

MAT426: Advanced Calculus

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2.38 Theorem

Theorem

If $\{I_n\}$ is a sequence of intervals in \mathbb{R}^1 , such that $I_n \supset I_{n+1}$ ($n = 1, 2, 3, \dots$), then $\bigcap_{n=1}^{\infty} I_n$ is not empty.

Proof:

2.39 Theorem

Theorem

Let k be a positive integer. If $\{I_n\}$ is a sequence of k -cells such that $I_n \supset I_{n+1}$ ($n = 1, 2, 3, \dots$), then $\bigcap_{i=1}^{\infty} I_n$ is not empty.

Proof:

Theorem

Theorem

Every k -cell is compact.

Proof:

2.41 Theorem

Theorem

If a set E in \mathbb{R}^k has one of the following three properties, then it has the other two:

- (a) E is closed and bounded.
- (b) E is compact.
- (c) Every infinite subset of E has a limit point in E .

Proof: