MT222: Calculus II

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7.3 - Trigonometric Substitution

Why we need this?

Think about finding the area under the curve of a semi-circle

Table of Trigonometric Substitiution

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Expression	Substitution	Identity
$\sqrt{a^2 - x^2}$	$x = a \sin\theta, \quad -\frac{\pi}{2} \leqslant \theta \leqslant \frac{\pi}{2}$	$1 - \sin^2\theta = \cos^2\theta$
$\sqrt{a^2 + x^2}$	$x = a \tan\theta, \quad -\frac{\pi}{2} < \theta < \frac{\pi}{2}$	$1 + \tan^2\theta = \sec^2\theta$
$\sqrt{x^2 - a^2}$	$x = a \sec\theta, \quad 0 \leqslant \theta < \frac{\pi}{2}$ or $\pi \leqslant \theta < \frac{3\pi}{2}$	$\sec^2\theta - 1 = \tan^2\theta$

Example 1

Evaluate

$$\int \frac{\sqrt{9-x^2}}{x^2}\,dx$$

Example 2

Find the area enclosed by the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

Example 3

Find

$$\int \frac{1}{x^2\sqrt{x^2+4}}\,dx$$

Example 4

Find

$$\int \frac{x}{\sqrt{x^2+4}}dx$$

Example 5

Evaluate

$$\int \frac{dx}{\sqrt{x^2 - a^2}},$$

where $a > 0$.

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Evaluate

$$\int \frac{dx}{\sqrt{x^2 - a^2}},$$

where $a > 0$. (Use Hyperbolic functions)

Example 6

Find

$$\int_0^{3\sqrt{3}/2} \frac{x^3}{(4x^2+9)^{3/2}}\,dx$$

Example 7

Evaluate

$$\int \frac{x}{\sqrt{3-2x-x^2}}\,dx$$