Tougaloo College
MAT426 - Advanced Calculus
Howework 05 - Spring, 2025

Due Date : 03/28/2025

Basic Topology - Exercises

1. (Problem 9) Let E^0 denote the set of all interior points of s set E.
   (a) Prove that E^0 is always an open set.

   Solution: Use definition 2.18(e) and theorem 2.19.

   (3 Points)

   (b) Prove that E is open if and only if $E^0 = E$.

   (3 Points)

   (c) If $G \subset E$ and G is open, prove that $G \subset E^0$. (3 Points)

   (d) Prove that the complement of E^0 is the closure of the complement of E. (3 Points)

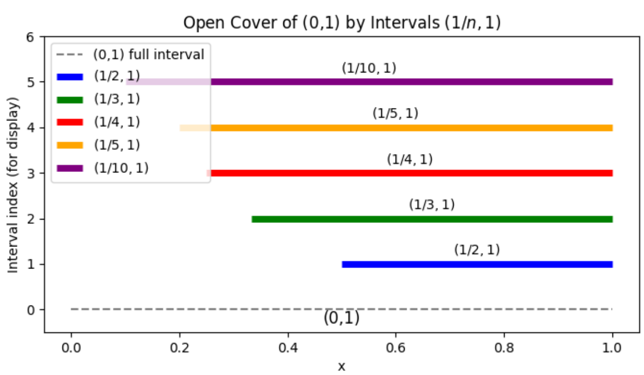
   (e) Do E and $\overline{E}$ always have the same interior points? (4 Points)

   (f) Do E and E^0 always have the same closure points? (4 Points)

   Total for Question 1: 20 Points

2. (Problem 14) Give an example of an open cover of the segment $(0, 1)$ which has no finite sub cover.

Solution:



The collection of open intervals

$$\mathcal{U} = \left\{ \left(\frac{1}{n}, 1 \right) | n \in \mathbb{N}, n \geq 2 \right\}$$

is an open cover of $(0,1)$ because for any $x \in (0,1)$, there exists a sufficiently large n such that $x \in (1/n, 1)$. However, it does not have a finite subcover; if it did, some points near 0 would be missed.

3. (Problem 16) Regard $\mathbb{Q}$, the set of all rational numbers, as a metric space, with $d(p,q) = |p-q|$. Let E be the set of all $p \in \mathbb{Q}$ such that $2 < p^2 < 3$. Show that E is closed and bounded in $\mathbb{Q}$, but that E is not compact. Is E open in $\mathbb{Q}$?

Solution:

$$E = \{p \in \mathbb{Q} | 2 < p^2 < 3\} = \{\mathbb{Q} \cap \{(-\sqrt{3}, -\sqrt{2}) \cup (\sqrt{2}, \sqrt{3})\}\}$$

Since $|p| < \sqrt{3}$, E is bounded.
For any sequence in E, converge within E since the $\pm\sqrt{2}, \pm\sqrt{3}$ are irrational.

Complete!