

Tougaloo College  
MAT426 - Advanced Calculus  
Howework 05 - Spring, 2025

Due Date : 03/28/2025

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## Basic Topology - Exercises

1. (Problem 9) Let  $E^0$  denote the set of all interior points of a set  $E$ .

(a) Prove that  $E^0$  is always an open set.

**Solution:** Use definition 2.18(e) and theorem 2.19.

(3 Points)

(b) Prove that  $E$  is open if and only if  $E^0 = E$ .

(3 Points)

(c) If  $G \subset E$  and  $G$  is open, prove that  $G \subset E^0$ .

(3 Points)

(d) Prove that the complement of  $E^0$  is the closure of the complement of  $E$ .

(3 Points)

(e) Do  $E$  and  $\overline{E}$  always have the same interior points?

(4 Points)

(f) Do  $E$  and  $E^0$  always have the same closure points?

(4 Points)

Total for Question 1: 20 Points

2. (Problem 36) Let  $G$  be a group and let  $a \in G$ . Prove that  $C(a) = C(a^{-1})$ .