

Transformation.

$$\hat{y} = 1 + 0.9x + \hat{e}$$

$$1. - x^* = x - 10$$

$$x = x^* + 10$$

$$\hat{y} = 1 + 0.9(x^* + 10) + \hat{e}$$

$$\hat{y} = 10 + 0.9x^* + \hat{e}$$

$$\Rightarrow \boxed{\hat{\alpha}^* = 10, \hat{\beta}^* = 0.9}$$

$$\text{Since } \hat{e}^* = \hat{e} \Rightarrow \boxed{\hat{\sigma}^* = \hat{\sigma} = 2}$$

$$r = \sqrt{R^2} = \sqrt{1 - \frac{\sum (y_i - \hat{y}_i)^2}{\sum (y_i - \bar{y})^2}}, \hat{y}_i^* = \hat{y}_i$$

$$\Rightarrow \boxed{r^* = r = 0.3}$$

$$- x^* = 10x$$

$$x = \frac{x^*}{10}$$

$$\hat{y} = 1 + 0.9 \cdot \frac{x^*}{10} + \hat{e}$$

$$\Rightarrow \boxed{\hat{\alpha}^* = 1, \hat{\beta}^* = \frac{0.9}{10} = 0.09, \hat{\sigma}^* = \hat{\sigma} = 2, r^* = r = 0.3}$$

$$- x^* = 10(x - 1)$$

$$x = \frac{x^*}{10} + 1$$

$$\hat{y} = 1 + 0.9\left(\frac{x^*}{10} + 1\right) + \hat{e}$$

$$= 10 + 0.09x^* + \hat{e}$$

$$\Rightarrow \boxed{\hat{\alpha}^* = 10, \hat{\beta}^* = 0.09, \hat{\sigma}^* = \hat{\sigma} = 2, r^* = r = 0.3}$$

$$2. - y = y^{**} - 10 = 1 + 0.9x + \hat{e}$$

$$y^{**} = 11 + 0.9x + \hat{e}$$

$$\Rightarrow \boxed{\hat{\alpha}^{**} = 11, \hat{\beta}^{**} = 0.9, \hat{\sigma}^{**} = \hat{\sigma} = 2}$$

$$\text{since } \hat{y}_i^{**} = \hat{y}_i + 10, \bar{y}^{**} = \bar{y} + 10$$

$$\text{we know } R^{2**} = R^2, \text{ so } \boxed{r^{**} = r = 0.3}$$

$$- y^{**} = 5y \Rightarrow y = \frac{y^{**}}{5} = 1 + 0.9x + \hat{e}$$

$$y^{**} = 5 + 4.5x + 5\hat{e}$$

$$\Rightarrow \boxed{\hat{\alpha}^{**} = 5, \hat{\beta}^{**} = 4.5, \text{ since } \hat{e}^{**} = 5\hat{e}, \text{ so } \boxed{\hat{\sigma}^{**} = 5\hat{\sigma} = 10}}$$

$$- y^{**} = 5(y+2)$$

$$y = \frac{y^{**}}{5} + 2 = 1 + 0.9x + \hat{e}$$

$$y^{**} = 5 + 4.5\hat{x} + 5\hat{e}$$

$$\Rightarrow \begin{cases} \hat{\alpha}^{**} = 5, \hat{\beta}^{**} = 4.5, \hat{\sigma}^{**} = 5\sigma = 10 \\ r^{**} = r = 0.3 \end{cases}$$

3. Transformation of  $X$ :

-  $x+c$ :  $\alpha$  will change to  $\alpha - c\beta$   
 $\beta$ ,  $r$  and  $\sigma$  will not change

-  $cx$ :  $\beta$  will change to  $\frac{\beta}{c}$   
 $\alpha$ ,  $r$  and  $\sigma$  will not change

Transformation of  $y$ :

-  $y+c$ :  $\alpha$  will change to  $\alpha + c$   
 $\beta$ ,  $r$ , and  $\sigma$  will not change

-  $cy$ :  $\alpha$  will change to  $c\alpha$ ,  $\beta$  will change to  $c\beta$ ,  
 $\sigma$  will change to  $c\sigma$   
 $r$  will not change.

$$4. \beta^* = \frac{\hat{\beta}}{10} = \frac{0.9}{10} = 0.09$$

$$SE(\hat{\beta}^*) = \frac{SE(\hat{\beta})}{10} = \frac{0.03}{10} = 0.003$$

$$t_0^* = \frac{\hat{\beta}^*}{SE(\hat{\beta}^*)} = \frac{0.09}{0.003} = 30$$

$$5. \hat{\beta}^{**} = 5\hat{\beta} = 5 \cdot 0.9 = 4.5$$

$$SE(\hat{\beta}^{**}) = 5 \cdot SE(\hat{\beta}) = 5 \cdot 0.03 = 0.15$$

$$t_0^{**} = \frac{4.5}{0.15} = 30$$

$$b. [\bar{\beta} - t_{\alpha/2} \cdot SE(\beta), \bar{\beta} + t_{\alpha/2} \cdot SE(\beta)]$$

transformation of  $X$ :

-  $x+c$ : will not change CI or hypothesis test.

-  $cx$ :  $\bar{\beta}^* = \frac{\bar{\beta}}{c}$ , CI will change to  $[\frac{\bar{\beta}}{c} - t_{\alpha/2} \cdot \frac{SE(\beta)}{c}, \frac{\bar{\beta}}{c} + t_{\alpha/2} \cdot \frac{SE(\beta)}{c}]$   
hypothesis test result will not change.

transformation of  $y$ :

-  $y+c$ : will not change CI or hypothesis test.

-  $cy$ : CI will change to  $[c\bar{\beta} - t_{\alpha/2} \cdot SE(\beta) \cdot c, c\bar{\beta} + t_{\alpha/2} \cdot SE(\beta) \cdot c]$   
hypothesis test result will not change.