MAT426: Advanced Calculus

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2.35 Theorem

Theorem

Closed subset of compact sets are compact.

Proof:

Corollary

Corollary

If F is closed and K is compact, then $F \cap K$ is compact.

Proof:

2.36 Theorem

Theorem

If $\{K_\alpha\}$ is a collection of compact subsets of a metric space X such that the intersection of every finite subcollection of $\{K_\alpha\}$ is non-empty, then $\cap K_\alpha$ is non-empty.

Proof:

Corollary

Corollary

If $\{K_n\}$ is a sequence of non-empty compact sets such that $K_n \supset K_{n+1}$, $(n = 1, 2, 3, \dots)$, then $\cap_{n=1}^{\infty} K_n$ is not empty.

2.37 Theorem

Theorem

If E is an infinite subset of a compact set K, then E has a limit point in K.

Proof:

2.38 Theorem

Theorem

If $\{I_n\}$ is a sequence of intervals in $\mathbb{R}^1$, such that $I_n \supset I_{n+1}$ $(n = 1, 2, 3, \dots)$, then $\cap_{n=1}^{\infty} I_n$ is not empty.

Proof:

2.39 Theorem

Theorem

Let k be a positive integer. If $\{I_n\}$ is a sequence of $k-$cells such that $I_n \supset I_{n+1}$ $(n = 1, 2, 3, \dots)$, then $\cap_{i=1}^{\infty} I_n$ is not empty.

Proof: