

# MT222: Calculus II

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# Mid Exam Discussion

## Problem 4

Use the washer or cylindrical shell method to find the volume of the solid obtained by rotating the region bounded by the curves  $y^2 = x$  and  $x = 2y$  about the  $y$ -axis.

## Problem 5

Find the average value of the following function on the interval  $[-1, 1]$ .

$$f(x) = \frac{x^2}{(x^3 + 3)^2}$$

## Problem 6

Evaluate the following integral using integration by parts.

$$\int t^2 \sin \beta t \, dt,$$

where  $\beta$  is a constant.

## 7.3 - Trigonometric Substitution

# Why we need this?

Think about finding the area under the curve of a semi-circle

# Table of Trigonometric Substitution

**Table of Trigonometric Substitutions**

Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin \theta, \quad -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$	$1 - \sin^2 \theta = \cos^2 \theta$
$\sqrt{a^2 + x^2}$	$x = a \tan \theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2 \theta = \sec^2 \theta$
$\sqrt{x^2 - a^2}$	$x = a \sec \theta, \quad 0 \leq \theta < \frac{\pi}{2} \text{ or } \pi \leq \theta < \frac{3\pi}{2}$	$\sec^2 \theta - 1 = \tan^2 \theta$



## Example 1

Evaluate

$$\int \frac{\sqrt{9 - x^2}}{x^2} dx$$

## Example 2

Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

## Example 3

Find

$$\int \frac{1}{x^2 \sqrt{x^2 + 4}} dx$$

## Example 4

Find

$$\int \frac{x}{\sqrt{x^2 + 4}} dx$$

## Example 5

Evaluate

$$\int \frac{dx}{\sqrt{x^2 - a^2}},$$

where  $a > 0$ .

## Example 5

Evaluate

$$\int \frac{dx}{\sqrt{x^2 - a^2}},$$

where  $a > 0$ . (Use Hyperbolic functions)

## Example 6

Find

$$\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2 + 9)^{3/2}} dx$$

## Example 7

Evaluate

$$\int \frac{x}{\sqrt{3 - 2x - x^2}} dx$$