MAT426: Advanced Calculus

Miraj Samarakkody

Tougaloo College

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Finite, Countable and Uncountable Sets

Definition

For any positive integer n, let J_n be the set whose elements are the integers $1, 2, \ldots, n$. Let J be the set consisting of all positive integers. For any set A we say,

1. A is finite $A \sim J_n$ for some n. (the empty set is also considered to be finite)
2. A is infinite if A is not finite.
3. A is countable if $A \sim J$.
4. A is uncountable if A is neither finite nor countable.
5. A is atmost countable if A is either finite or countable.

A set A is said to be **finite** if it is empty or if there is a one-to-one correspondence between A and J_n for some positive integer n. A set that is not finite is said to be **infinite**.

Example

Let A be the set of integers. Then A is countable. Consider the following arrengement of the sets A and J.

$$A : 0, 1, -1, 2, -2, 3, -3, \ldots$$

$$J : 1, 2, 3, 4, \ldots$$

Find a explicit formula for a function f from J to A.

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Importance of this example

Definition

By a sequence we mean a function f defined on the set J of all positive integers. If $f(n) = a_n$ for all n, we write $\{a_n\}$ for the sequence $\{a_1, a_2, a_3, \ldots\}$. The number a_n is called the nth term of the sequence.