

MAT434: Theory of Mathematical Statistics

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The Binomial Distribution

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Thus

$$P(X = k) = p(k) = \binom{n}{k} p^k (1 - p)^{n-k}$$

Example

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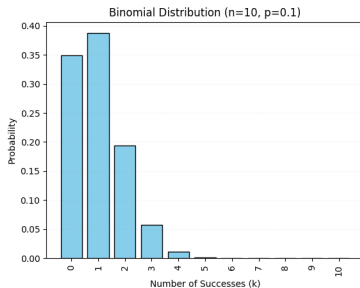


Figure: The Binomial Distribution for $n = 10$ and $p = 0.1$

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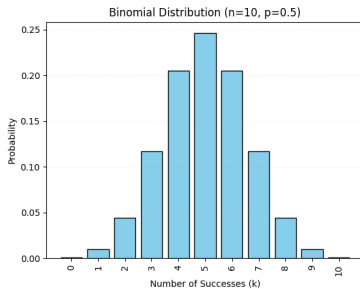


Figure: The Binomial Distribution for $n = 10$ and $p = 0.5$

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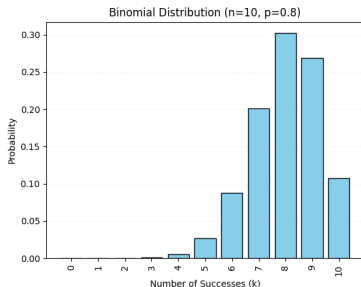


Figure: The Binomial Distribution for $n = 10$ and $p = 0.8$

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Note that these probabilities sum to 1.

$$\sum_{k=1}^{\infty} (1 - p)^{k-1}p = p \sum_{j=0}^{\infty} (1 - p)^j = 1$$

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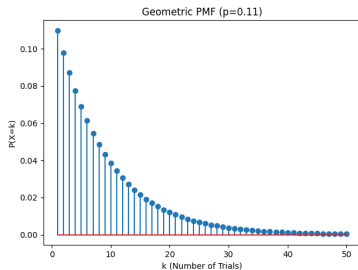


Figure: The probability mass function of a geometric random variable with $p = 1/9$

Negative Binomial Distribution

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