MA684 homework 08

Xinyi Wang

November 10, 2016

Getting to know stan

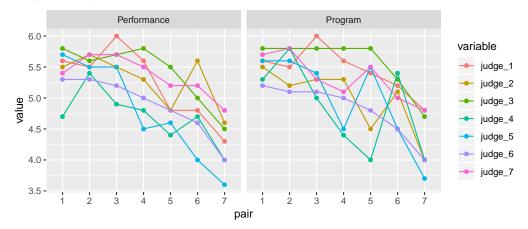
Read through the tutorial on Stan https://github.com/stan-dev/rstan/wiki/RStan-Getting-Started

• Explore Stan website and Stan reference manual and try to connect them with Gelman and Hill 16 - 17.

Data analysis

Using stan:

The folder olympics has seven judges' ratings of seven figure skaters (on two criteria: "technical merit" and "artistic impression") from the 1932 Winter Olympics. Take a look at http://www.stat.columbia.edu/~gelman/arm/examples/olympics/olympics1932.txt



##		Program	${\tt Performance}$	pair	Judge
##	1:	5.6	5.6	1	judge_1
##	2:	5.5	5.5	1	judge_2
##	3:	5.8	5.8	1	judge_3
##	4:	5.3	4.7	1	judge_4
##	5:	5.6	5.7	1	judge_5
##	6:	5.2	5.3	1	judge_6

use stan to fit a non-nested multilevel model (varying across skaters and judges) for the technical merit ratings.

$$y_i \sim N(\mu + \gamma_{j[i]} + \delta_{k[i]}, \sigma_y^2), \text{ for } i = 1, \dots, n$$
 (1)

$$\gamma_j \sim N(0, \sigma_\gamma^2) j = 1, \dots, 7 \tag{2}$$

$$\delta_k \sim N(0, \sigma_{\delta}^2)k = 1, \dots, 7 \tag{3}$$

 $https://github.com/stan-dev/example-models/blob/master/ARM/Ch.17/17.3_flight_simulator.stan\ https://github.com/stan-dev/example-models/blob/master/ARM/Ch.17/17.3_non-nested_models.R$

```
fit_program<-lmer(Program~1+(1|pair) + (1|Judge),olympics_long)</pre>
dataList.1 <- list(N=49, n_judges=7, n_pairs=7, judge=as.integer(olympics_long$Judge), pair=as.integer
skating_stan<-"
data {
  int<lower=0> N;
  int<lower=0> n_judges;
  int<lower=0> n_pairs;
  int<lower=0,upper=n_judges> judge[N];
  int<lower=0,upper=n_pairs> pair[N];
  vector[N] y;
}
parameters {
  real<lower=0> sigma;
  real<lower=0> sigma_gamma;
  real<lower=0> sigma delta;
  vector[n_judges] gamma;
  vector[n_pairs] delta;
  real mu;
}
model {
  vector[N] y_hat;
  sigma ~ uniform(0, 100);
  sigma_gamma ~ uniform(0, 100);
  sigma_delta ~ uniform(0, 100);
  mu ~ normal(0, 100);
  gamma ~ normal(0, sigma_gamma);
  delta ~ normal(0, sigma_delta);
  for (i in 1:N)
    y_hat[i] = mu + gamma[judge[i]] + delta[pair[i]];
  y ~ normal(y_hat, sigma);
}
```

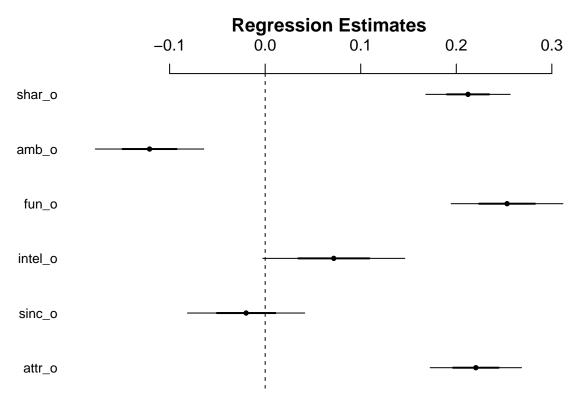
pilots <- read.table ("http://www.stat.columbia.edu/~gelman/arm/examples/pilots/pilots.dat", header=TRUE)

flight simulator.sf1 <- stan(model code=skating stan, data=dataList.1, iter=2000, chains=4)

Multilevel logistic regression

The folder speed.dating contains data from an experiment on a few hundred students that randomly assigned each participant to 10 short dates with participants of the opposite sex (Fisman et al., 2006). For each date, each person recorded several subjective numerical ratings of the other person (attractiveness, compatibility, and some other characteristics) and also wrote down whether he or she would like to meet the other person again. Label $y_{ij}=1$ if person i is interested in seeing person j again 0 otherwise. And r_{ij1},\ldots,r_{ij6} as person i's numerical ratings of person j on the dimensions of attractiveness, compatibility, and so forth. Please look at http://www.stat.columbia.edu/~gelman/arm/examples/speed.dating/Speed%20Dating%20Data%20Key.doc for details.

```
dating<-fread("http://www.stat.columbia.edu/~gelman/arm/examples/speed.dating/Speed%20Dating%20Data.csv
dating_pooled <- glm(match~attr_o +sinc_o +intel_o +fun_o +amb_o +shar_o,data=dating,family=binomial)
dating_pooled <- glmer(match~gender + attr_o +sinc_o +intel_o +fun_o +amb_o +shar_o+(1|iid)+(1|pid),dat
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.67667 (tol
## = 0.001, component 1)
  1. Fit a classical logistic regression predicting Pr(y_{ij}=1) given person i's 6 ratings of person j. Discuss
    the importance of attractiveness, compatibility, and so forth in this predictive model.
m1 <- glm(match~attr_o +sinc_o +intel_o +fun_o +amb_o +shar_o,data=dating,family=binomial)
summary(m1)
##
## Call:
## glm(formula = match ~ attr_o + sinc_o + intel_o + fun_o + amb_o +
##
       shar_o, family = binomial, data = dating)
##
## Deviance Residuals:
       Min
                 10
                      Median
                                   3Q
                                           Max
## -1.5300 -0.6362 -0.4420 -0.2381
                                        3.1808
## Coefficients:
##
               Estimate Std. Error z value Pr(>|z|)
## (Intercept) -5.62091 0.21859 -25.714 < 2e-16 ***
## attr_o
               0.22047
                           0.02388
                                     9.233 < 2e-16 ***
## sinc_o
               -0.01996
                           0.03067 -0.651
                                             0.5152
## intel_o
               0.07176
                           0.03716
                                     1.931
                                             0.0535 .
## fun o
               0.25315
                           0.02922
                                    8.665 < 2e-16 ***
                           0.02838 -4.264 2.01e-05 ***
## amb o
              -0.12099
## shar o
               0.21225
                           0.02209
                                    9.608 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 6466.6 on 7030 degrees of freedom
## Residual deviance: 5611.0 on 7024 degrees of freedom
     (1347 observations deleted due to missingness)
## AIC: 5625
## Number of Fisher Scoring iterations: 5
coefplot(m1)
```



The result of this model can be written like:

$$P(match = 1) = logit^{-1}(-5.6 + 0.22attr - 0.02sinc + 0.07intel + 0.25fun - 0.12amb + 0.21shar)$$

Attractiveness: One point higher in attractiveness will lead to 5.5%(0.22/4=0.055) higher willingness of another date.

Sincerity: One point higher in sincerity will lead to 0.5%(-0.02/4=-0.005) lower willingness of another date, which is contrary to our expectation.

Intelligence: One point higher in intelligence will lead to 1.75%(0.07/4=0.0175) higher willingness of another date.

Fun: One point higher in humor will lead to 6.25%(0.25/4=0.0625) higher willingness of another date.

Ambition: One point higher in ambition will lead to 3%(0.12/4=-0.03) lower willingness of another date.

Shared interest: One point higher in shared interest will lead to 5.25%(0.21/4=0.0525) higher willingness of another date.

2. Expand this model to allow varying intercepts for the persons making the evaluation; that is, some people are more likely than others to want to meet someone again. Discuss the fitted model.

```
m2 <- lmer(match~gender+scale(attr_o) +scale(sinc_o) +scale(intel_o) +scale(fun_o) +scale(amb_o) +scale
summary(m2)</pre>
```

```
## Generalized linear mixed model fit by maximum likelihood (Laplace
## Approximation) [glmerMod]
## Family: binomial ( logit )
## Formula:
## match ~ gender + scale(attr_o) + scale(sinc_o) + scale(intel_o) +
## scale(fun_o) + scale(amb_o) + scale(shar_o) + (1 | iid)
```

```
Data: dating
## Control:
## structure(list(optimizer = c("bobyqa", "Nelder_Mead"), calc.derivs = TRUE,
       use.last.params = FALSE, restart_edge = FALSE, boundary.tol = 1e-05,
##
       tolPwrss = 1e-07, compDev = TRUE, nAGQOinitStep = TRUE, checkControl = list(
           check.nobs.vs.rankZ = "ignore", check.nobs.vs.nlev = "stop",
##
           check.nlev.gtreq.5 = "ignore", check.nlev.gtr.1 = "stop",
##
           check.nobs.vs.nRE = "stop", check.rankX = "message+drop.cols",
##
           check.scaleX = "warning", check.formula.LHS = "stop",
##
           check.response.not.const = "stop"), checkConv = list(
##
           check.conv.grad = list(action = "warning", tol = 0.001,
              relTol = NULL), check.conv.singular = list(action = "ignore",
##
##
              tol = 1e-04), check.conv.hess = list(action = "warning",
              tol = 1e-06)), optCtrl = list()), class = c("glmerControl",
## "merControl"))
##
##
        AIC
                BIC
                       logLik deviance df.resid
##
             5605.0 -2762.6
                               5525.2
##
## Scaled residuals:
               1Q Median
      Min
                               3Q
## -1.7458 -0.4453 -0.2877 -0.1454 10.3764
##
## Random effects:
## Groups Name
                       Variance Std.Dev.
           (Intercept) 0.4294
                              0.6553
## Number of obs: 7031, groups: iid, 551
## Fixed effects:
                 Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                 -2.13226
                              0.07079 -30.122 < 2e-16 ***
## gender
                  0.15452
                              0.09322
                                       1.658
                                               0.0974 .
## scale(attr_o)
                  0.46047
                              0.05203
                                       8.850 < 2e-16 ***
## scale(sinc_o) -0.02474
                              0.05728 -0.432
                                               0.6658
## scale(intel o) 0.10874
                              0.06203
                                       1.753
                                               0.0796 .
                              0.06192
## scale(fun_o)
                  0.51341
                                       8.291 < 2e-16 ***
## scale(amb o)
                 -0.23570
                              0.05468 -4.311 1.63e-05 ***
## scale(shar_o)
                 0.48474
                              0.05045
                                       9.609 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Correlation of Fixed Effects:
               (Intr) gender scl(t_) scl(sn_) scl(n_) scl(f_) scl(m_)
## gender
              -0.672
## scale(ttr_) -0.202 0.109
## scale(snc_) -0.022 0.048 -0.123
## scale(ntl_) 0.026 -0.055 -0.039 -0.466
## scale(fun_) -0.156 0.015 -0.246 -0.150
                                              -0.132
## scale(amb_) 0.143 -0.092 -0.062 -0.014
                                              -0.370 -0.187
## scale(shr_) -0.135 0.009 -0.100 -0.054
                                              -0.005 -0.268 -0.203
## convergence code: 0
## unable to evaluate scaled gradient
## Model failed to converge: degenerate Hessian with 5 negative eigenvalues
```

```
P(match = 1) = logit^{-1}(-2.13 + 0.15gender + 0.46scale(attr) - 0.02scale(sinc) + 0.11scale(intel) +
0.51scale(fun) - 0.23scale(amb) + 0.48scale(shar) + iid_i
  3. Expand further to allow varying intercepts for the persons being rated. Discuss the fitted model.
m3 <- glmer(match~gender+scale(attr_o) +scale(sinc_o) +scale(intel_o) +scale(fun_o) +scale(amb_o) +scale
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge with max|grad| = 0.263517
## (tol = 0.001, component 1)
summary(m3)
## Generalized linear mixed model fit by maximum likelihood (Laplace
     Approximation) [glmerMod]
  Family: binomial (logit)
## Formula:
## match ~ gender + scale(attr_o) + scale(sinc_o) + scale(intel_o) +
       scale(fun_o) + scale(amb_o) + scale(shar_o) + (1 | iid) +
##
##
       (1 | pid)
##
      Data: dating
##
##
                 BIC
                       logLik deviance df.resid
        AIC
##
     5257.6
              5326.1 -2618.8
                                5237.6
                                            7021
##
## Scaled residuals:
##
       Min
                1Q Median
                                3Q
                                        Max
## -3.7847 -0.3824 -0.2194 -0.0917
##
## Random effects:
## Groups Name
                       Variance Std.Dev.
## iid
           (Intercept) 0.595
                                0.7713
           (Intercept) 1.262
                                1.1235
## Number of obs: 7031, groups: iid, 551; pid, 537
##
## Fixed effects:
                  Estimate Std. Error z value Pr(>|z|)
## (Intercept)
                  -2.53475
                              0.11733 -21.604 < 2e-16 ***
## gender
                   0.16773
                              0.14956
                                        1.121
                                                0.2621
                              0.06376 10.023 < 2e-16 ***
## scale(attr_o)
                   0.63906
## scale(sinc_o)
                   0.03499
                              0.06786
                                        0.516
                                                 0.6061
                              0.07360
                                        2.327
                                                 0.0200 *
## scale(intel_o) 0.17125
## scale(fun_o)
                   0.57661
                              0.07099
                                        8.122 4.59e-16 ***
## scale(amb_o)
                              0.06466
                                       -2.559
                                               0.0105 *
                  -0.16544
## scale(shar o)
                   0.58881
                              0.06158
                                        9.561 < 2e-16 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Correlation of Fixed Effects:
##
               (Intr) gender scl(t_) scl(sn_) scl(n_) scl(f_) scl(m_)
               -0.646
## gender
## scale(ttr_) -0.221
                      0.092
## scale(snc_) -0.049 0.036 -0.064
                                     -0.438
## scale(ntl ) -0.009 -0.044 -0.024
## scale(fun_) -0.139 0.008 -0.220 -0.123
                                               -0.098
## scale(amb_) 0.072 -0.070 -0.051
                                     0.011
                                              -0.334 -0.167
```

```
## scale(shr_) -0.138  0.004 -0.072  -0.057  -0.020 -0.234  -0.158
## convergence code: 0
## Model failed to converge with max|grad| = 0.263517 (tol = 0.001, component 1)
a_iid <- data.frame(ranef(m3))[1:5,4]
a_iid
## [1] 0.452243548 -0.465241025 -0.797514626 -0.324989639 -0.005380029
a_pid <- data.frame(ranef(m3))[552:556,4]</pre>
a_pid
## [1] 0.97842409 0.11652972 -1.79408020 -0.63282281 0.05274865
  4. You will now fit some models that allow the coefficients for attractiveness, compatibility, and the other
     attributes to vary by person. Fit a no-pooling model: for each person i, fit a logistic regression to the
     data y_{ij} for the 10 persons j whom he or she rated, using as predictors the 6 ratings r_{ij1}, \ldots, r_{ij6}.
     (Hint: with 10 data points and 6 predictors, this model is difficult to fit. You will need to simplify it in
     some way to get reasonable fits.)
m4 <- glm(match~attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o + factor(iid)-1,data=dating)
  5. Fit a multilevel model, allowing the intercept and the coefficients for the 6 ratings to vary by the rater i.
m5 <- glmer(match~(1+attr_o+sinc_o+intel_o+fun_o+amb_o+shar_o|iid) + attr_o + sinc_o + intel_o + fun_o
## Warning in optwrap(optimizer, devfun, start, rho$lower, control =
## control, : convergence code 1 from bobyqa: bobyqa -- maximum number of
## function evaluations exceeded
## Warning in (function (fn, par, lower = rep.int(-Inf, n), upper =
## rep.int(Inf, : failure to converge in 10000 evaluations
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : unable to evaluate scaled gradient
## Warning in checkConv(attr(opt, "derivs"), opt$par, ctrl =
## control$checkConv, : Model failed to converge: degenerate Hessian with 1
## negative eigenvalues
  6. Compare the inferences from the multilevel model in (5) to the no-pooling model in (4) and the
     complete-pooling model from part (1) of the previous exercise.
anova(m5, m1, m4)
## Data: dating
## Models:
## m1: match ~ attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o
## m5: match ~ (1 + attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o |
           iid) + attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o
## m4: match ~ attr_o + sinc_o + intel_o + fun_o + amb_o + shar_o +
## m4:
           factor(iid) - 1
       Df
                     BIC logLik deviance
                                             Chisq Chi Df Pr(>Chisq)
        7 5625.0 5673.0 -2805.5
## m1
                                    5611.0
## m5 35 5576.8 5816.8 -2753.4
                                    5506.8 104.23
                                                        28 1.034e-10 ***
## m4 558 5607.8 9434.6 -2245.9
                                    4491.8 1014.93
                                                       523 < 2.2e-16 ***
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
```