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8) X, Y – индикаторы событий A, B, которые означают полож. ответы на вопросы α, β соц. анкеты

таблица распределения (X,Y)

X \ Y	0	1
0	P_{11}	P_{12}
1	P_{21}	P_{22}

положительному ответу присвоен ранг 1, отриц. – 0

$P_{11} = 0{,}03 \quad P_{21} = 0{,}05 \quad P_{12} = 0{,}02 \quad P_{22} = 0{,}9$

Найти коэффициент корреляции ρ_{XY}

8.1 таблицы распределения для X, Y используя ф-лы согласованности

X	0	1
p	0,05	0,95

Y	0	1
p	0,08	0,92

X \ Y	$y_1=0$	$y_2=1$	p_i
$x_1=0$	0,03	0,02	0,05
$x_2=1$	0,05	0,9	0,95
q_j	0,08	0,92	1 \ 1

общая таблица

(1) 8.2 m_X, m_Y

$m_X = 0 \cdot 0{,}05 + 1 \cdot 0{,}95 = \underline{\underline{0{,}95}}$

$m_Y = 0 \cdot 0{,}08 + 1 \cdot 0{,}92 = \underline{\underline{0{,}92}}$

8.3 $\alpha_{2X} = M[X^2]$, $\alpha_{2Y} = M[Y^2]$

$\alpha_{2X} = 0^2 \cdot 0{,}05 + 1^2 \cdot 0{,}95 = 0{,}95$

$\alpha_{2Y} = 0^2 \cdot 0{,}08 + 1^2 \cdot 0{,}92 = 0{,}92$

(2) 8.4 $D_X, D_Y, \sigma_X\ \sigma_Y$

$D_X = \alpha_{2X} - m_X^2 = 0{,}95 - 0{,}95^2 = \underline{\underline{0{,}0475}}$

$D_Y = \alpha_{2Y} - m_Y^2 = 0{,}92 - 0{,}92^2 = \underline{\underline{0{,}0736}}$

$\sigma_X = \sqrt{D_X} = \sqrt{0{,}0475} \simeq \underline{\underline{0{,}2179}}$

$\sigma_Y = \sqrt{D_Y} = \sqrt{0{,}0736} \simeq \underline{\underline{0{,}2713}}$

8.5 $M[XY]$

$M[XY] = \cancel{0 \cdot 0 \cdot 0{,}03} + \cancel{0 \cdot 1 \cdot 0{,}02} + \cancel{1 \cdot 0 \cdot 0{,}05} + 1 \cdot 1 \cdot 0{,}9 = 0{,}9$

3) 8.6 K_{XY}

$K_{XY} = M[XY] - m_X \cdot m_Y = 0{,}9 - 0{,}95 \cdot 0{,}92 = 0{,}9 - 0{,}874 = \underline{\underline{0{,}026}}$

4) 8.7 $\rho_{XY} = K_{XY}/(\sigma_X \sigma_Y) = 0{,}026/(0{,}2179 \cdot 0{,}2713) \simeq 0{,}026/0{,}059 \simeq \underline{\underline{0{,}4407}}$

События A, B имеют значительную положительную связь.

5) 8.доп Условие распределения $p(x_i/y_j)$, $p(y_j/x_i)$

$$p(x_i/y_j) = \frac{P(X=x_i, Y=y_j)}{P(Y=y_j)} = \frac{p_{ij}}{q_j}$$

$$p\left(\frac{x_1}{y_1}\right) = \frac{0{,}03}{0{,}08} = \underline{\underline{0{,}375}} \quad p\left(\frac{x_1}{y_2}\right) = \frac{0{,}02}{0{,}92} \approx \underline{\underline{0{,}022}} \quad p\left(\frac{x_2}{y_1}\right) = \frac{0{,}05}{0{,}08} \approx \underline{\underline{1{,}67}} \quad p\left(\frac{x_2}{y_2}\right) = \frac{0{,}9}{0{,}92} \approx \underline{\underline{0{,}978}}$$

$$q(y_j/x_i) = \frac{P(X=x_i, Y=y_j)}{P(X=x_i)} = \frac{p_{ij}}{p_i}$$

$$q\left(\frac{y_1}{x_1}\right) = \frac{0{,}03}{0{,}05} = \underline{\underline{0{,}6}} \quad q\left(\frac{y_1}{x_2}\right) = \frac{0{,}05}{0{,}95} \approx \underline{\underline{0{,}053}} \quad q\left(\frac{y_2}{x_1}\right) = \frac{0{,}02}{0{,}05} = \underline{\underline{0{,}4}} \quad q\left(\frac{y_2}{x_2}\right) = \frac{0{,}9}{0{,}95} \approx \underline{\underline{0{,}947}}$$

6) 8.доп

$$\mathrm{cov}_{XY} = \begin{bmatrix} \mathrm{cov}(X,X) & \mathrm{cov}(X,Y) \\ \mathrm{cov}(X,Y) & \mathrm{cov}(YX) \end{bmatrix} = \begin{bmatrix} DX & K_{XY} \\ K_{XY} & DY \end{bmatrix} = \begin{bmatrix} 0{,}0475 & 0{,}026 \\ 0{,}026 & 0{,}0736 \end{bmatrix}$$

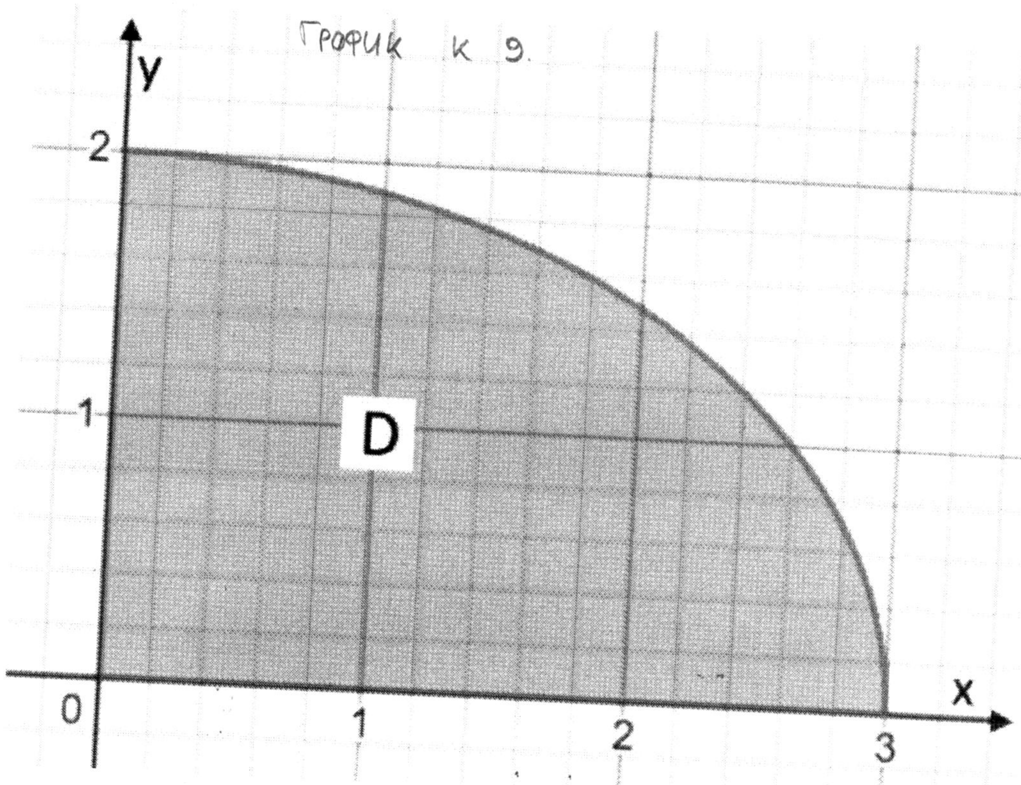
$$\det(\mathrm{cov}_{XY}) = 0{,}0475 \cdot 0{,}0736 - (0{,}026)^2 = 0{,}00282$$

$$(X,Y) \to m = (0{,}95,\ 0{,}92);\ |\Sigma| = |\mathrm{cov}_{XY}| = 0{,}00282$$

7) 8.доп

$$\mathrm{cor}_{XY} = \begin{bmatrix} 1 & 0{,}4407 \\ 0{,}4407 & 1 \end{bmatrix}$$

График к 9.



9) (X,Y) распределена равномерно в области D

D - четверть эллипса $\frac{x^2}{9}+\frac{y^2}{4}\le 1$ $x\ge 0,\ y\ge 0$, $y=\frac{2\sqrt{9-x^2}}{3}$ $x=\frac{3\sqrt{4-y^2}}{2}$

9.1 Плотность вероятности $f_{XY}(x,y)$ $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ ← эллипс, его площадь $S=a\cdot b\cdot\pi$

$$f_{XY}(x,y)=\begin{cases}\frac{2}{3\pi} & (x,y)\in D\\ 0, & (x,y)\notin D\end{cases}$$

$S_D=\frac{\sqrt{9}\cdot\sqrt{4}\cdot\pi}{4}=\frac{3\pi}{2}$

$f(x,y)=1/S_D\ [(x,y)\in D]=1/\frac{3\pi}{2}=\frac{2}{3\pi}$

9.2 $f_X(x), f_Y(y)$

$$f_X(x)=\int_{-\infty}^{+\infty}f_{XY}(x,y)\,dy=\int_{-\infty}^{0}0\,dy+\int_{0}^{\frac{2\sqrt{9-x^2}}{3}}\frac{2}{3\pi}\,dy+\int_{\frac{2\sqrt{9-x^2}}{3}}^{+\infty}0\,dy=$$

$=\frac{4\sqrt{9-x^2}}{9\pi}$ при $x\in[0;3]$ (0 при $x\notin[0;3]$)

$$f_Y(y)=\int_{-\infty}^{+\infty}f_{XY}(x,y)\,dx=\int_{-\infty}^{0}0\,dx+\int_{0}^{\frac{3\sqrt{4-y^2}}{2}}\frac{2}{3\pi}\,dx+\int_{\frac{3\sqrt{4-y^2}}{2}}^{+\infty}0\,dx=\frac{\sqrt{4-y^2}}{\pi}$$ при $y\in[0;2]$

(0 при $y\notin[0;2]$)

9.3 $$m_X=\int_{-\infty}^{+\infty}x f_X(x)\,dx=\int_0^3 x\frac{4\sqrt{9-x^2}}{9\pi}dx=\frac{4}{9\pi}\int_0^3 x\sqrt{9-x^2}\,dx=\begin{bmatrix}t=9-x^2\\dt=-2x\,dx\end{bmatrix}$$

$$=-\frac{2}{9\pi}\int_9^0\sqrt{t}\,dt=\frac{2}{9\pi}\int_0^9\sqrt{t}\,dt=\frac{4t^{3/2}}{27\pi}\Big|_0^9=\frac{4\cdot 9^{3/2}}{27\pi}-\cancel{\frac{4\cdot 0^{3/2}}{27\pi}}=\frac{4\cdot\cancel{27}}{\cancel{27}\pi}=$$

$=\frac{4}{\pi}$

$$m_Y=\int_{-\infty}^{+\infty}y f_Y(y)\,dy=\int_0^2 y\frac{\sqrt{4-y^2}}{\pi}dy=\frac{1}{\pi}\int_0^2 y\sqrt{4-y^2}\,dy=\begin{bmatrix}t=4-y^2\\dt=-2y\,dy\end{bmatrix}=$$

$$=-\frac{1}{2\pi}\int_4^0\sqrt{t}\,dt=\frac{1}{2\pi}\int_0^4\sqrt{t}\,dt=\frac{t^{3/2}}{3\pi}\Big|_0^4=\frac{4^{3/2}}{3\pi}-\cancel{\frac{0^{3/2}}{3\pi}}=\frac{8}{3\pi}$$

9.4 $D_X=M[X^2]-m_X^2$

$$M[X^2]=\int_{-\infty}^{+\infty}x^2 f_X(x)\,dx=\int_0^3 x^2\frac{4\sqrt{9-x^2}}{9\pi}dx=\frac{4}{9\pi}\int_0^3 x^2\sqrt{9-x^2}\,dx=$$

$$=\begin{bmatrix}x=3\sin(t)\\dx=3\cos(t)\,dt\end{bmatrix}=\frac{4}{3\pi}\int_0^{\pi/2}27\sin^2(t)\cos(t)\sqrt{\cos^2(t)}\,dt=$$

$$=\frac{36}{\pi}\int_0^{\pi/2}\sin^2(t)\cos(t)\sqrt{\cos^2(t)}\,dt=\frac{36}{\pi}\int_0^{\frac{\pi}{2}}(1-\cos^2(t))\cos(t)\sqrt{\cos^2(t)}\,dt=$$

$$= \frac{36}{\pi}\int_0^{\pi/2}(\cos^2(t)-\cos^4(t))dt = -\frac{36}{\pi}\int_0^{\pi/2}\cos^4(t)dt + \frac{36}{\pi}\int_0^{\frac{\pi}{2}}\cos^2(t)dt =$$

$$= \underbrace{-\frac{9\sin(t)\cos^3(t)}{\pi}\Big|_0^{\frac{\pi}{2}}}_{0} + \frac{9}{\pi}\int_0^{\frac{\pi}{2}}\cos^2(t)dt = \frac{9}{\pi}\int_0^{\frac{\pi}{2}}\cos^2(t)dt =$$

$$= \frac{9}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}\cos(2t)+\frac{1}{2}\right)dt = \frac{9}{2\pi}\int_0^{\frac{\pi}{2}}\cos(2t)dt + \frac{9}{2\pi}\int_0^{\frac{\pi}{2}}dt =$$

$$= \begin{bmatrix} k=2t \\ dk=2dt \end{bmatrix} = \frac{9}{4\pi}\int_0^{\pi}\cos(k)dk + \frac{9}{2\pi}\int_0^{\frac{\pi}{2}}dt = \underbrace{\frac{9\sin(k)}{4\pi}\Big|_0^{\pi}}_{0} + \frac{9}{2\pi}\int_0^{\frac{\pi}{2}}dt =$$

$$= \frac{9}{2\pi}\int_0^{\frac{\pi}{2}}dt = \frac{9t}{2\pi}\Big|_0^{\pi/2} = \frac{9\cancel{\pi}}{2\cancel{\pi}\cdot 2} - \cancel{\frac{9\cdot 0}{2\pi}} = \frac{9}{4}$$

$$D_x = \frac{9}{4} - \frac{16}{\pi^2} = \frac{9\pi^2-64}{4\pi^2}$$

$$\sigma_x = \sqrt{\frac{9\pi^2-64}{4\pi^2}} \simeq 0{,}793$$

$$M[Y^2] = \int_{-\infty}^{+\infty} y^2 f_Y(y)\,dy = \int_0^2 y^2\frac{\sqrt{4-y^2}}{\pi}dy = \frac{1}{\pi}\int_0^2 y^2\sqrt{4-y^2}\,dy =$$

$$= \begin{bmatrix} y=2\sin(t) \\ dy=2\cos(t)dt \end{bmatrix} = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} 8\sin^2(t)\cos(t)\sqrt{\cos^2(t)}dt = \frac{16}{\pi}\int_0^{\frac{\pi}{2}}\sin^2(t)\cos(t)\sqrt{\cos^2(t)}dt =$$

$$= \frac{16}{\pi}\int_0^{\frac{\pi}{2}}\cos(t)\sqrt{\cos^2(t)}(1-\cos^2(t))dt = \frac{16}{\pi}\int_0^{\frac{\pi}{2}}(\cos^2(t)-\cos^4(t))\,dt =$$

$$= -\frac{16}{\pi}\int_0^{\frac{\pi}{2}}\cos^4(t)\,dt + \frac{16}{\pi}\int_0^{\pi/2}\cos^2(t)\,dt = \underbrace{\left(-\frac{4\sin(t)\cos^3(t)}{\pi}\right)\Big|_0^{\frac{\pi}{2}}}_{0} +$$

$$+ \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\cos^2(t)dt = \frac{4}{\pi}\int_0^{\pi/2}\cos^2(t)dt = \frac{4}{\pi}\int_0^{\frac{\pi}{2}}\left(\frac{1}{2}\cos(2t)+\frac{1}{2}\right)dt =$$

$$= \frac{2}{\pi}\int_0^{\frac{\pi}{2}}\cos(2t)dt + \frac{2}{\pi}\int_0^{\frac{\pi}{2}}dt = \begin{bmatrix} k=2t \\ dk=2dt \end{bmatrix} = \frac{1}{\pi}\int_0^{\pi}\cos(k)dk +$$

$$+ \frac{2}{\pi}\int_0^{\frac{\pi}{2}}dt = \underbrace{\frac{\sin(k)}{\pi}\Big|_0^{\pi}}_{0} + \frac{2}{\pi}\int_0^{\frac{\pi}{2}}dt = \frac{2}{\pi}\int_0^{\frac{\pi}{2}}dt = \frac{2t}{\pi}\Big|_0^{\frac{\pi}{2}} =$$

$$= \frac{\cancel{2}\cancel{\pi}}{\cancel{\pi}\cancel{2}} - \frac{2\cdot\cancel{0}}{\cancel{\pi}} = 1$$

$$D_Y = 1 - \frac{64}{9\pi^2} = \frac{9\pi^2-64}{9\pi^2}$$

$$\sigma_Y = \sqrt{D_Y} = \sqrt{\frac{9\pi^2-64}{9\pi^2}} \approx 0{,}529$$

9.5 $K_{XY} = M[XY] - m_x m_Y$

$$M[XY] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xy\, f_{XY}(x,y)\,dx\,dy = \frac{2}{3\pi}\iint_D xy\,dx\,dy = \frac{2}{3\pi}\int_0^3 x\,dx \int_0^{\frac{2\cdot\sqrt{9-x^2}}{3}} y\,dy =$$

$$= \frac{2}{3\pi}\int_0^3 x\left(\frac{y^2}{2}\Big|_0^{\frac{2\cdot\sqrt{9-x^2}}{3}}\right)dx = \frac{1}{3\pi}\int_0^3 \frac{x\cdot 4\cdot(9-x^2)}{9}\,dx = \frac{4}{27\pi}\int_0^3 x(9-x^2)\,dx =$$

$$= \begin{bmatrix} t = x^2 \\ dt = 2x\,dx \end{bmatrix} = \frac{2}{27\pi}\int_0^9 (9-t)\,dt = -\frac{2}{27\pi}\int_0^9 t\,dt + \frac{2}{3\pi}\int_0^9 dt =$$

$$= \left(-\frac{t^2}{27\pi}\right)\Big|_0^9 + \frac{2}{3\pi}\int_0^9 dt = -\frac{81}{27\pi} + \frac{2t}{3\pi}\Big|_0^9 = -\frac{81}{27\pi} + \frac{18}{3\pi} =$$

$$= \frac{81}{27\pi} = \frac{3}{\pi} \approx 0{,}9549$$

$$K_{XY} = M[XY] - m_x \cdot m_y = \frac{3}{\pi} - \left(\frac{4}{\pi}\cdot\frac{8}{3\pi}\right) = \frac{3}{\pi} - \frac{32}{3\pi^2} \approx -0{,}12585$$

$$\rho_{XY} = \frac{K_{XY}}{(\sigma_X \sigma_Y)} = \frac{-0{,}12585}{0{,}793\cdot 0{,}529} \approx -0{,}300$$

9.6 X, Y зависимы, т.к. $\rho_{XY} \neq 0$.