# MAT434: Theory of Mathematical Statistics - Joint Distributions

## Independent Random Variables [1]

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# Independent Random Variables

## Definition

Random variables $X_1, X_2, X_3, \ldots, X_n$ are said to be *independent* if their joint cdf factors into the product of their marginal cdf's:

$$F(x_1, x_2, \ldots, x_n) = F_{X_1}(x_1)F_{X_2}(x_2)\ldots F_{X_n}(x_n)$$

for all $x_1, x_2, \ldots, x_n$.

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This definition holds for both continuous and discrete random variables:

- For discrete random variables, it is equivalent to their joint frequency function factors.
- For continuous random variables, it is equivalent to their joint density fuction factors.

If $X$ and $Y$ are independent, then

$$P(X \in A, Y \in B) = P(X \in A)P(Y \in B)$$

# Example A

Suppose that the point $(X, Y)$ is uniformly distributed on the square

$$S = \{(x, y) | -1/2 \leq x \leq 1/2, -1/2 \leq y \leq 1/2\}$$

$$f_{XY}(x, y) = \begin{cases} 1 & \text{for } (x, y) \in S \\ 0 & \text{otherwise} \end{cases}$$

Prove that $X$ and $Y$ are independent.

# Example B

Now consider rotating the square $S$ by $45^\circ$ to obtain the diamond shape.

# References


John A. Rice.
*Mathematical Statistics and Data Analysis*.
Cengage Learning, 3rd edition, 2006.