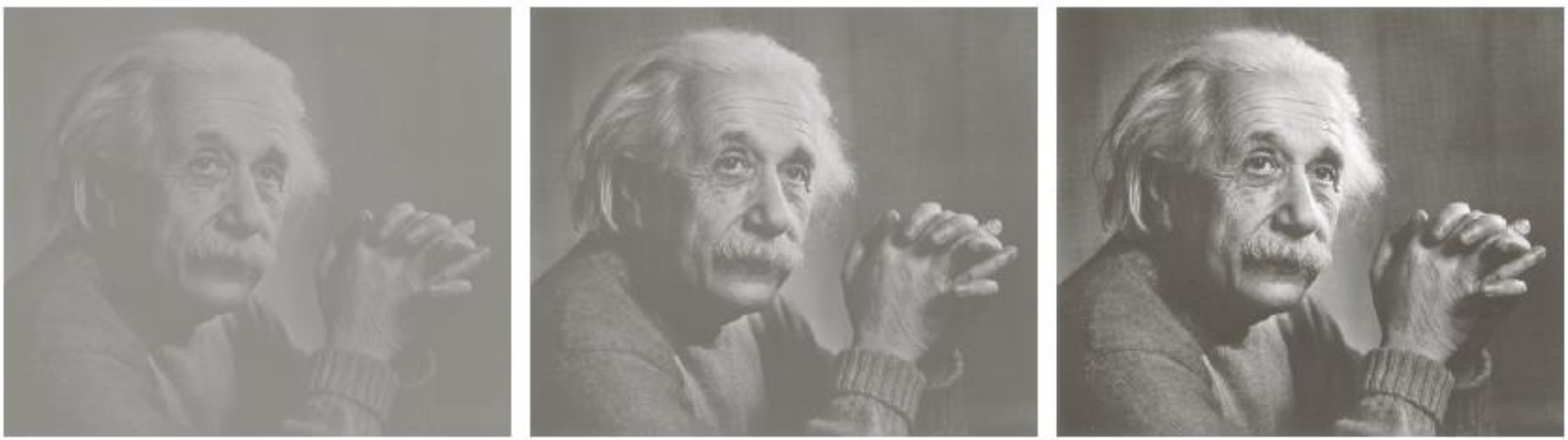
Point Processing Example – Contrast Stretching



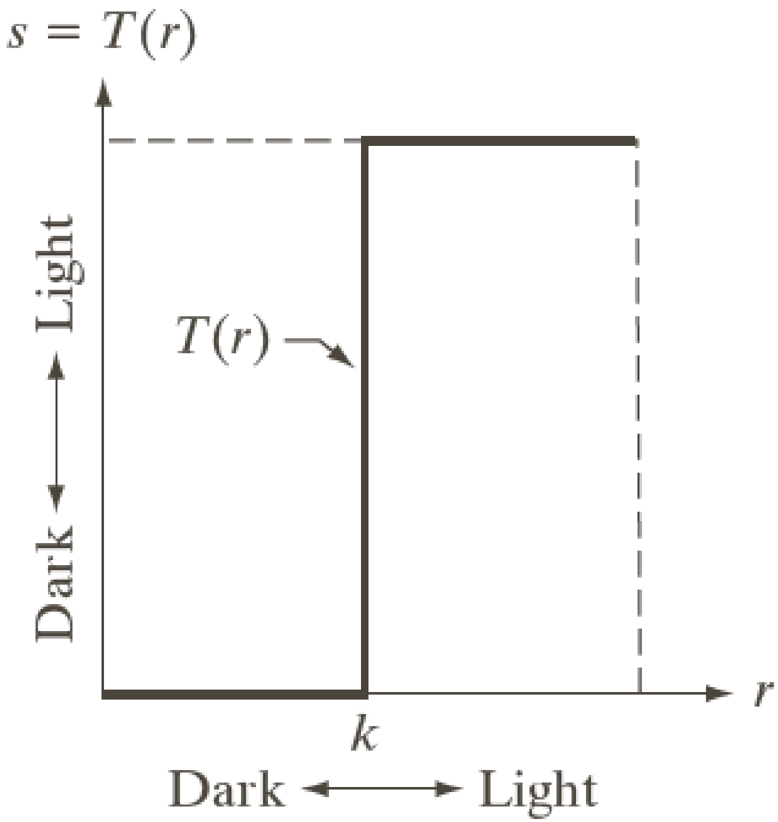
- Produce **higher contrast** than the original by
  - Darkening the levels below k in the original image
  - Brightening the levels above k in the original image

Point Processing Example –
Contrast Stretching (cont.)

Contrast stretching: is a process that expands the range of intensity levels in an image so that it spans the full intensity range of the recording medium or display device.



Point Processing Example –Thresholding

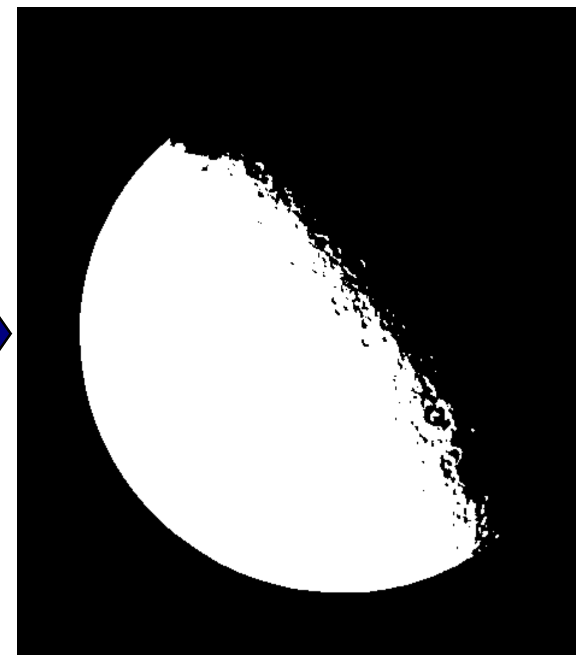


- Produce a two-level (binary) image

Point Processing Example - Thresholding

- Particularly useful for segmentation
  - isolate an object of interest from a background

$$s = \begin{cases} 1.0 & r > \text{threshold} \\ 0.0 & r <= \text{threshold} \end{cases}$$



Images are from the Digital Image Processing textbook, see slide 7

Basic Grey Level Transformations

- There are many different kinds of grey level transformations
- Three of the most common are shown here
  - Linear
    - Negative/Identity
  - Logarithmic
    - Log/Inverse log
  - Power law
    - n^{th} power/n^{th} root

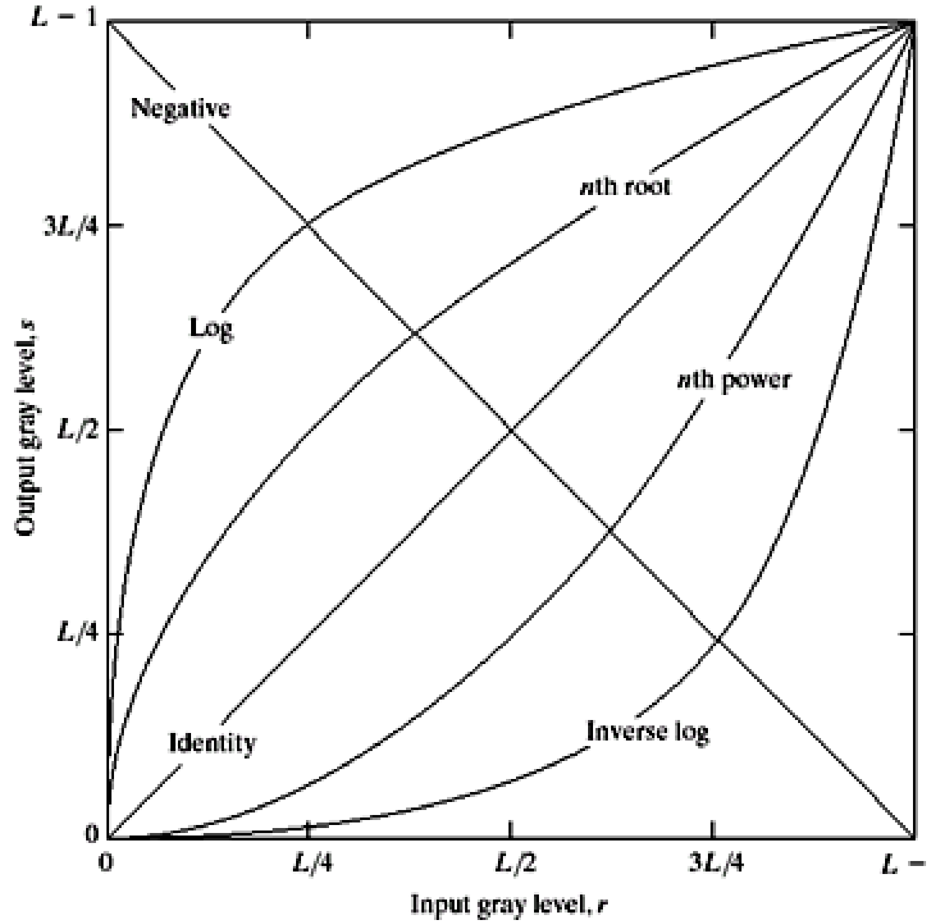
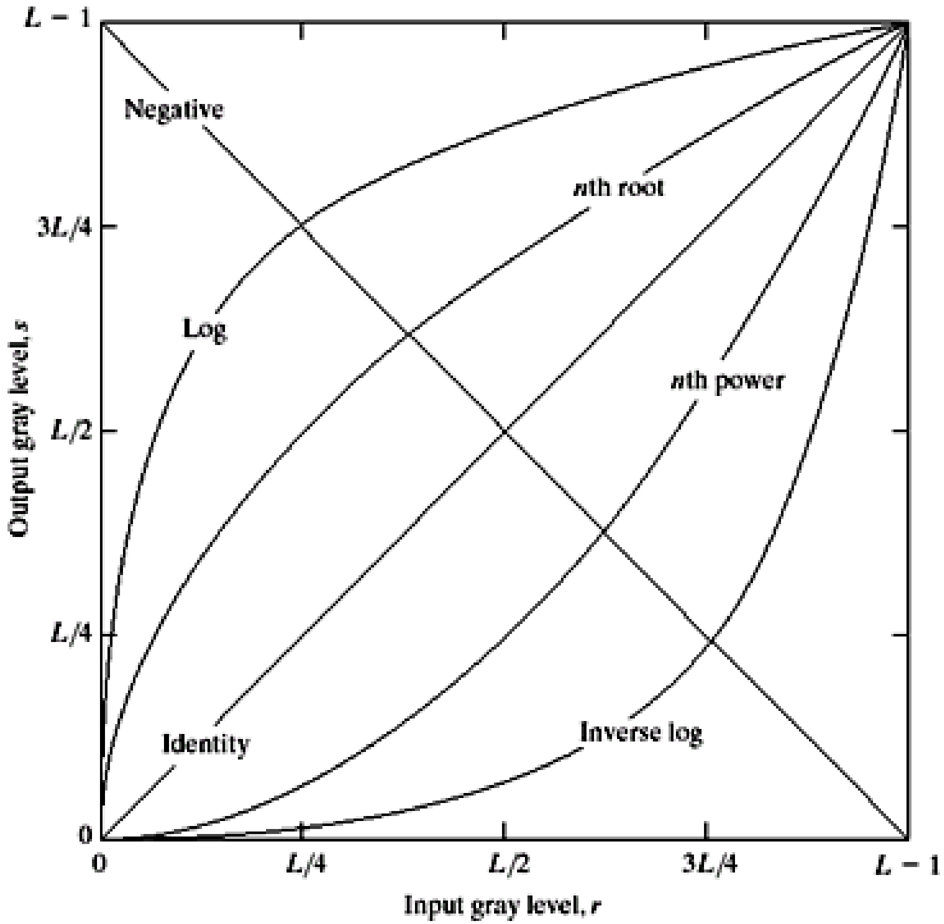


Image Negatives (Complement)

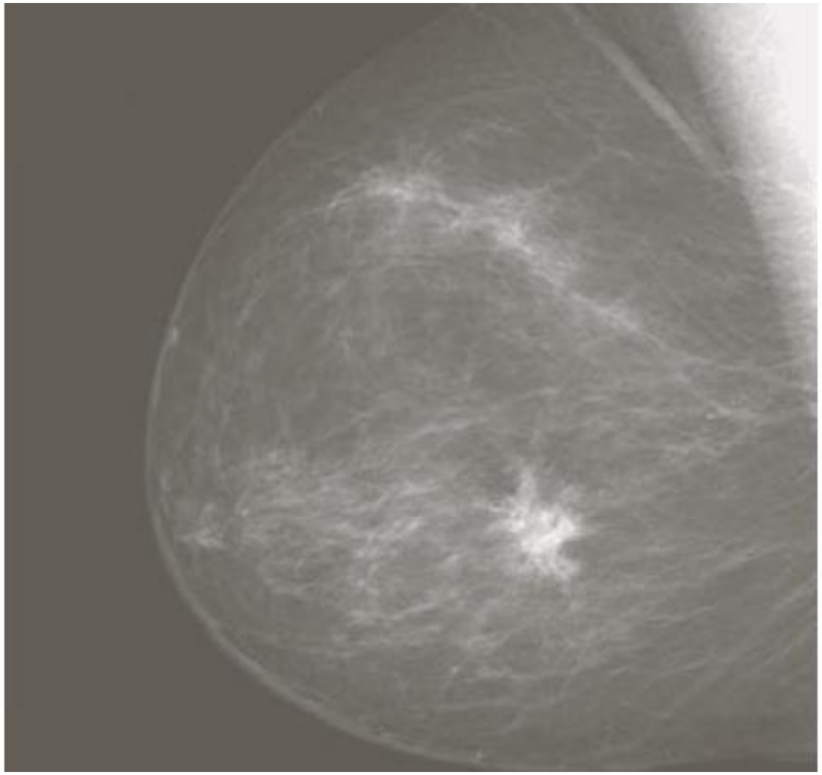


- An image with gray level in the range ***[0, L-1]*** where ***$L = 2^n$*** ; n = 1, 2…
- Negative transformation :

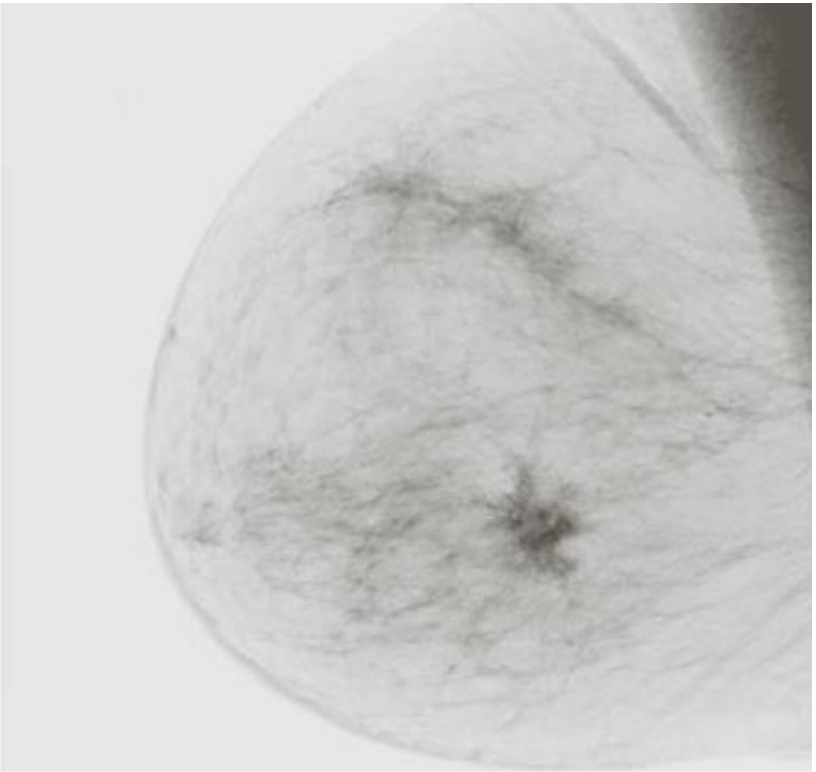
  $$s = L - 1 - r$$

- Reversing the intensity levels of an image.
- Suitable for enhancing white or gray detail in dark background.

Example- image negatives



Original Image showing a small lesion

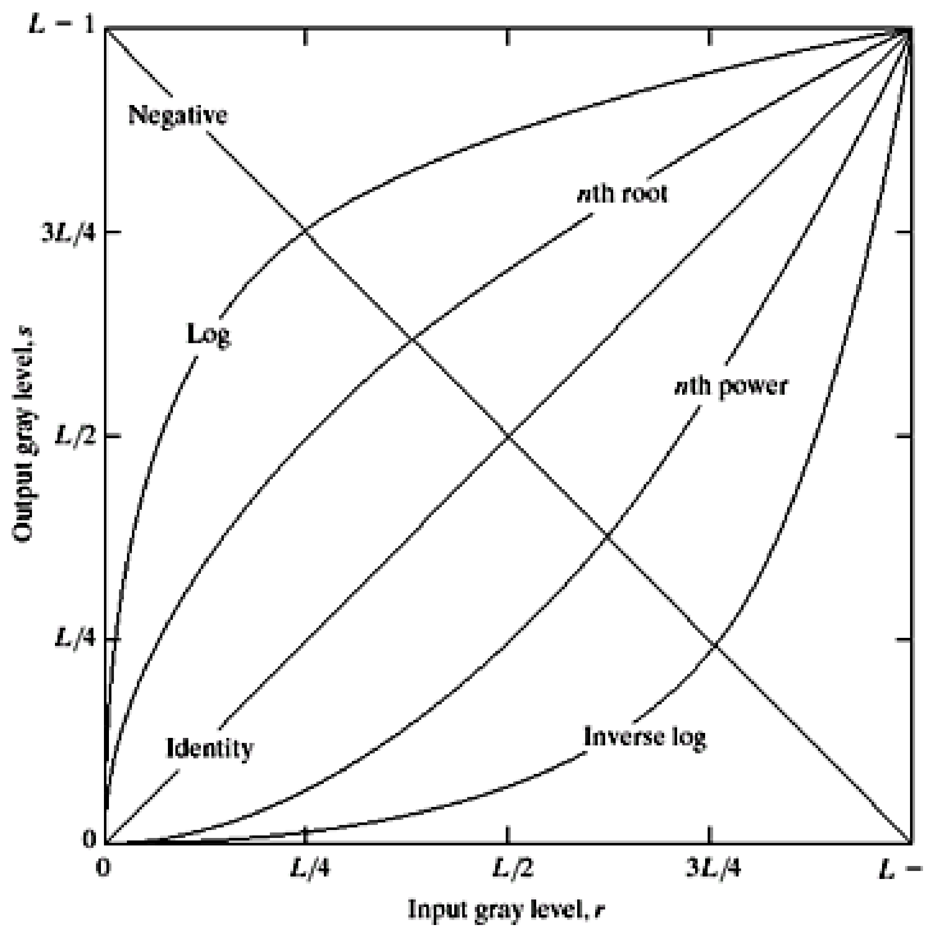


Negative Image : gives a better vision to analyze the image

Logarithmic Transformations

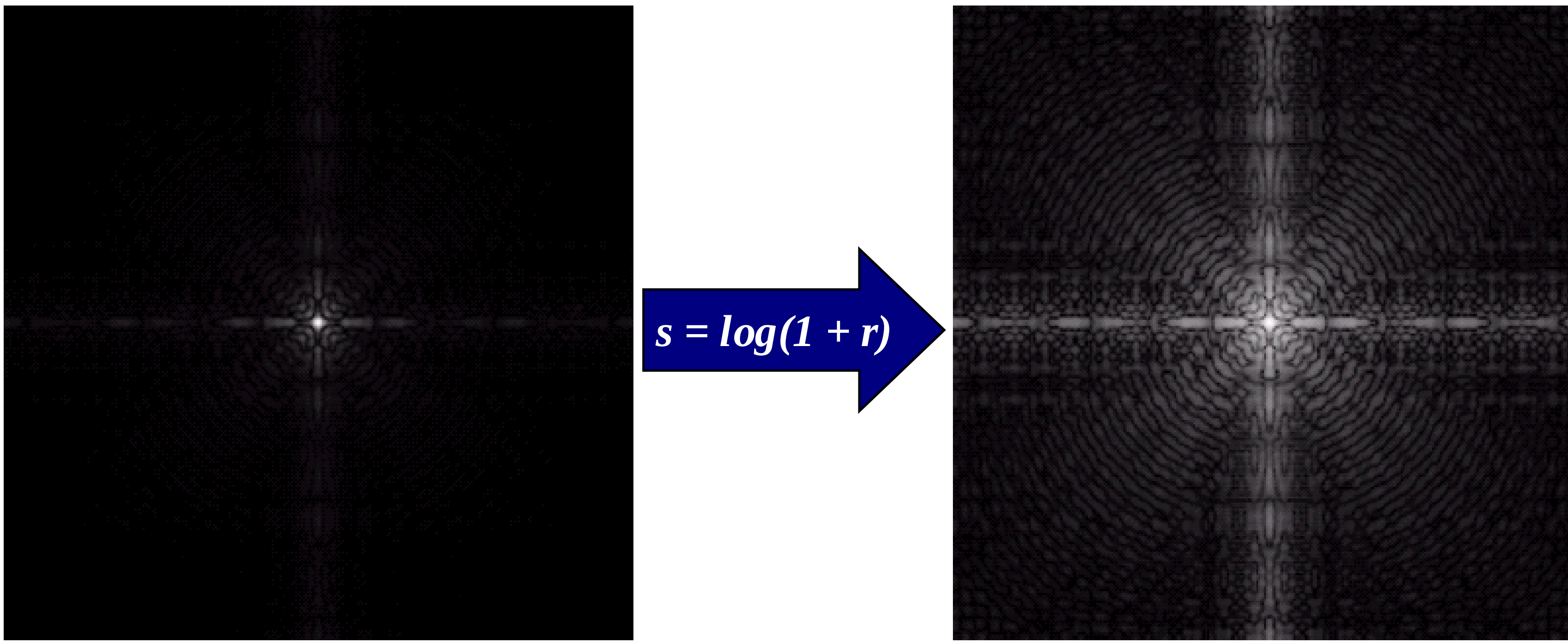
$$s = c * log(1 + r)$$

- c is a constant
- Maps a narrow range of low input grey level values into a wider range of output values
  - Expands dark values to **enhance details of dark area**
- The inverse log transformation performs the opposite transformation



Logarithmic Transformations (cont…)

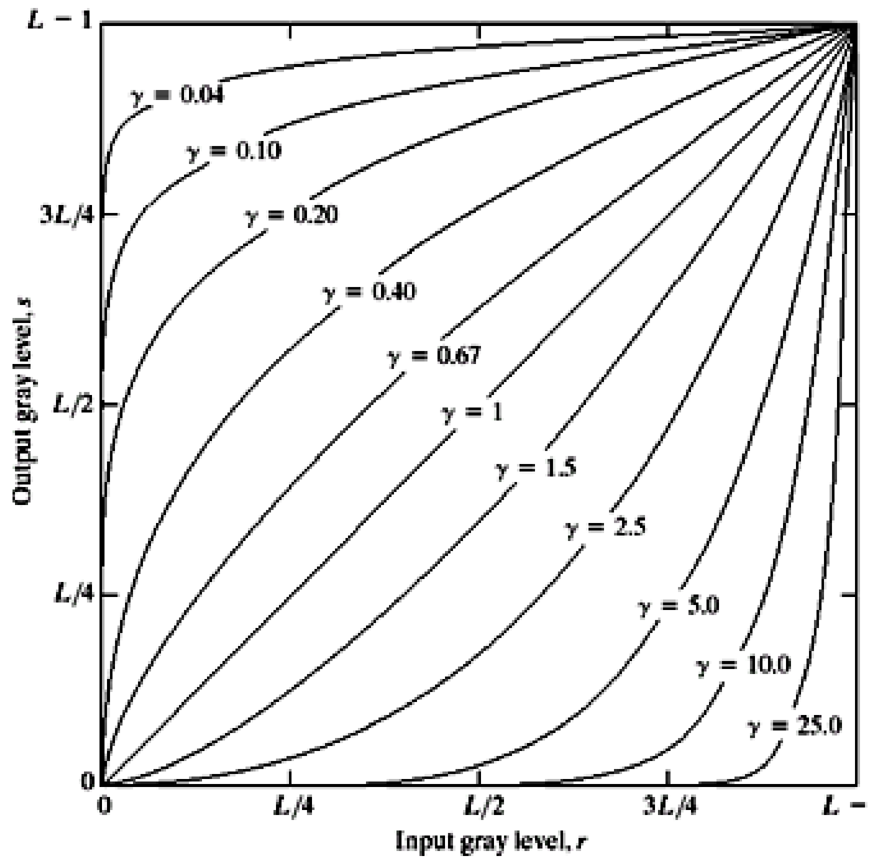
- In the following example the Fourier transform of an image is put through a log transform to reveal more detail



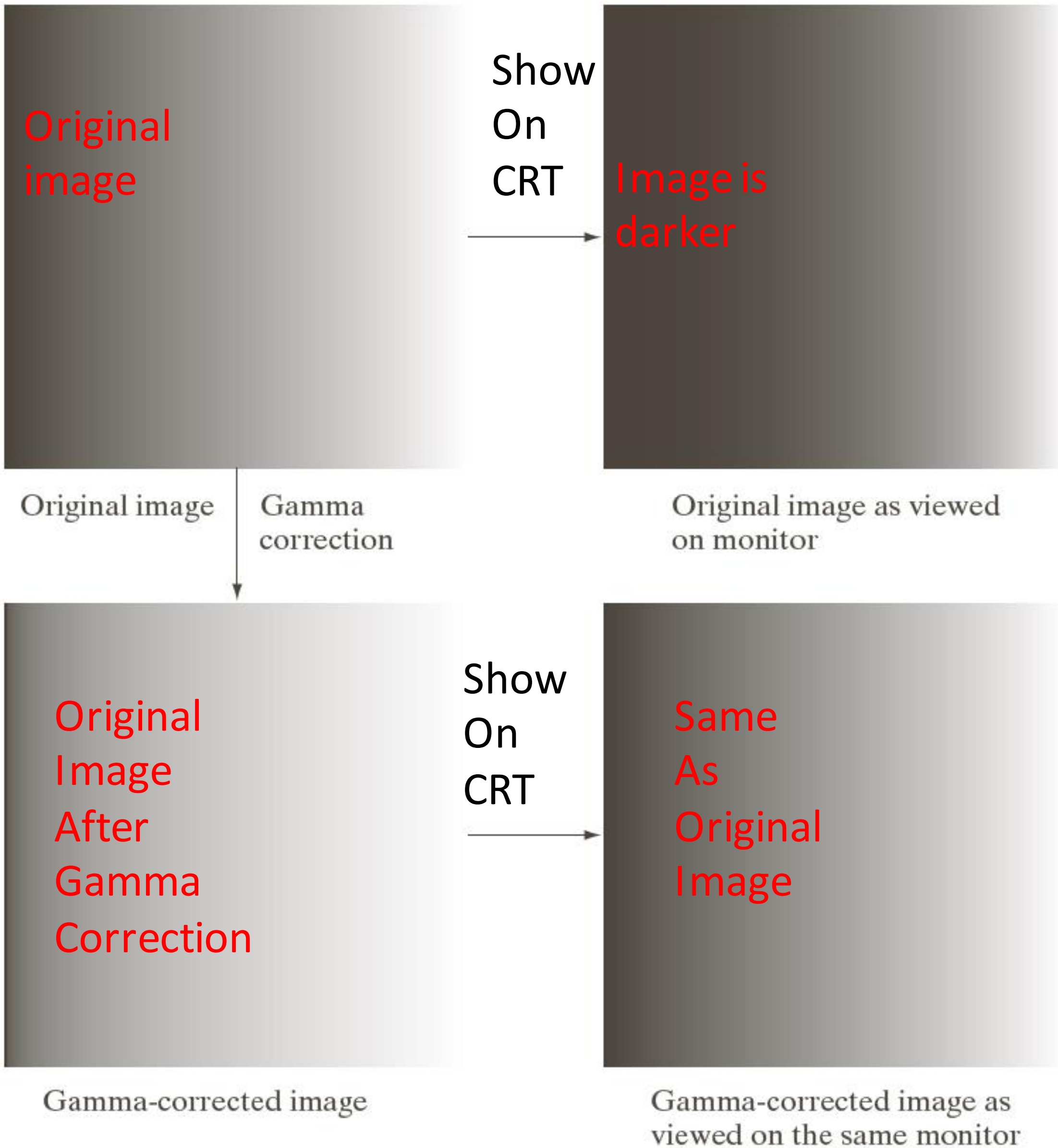
Power Law Transformations

$$s = c * r^{\gamma}$$

- c and γ are positive constants
- Maps a narrow range of dark input values into a wider range of output values or vice versa



Power-Law (Gamma) Transformations



- Cathode ray tube (CRT) devices have an intensity-to-voltage response that is a power function, with γ varying from 1.8 to 2.5
- LCD or LED display have same behavior
- The picture will become darker.
- **Gamma correction** is done by preprocessing the image before inputting it to the monitor with $\boldsymbol{s = cr^{1/\gamma}}$

Images are from the Digital Image Processing textbook, see slide 7

Power Law Example

- Magnetic resonance image (MRI) of a fractured human spine

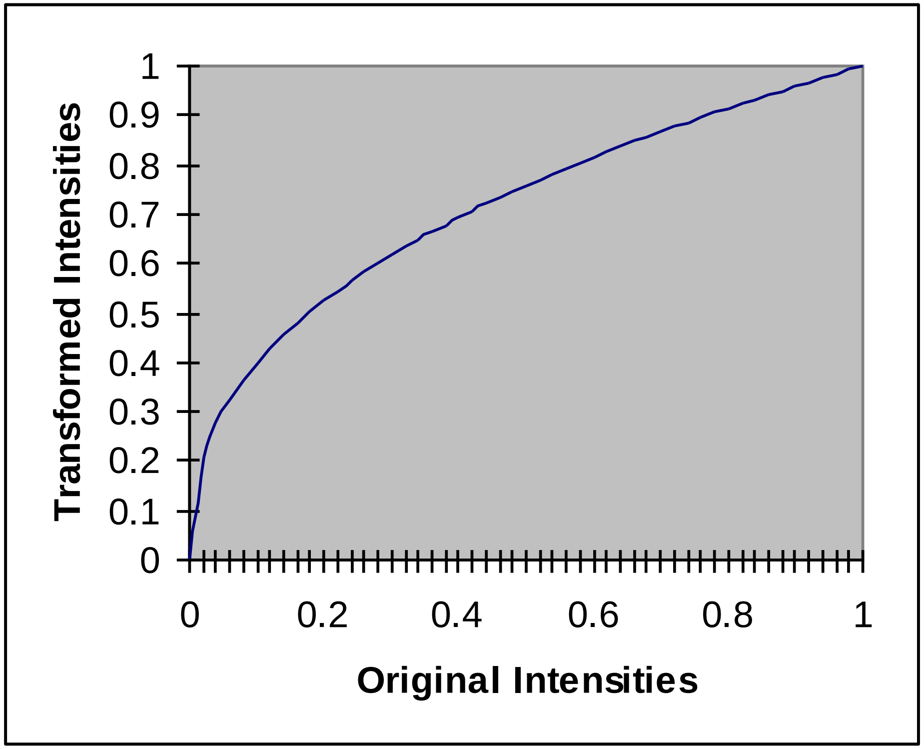
- Dark picture

Power Law Example (cont…)

$$\gamma = 0.6$$

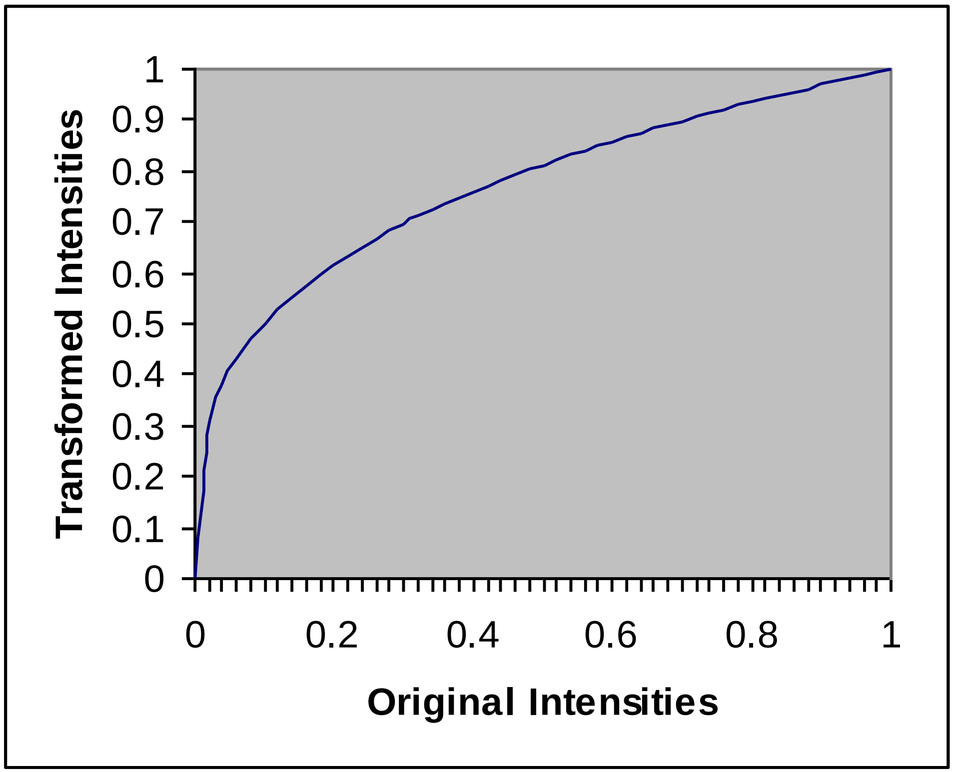
Power Law Example (cont…)

$$\gamma = 0.4$$



Power Law Example (cont…)

$\gamma = 0.3$



Power Law Example (cont…)

- Too small γ

$\rightarrow$

reduced contrast

$\rightarrow \rightarrow \rightarrow$

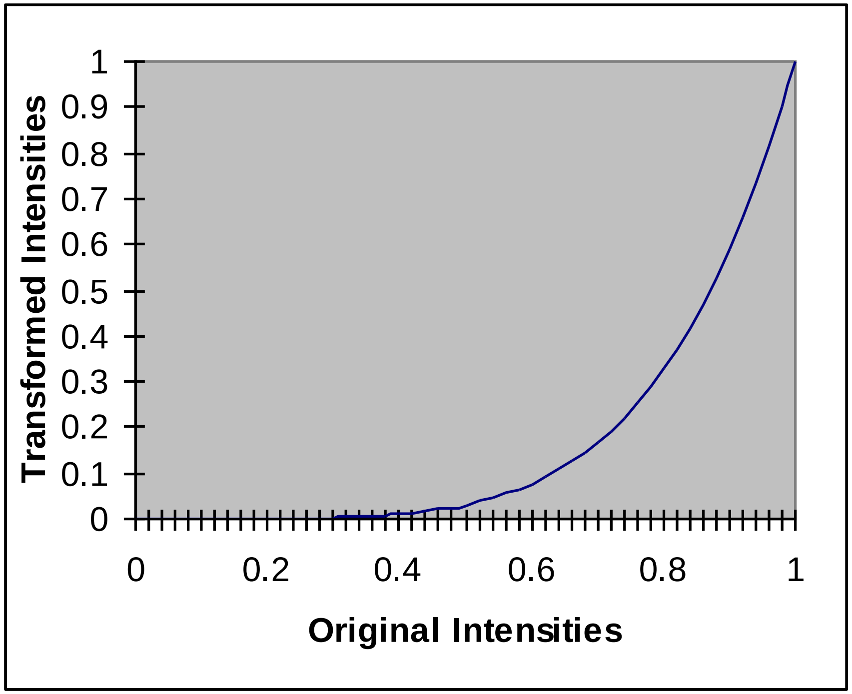
the image starts to have a “washout” look
  - especially in the background

Power Law Example

- Washed-out appearance

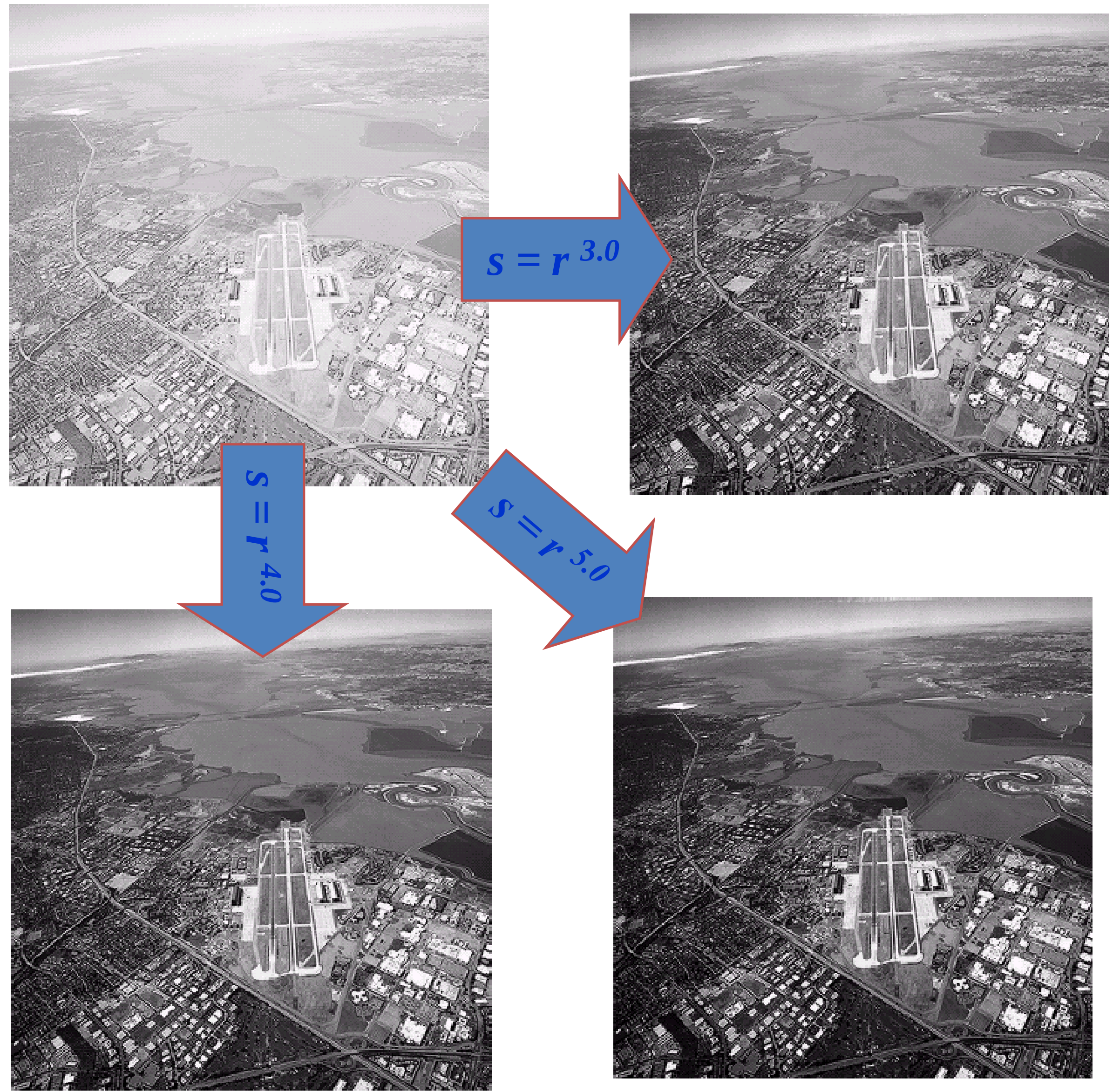
Power Law Example (cont…)

$\gamma = 5.0$



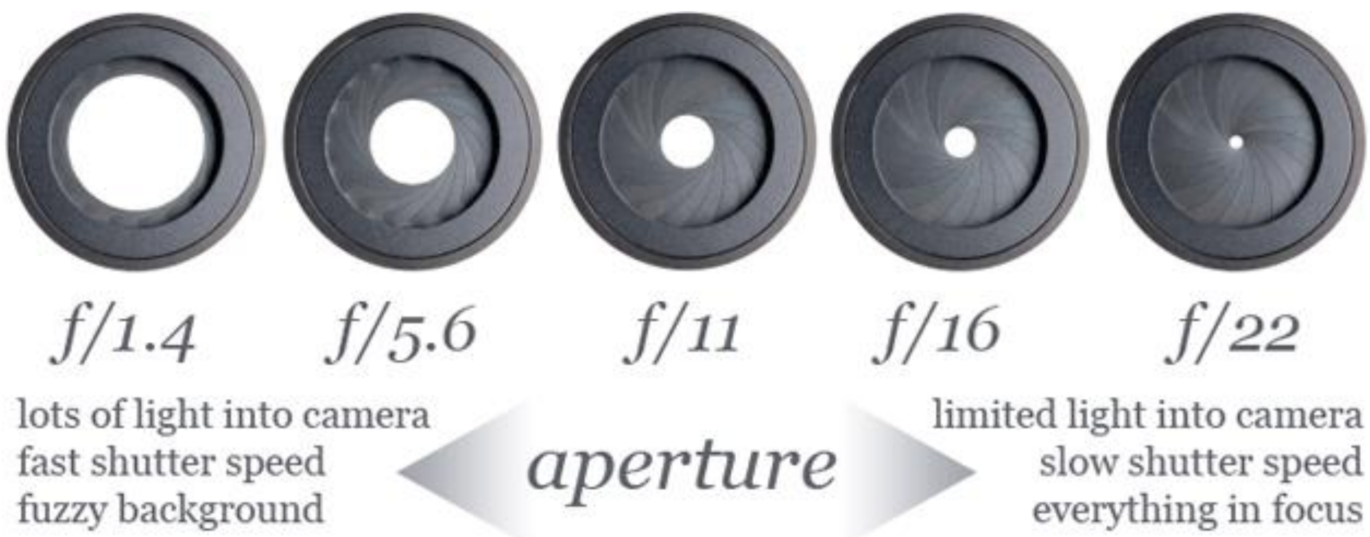
Power Law Transformations (cont…)

- The power law transforms are used to darken the image
- Different curves highlight different detail



Low-contrast images

- Can result from:
  - poor illumination
  - lack of dynamic range in the imaging sensor
  - wrong setting of a lens aperture during image acquisition.



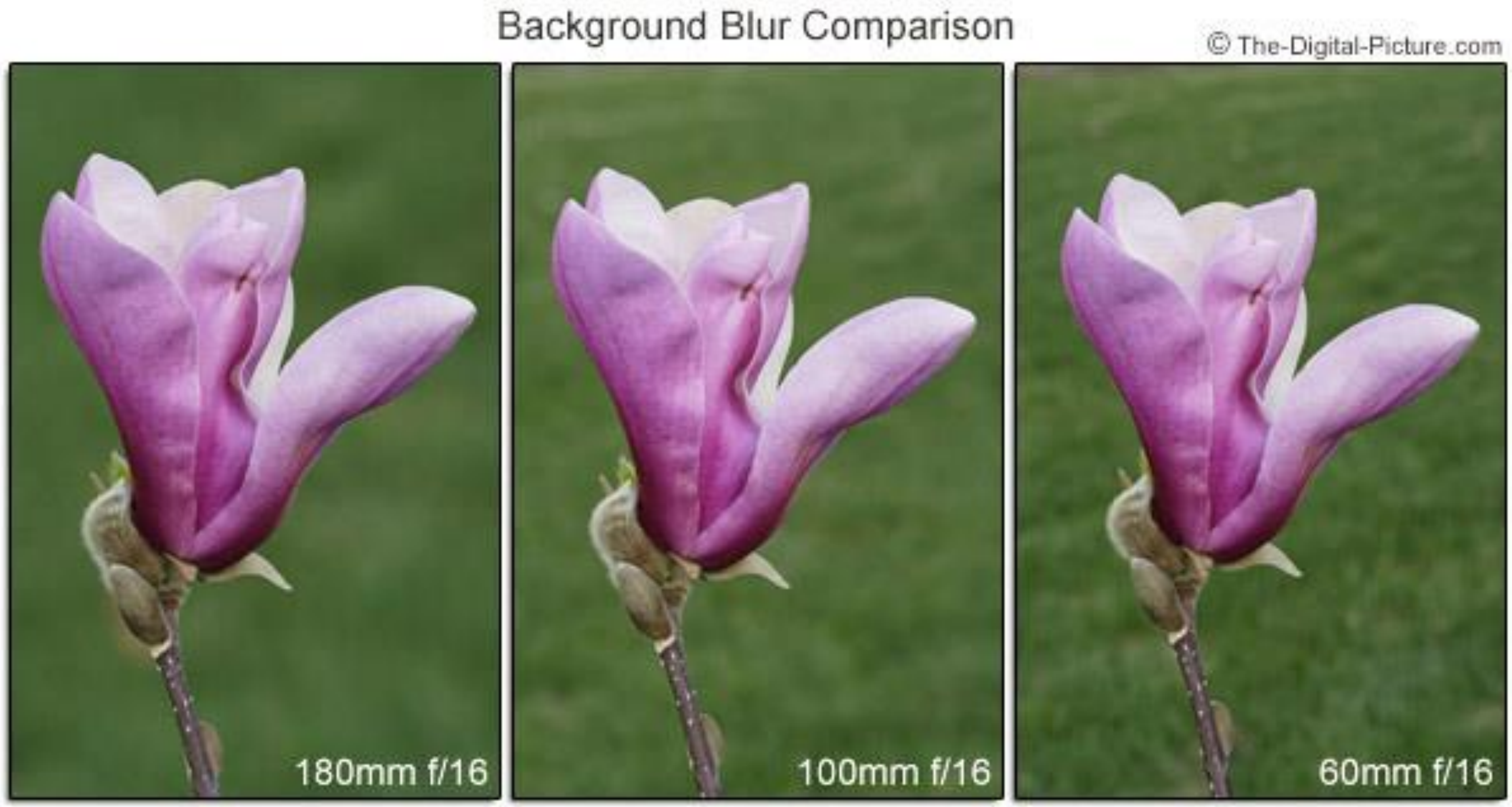
Images are from the Digital Image Processing textbook, see slide 7

Different setting of a lens aperture during image acquisition

Aperture set at f/2.8 - Less depth of field

Aperture set at f/11 - Greater depth of field

Different focal length imaging acquisition



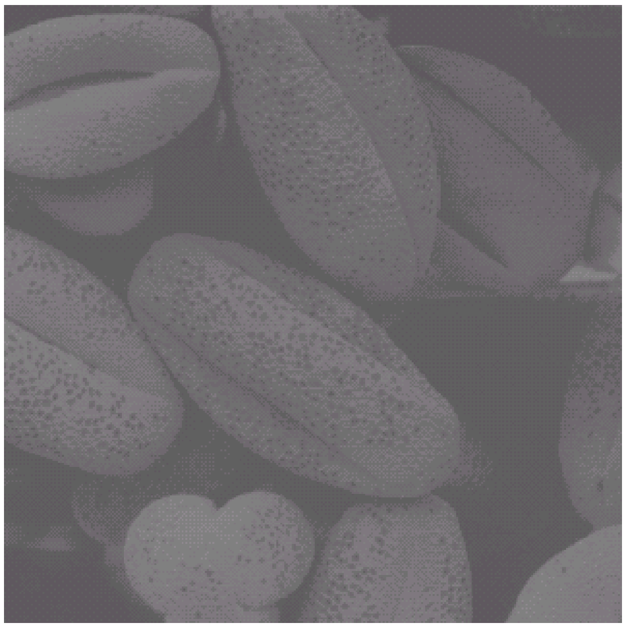
- The distance between the lens and the image sensor when the subject is in focus
  - Shorter focal length → focus in a shorter distance

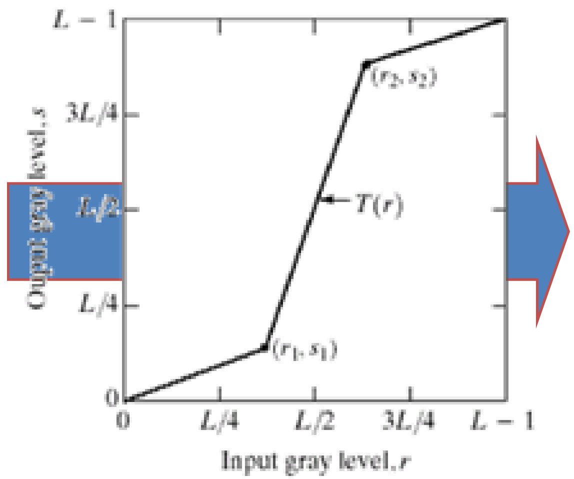
Piecewise Linear Transformation Functions

- Rather than using a well defined mathematical function we can use **arbitrary user-defined transforms**
- Advantage: Allow **more control** on the complexity of T(r).
- Disadvantage: Their specification requires considerably **more user input**
- Applications:
  - Contrast stretching.
  - Gray-level slicing.
  - Bit-plane slicing.

Contrast stretching

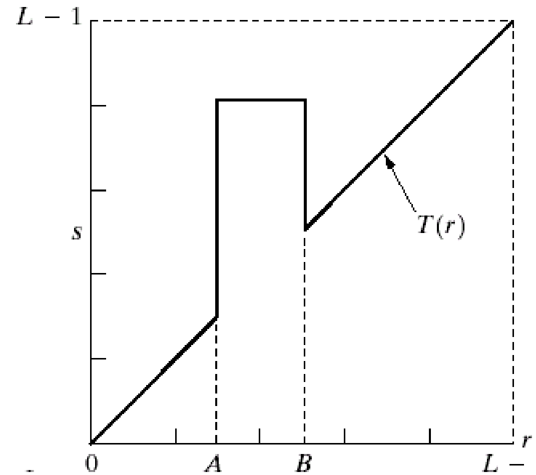
- Increases the dynamic range of gray levels
- Contrast stretching linear

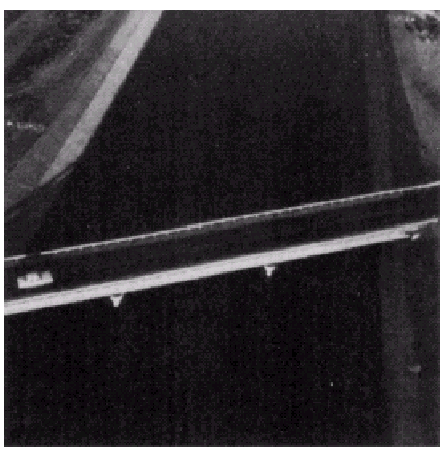


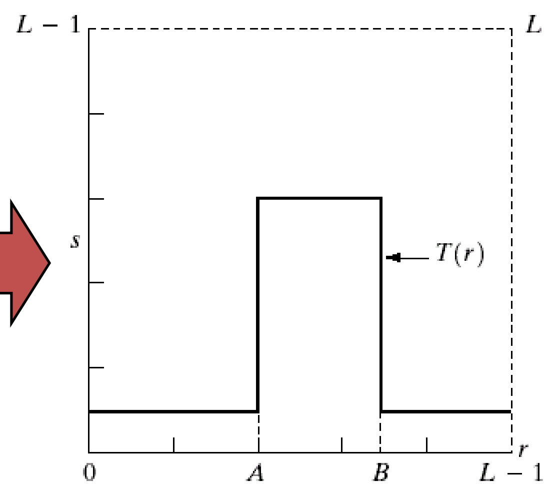


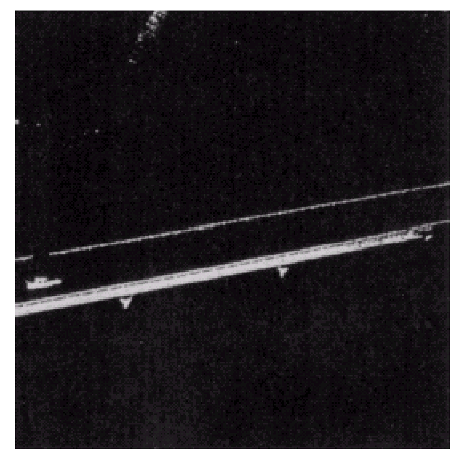
Gray Level Slicing

- Highlights a specific range of grey levels
  - Other levels can be suppressed or maintained
  - Useful for highlighting features in an image



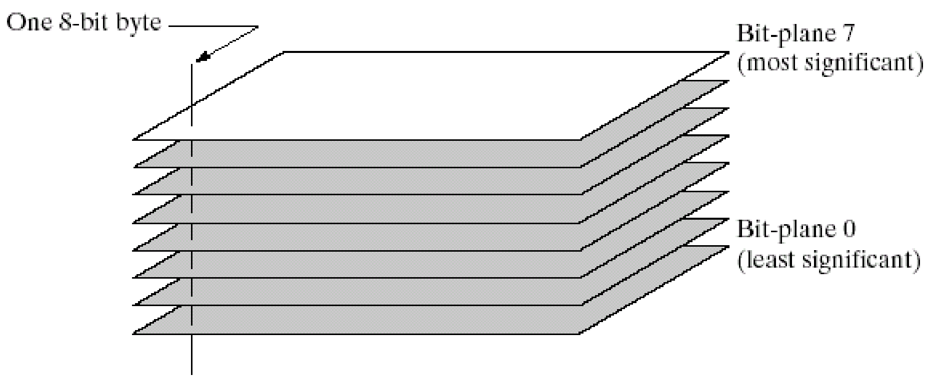


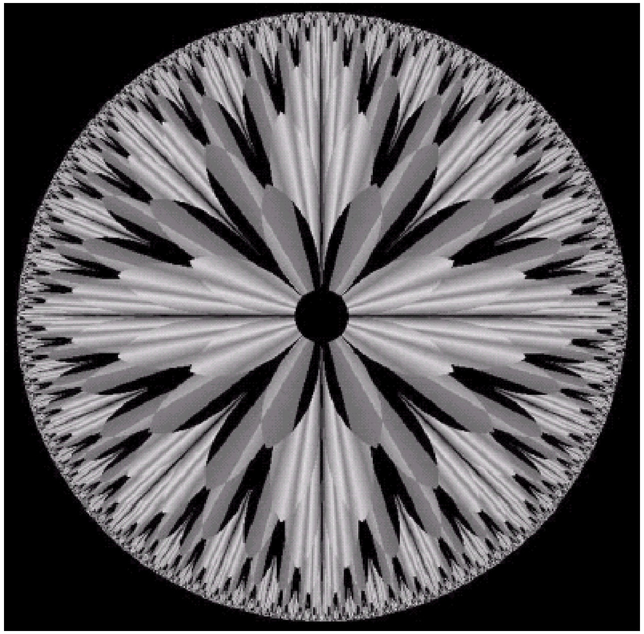




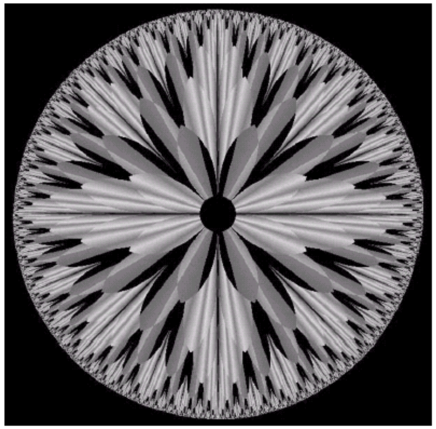
Bit Plane Slicing

- Often by isolating particular bits of the pixel values in an image we can highlight interesting aspects of that image
  - Higher-order bits usually contain most of the significant visual information
  - Lower-order bits contain subtle details
- Analyzing the relative importance of each bit

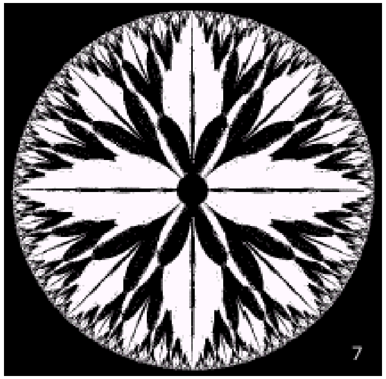


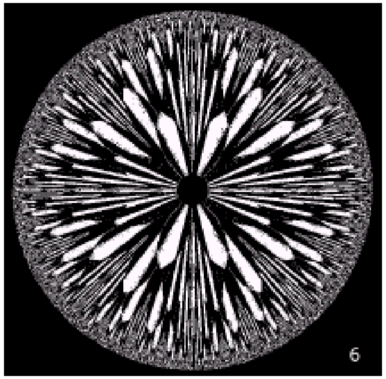


Bit Plane Slicing (cont...)



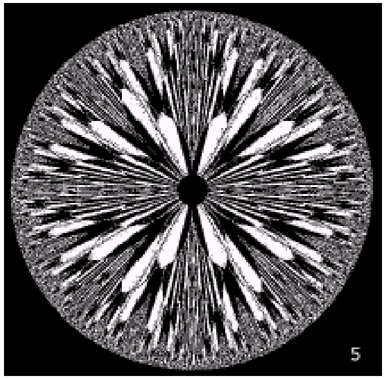
[10000000]

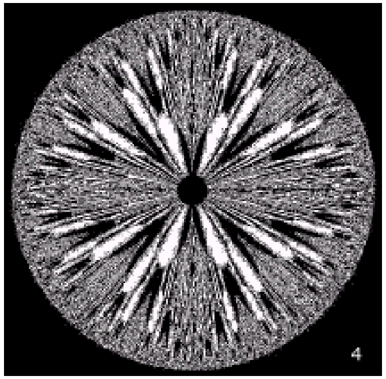


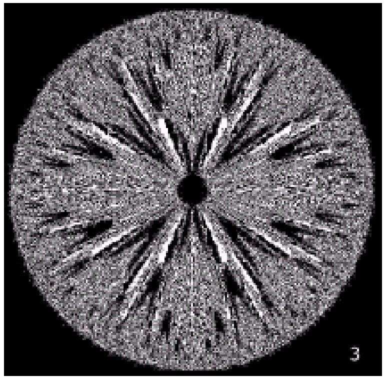


[01000000]

[00100000]

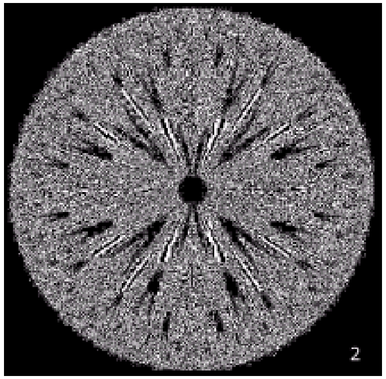


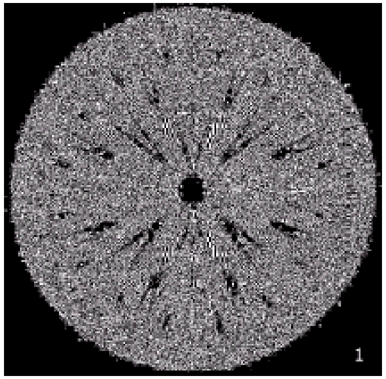


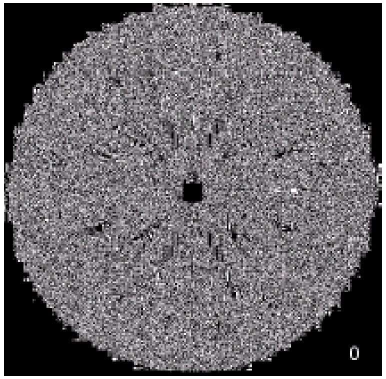


[00001000]

[00000100]







[00000001]

Bit Plane Slicing (cont…)

a	b	c
d	e	f
g	h	i

FIGURE 3.14 (a) An 8-bit gray-scale image of size 500 × 1192 pixels. (b) through (i) Bit planes 1 through 8, with bit plane 1 corresponding to the least significant bit. Each bit plane is a binary image.

Bit Plane Slicing (cont…)

Reconstructed image using only bit planes 8 and 7

Reconstructed image using only bit planes 8, 7 and 6

Reconstructed image using only bit planes 7, 6 and 5