MAT434: Theory of Mathematical Statistics

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What is closure property?

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Identity

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Identity
Inverse

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What is closure property?
Identity
Inverse
Commutative or Abelian

The Dihedral Group

The dihedral group of order $2n$ is often called the group of symmetries of a regular n-gon.

Groups

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1. **Associativity**: $(ab)c = a(bc)$ for all $a, b \in G$.
2. **Identity**: There is an element e (called identity) in G such that $ae = ea = a$ for all $a \in G$.
3. **Inverses**: For each element $a \in G$, there is an element $b \in G$ such that $ab = ba = e$.

If for any $a, b \in G$, $ab = ba$ then group is an abelian group.

Examples

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3. The set S of positive irrational numbers together with 1 under multiplication is now a group.
4. A rectangular array of the form $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ is called a 2×2 matrix. The set of all 2×2 matrices with real entries is a group under componentwise addition.

$$\begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} + \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} = \begin{bmatrix} a_1 + a_2 & b_1 + b_2 \\ c_1 + c_2 & d_1 + d_2 \end{bmatrix}$$

Examples Cont.

5. The set $\mathbb{Z}_n = \{0, 1, \ldots, n-1\}$ for $n \geq 1$ is a group under addition modulo n. This group is ususally reffered to as the group of integers modulo n.

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7. The set of all invertible $n \times n$ matrices with real entries is a group under matrix multiplication. This group is denoted by $GL(n, \mathbb{R})$.

$$GL(2, \mathbb{R}) = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} \mid ad - bc \neq 0 \text{ for } a, b, c, d \in \mathbb{R} \right\}$$

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What happens when $ad - bc = 0$?