MT222: Calculus II

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7.3 - Trigonometric Substitution

Why we need this?

Think about finding the area under the curve of a semi-circle

Table of Trigonometric Substitiution

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Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin \theta, \quad -\dfrac{\pi}{2} \leqslant \theta \leqslant \dfrac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta, \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta, \quad 0 \leqslant \theta < \dfrac{\pi}{2} \text{ or } \pi \leqslant \theta < \dfrac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Example 1

Evaluate

$$\int \frac{\sqrt{9 - x^2}}{x^2} \, dx$$

Example 2

Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$