MAT222: Calculus II

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Integration of Rational Functions by Partial Fractions

Motivation

In this section, we show how to integrate any rational function by expressing it as a sum of simpler fractions, called *partial fraction*, that we already know how to integrate.

Example 1

Find

$$\int \frac{x^3 + x}{x - 1}\, dx$$

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Next we study some different cases.

CASE I: The denominator $Q(x)$ is a product of distinct linear factors.

We can write

$$Q(x) = (a_1x + b_1)(a_2x + b_2)\dots(a_kx + b_k),$$

where no factor is repeated.

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In this case the partial fraction theorem states that there exist constants $A_1, A_2, A_3, \dots, A_k$ such that

$$\frac{R(x)}{Q(x)} = \frac{A_1}{a_1x + b_1} + \frac{A_2}{a_2x + b_2} + \dots + \frac{A_k}{a_kx + b_k}.$$

Example 2

Write the partial fraction decomposition of

$$\frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x}$$

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Use and alternative method to write the partial fraction decomposition of

$$\frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x}$$