MAT434: Theory of Mathematical Statistics

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The Binomial Distribution

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Thus

$$P(X=k) = p(k) = \binom{n}{k} p^k (1-p)^{n-k}$$

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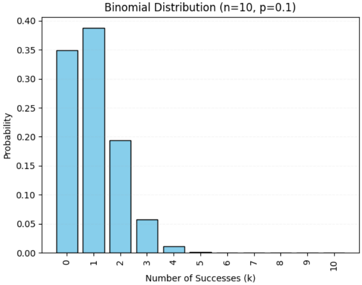

Figure: The Binomial Distribution for $n = 10$ and $p = 0.1$

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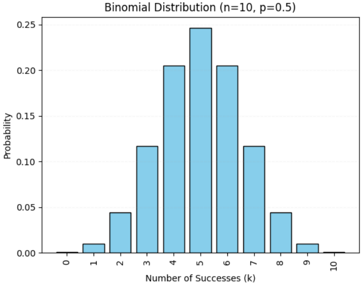

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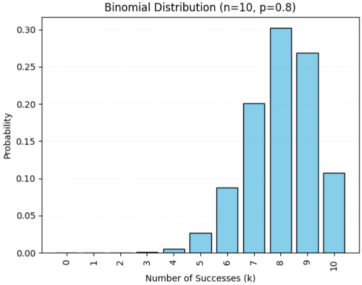

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Note that these probabilities sum to 1.

$$\sum_{k=1}^{\infty}(1 - p)^{k-1}p = p\sum_{j=0}^{\infty}(p - 1)^{j} = 1$$

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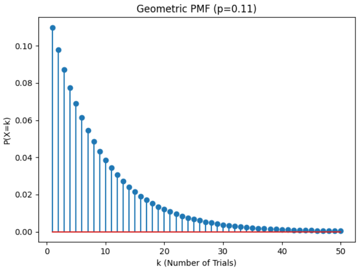

Figure: The probability mass function of a geometric random variable with $p = 1/9$