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HW 2 :

1. $Y = 1, 2, 3, 4, 6, 7, 8$

$$P_Y(k) = \begin{cases} \frac{1}{42} & k=1 \\ \frac{2}{42} = \frac{1}{21} & k=2 \\ \frac{6}{42} = \frac{1}{7} & k=3 \\ \frac{4}{42} = \frac{2}{21} & k=4 \\ \frac{6}{42} = \frac{1}{7} & k=6 \\ \frac{7}{42} = \frac{1}{6} & k=7 \\ \frac{16}{42} = \frac{8}{21} & k=8 \end{cases}$$

$$E(Y) = \sum_k k \cdot P_Y(k)$$

$$= \frac{1}{42} + \frac{4}{42} + \frac{18}{42} + \frac{16}{42} + \frac{36}{42} + \frac{49}{42} + \frac{128}{42}$$

$$= \frac{252}{42} = 6$$

2. $E(XY) = \int_0^1 \int_0^1 xy \, f(x,y) \, dy \, dx$

$$= \int_0^1 \int_0^x xy (12y^2) \, dy \, dx$$

$$= \int_0^1 [12x \cdot \frac{y^4}{4}]_0^x \, dx$$

$$= \int_0^1 3x^5 \, dx = \frac{1}{2}$$

3. $E[(X_1 - 2X_2 + X_3)^2] = \int_0^1 \int_0^1 \int_0^1 (X_1 - 2X_2 + X_3)^2 \, dX_1 \, dX_2 \, dX_3$

$$= \int_0^1 \int_0^1 \frac{1}{3} - 2X_2 + X_3 + 4X_2^2 - 4X_2X_3 + X_3^2 \, dX_2 \, dX_3$$

$$= \int_0^1 \frac{1}{3} - 1 + X_3 + \frac{4}{3} - 2X_3 + X_3^2 \, dX_3$$

$$= \frac{1}{3} - 1 + \frac{1}{2} + \frac{4}{3} - 1 + \frac{1}{3}$$

$$= \frac{6}{3} - 2 + \frac{1}{2}$$

$$= \frac{1}{2}$$

4. $E(e^{\frac{3}{4}x}) = \int_0^{+\infty} e^{\frac{3}{4}x} \cdot e^{-x}\, dx$

$= \int_0^{+\infty} e^{-\frac{1}{4}x}\, dx$

$= \left[-4e^{-\frac{1}{4}x}\right]_0^{\infty}$

$= 4$

5. $E(Y) = \frac{1}{6}(3+9+19+33+51+73)$

$= \frac{94}{3}$

6. $E(Y^2) = E((2X+1)^2) = E(4X^2+4X+1)$

$= 4E(X^2) + 4E(X) + 1$

$E(X) = \int_0^1 x \cdot 2(1-x)\, dx = \left[-\frac{2}{3}x^3\right]_0^1 = \frac{1}{3}$

$E(X^2) = \int_0^1 x^2 \cdot 2(1-x)\, dx = \left[-\frac{1}{2}x^4\right]_0^1 + \left[\frac{2}{3}x^3\right]_0^1 = \frac{1}{6}$

$E(Y^2) = \frac{4}{6} + \frac{4}{3} + 1 = 3$

7. $E[(aX+b)^n] = E\left(\sum_{k=0}^{n} C_n^k b^k (aX)^{n-k}\right)$

$= \sum_{k=0}^{n} C_n^k b^k E[aX^{n-k}]$

$= \sum_{i=0}^{n} C_n^i a^{n-i} b^i E(X^{n-i})$

8. $P(X=x) = C_n^x p^x (1-p)^{n-x}$

$P(Y=y) = P(X=n-y) = C_n^{n-y} p^{n-y} (1-p)^y$

let $Z = X-Y$, $Z = X-(n-X) = 2X-n$

$E(X-Y) = E(2X-n) = 2E(X)-n = 2np-n$

when sample size is 20, $p = 5\%$,

$E(X-Y) = 2\times 20 \times 0.05 - 20 = -18$