MAT434: Theory of Mathematical Statistics

Conditional Distributions [1]

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The Discrete Case

If X and Y are jointly distributed discrete random variables, the conditional probability that $X = x_i$ given that $Y = y_j$ is, if $p_Y(y_j) > 0$,

$$P(X = x_i | Y = y_j) = \frac{P(X = x_i, Y = y_j)}{P(Y = y_j)} = \frac{P_{XY}(x_i, y_j)}{p_Y(y_j)}$$

This probability is defined to be zero if $p_Y(y_j) = 0$.

Example

A fair coin is tossed three times: let X denote the number of heads on the first toss and Y the total number of heads.

	y			
x	0	1	2	3
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

Find the conditional frequency function of X given $Y = 1$.

Total Probability

The definition of the conditional frequency function can be reexpressed as

$$p_{XY}(x,y) = p_{X|Y}(x|y)p_Y(y)$$

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Summing both sides over all values of y, we have an extreamly useful application of the law jof total probability:

$$p_X(x) = \sum_Y p_{X|Y}(x|y)p_Y(y)$$

Example B

Suppose that a particle counter is imperfect and independently detects each incoming particle with probability p. If the distribution of the number of incoming particles in a unit time is a Poisson distribution with parameter λ, what is the distribution of the number of counted particles?

The Continuous Case

Definition

References

John A. Rice.
Mathematical Statistics and Data Analysis.
Cengage Learning, 3rd edition, 2006.