MT222: Calculus II

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7.3 - Trigonometric Substitution

Table of Trigonometric Substitiution

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Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin \theta, \quad -\dfrac{\pi}{2} \leqslant \theta \leqslant \dfrac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta, \quad -\dfrac{\pi}{2} < \theta < \dfrac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta, \quad 0 \leqslant \theta < \dfrac{\pi}{2} \text{ or } \pi \leqslant \theta < \dfrac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Example 5

Evaluate

$$\int \frac{dx}{\sqrt{x^2 - a^2}},$$

where $a > 0$. (Use Hyperbolic functions)

Example 6

Find

$$\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}} dx$$

Example 7

Evaluate

$$\int \frac{x}{\sqrt{3 - 2x - x^2}}\, dx$$