MAT434: Theory of Mathematical Statistics

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2.3 Functions of a Random Variable

Proposition C

Proposition

Let $Z = F(X)$; then Z has a uniform distribution of $[0, 1]$.

Proof:

Proposition D

Proposition

Let U be uniform on $[0, 1]$, and let $X = F^{-1}(U)$. Then the cdf of X is F.

Proof:

Joint Distributions

3.1 Introduction

Definition

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- A model for the joint distribution of age and length in a population of fish can be used to estimate the age distribution from the length distribution.

Introduction

The joint behavior of two random variables, X and Y, is determined by the cumulative distribution function

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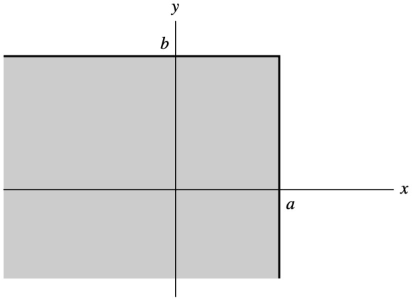
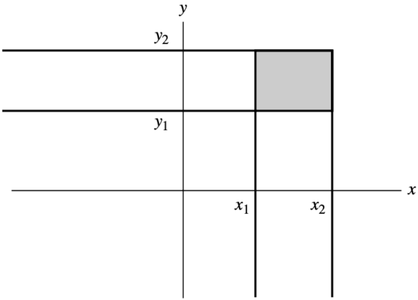
The cdf gives the probability that the point (X, Y) beliongs to a semi-infinite rectangle in the plane.



Introduction

The probability that (X, Y) belongs to a rectangle if

$$\begin{aligned} P(x_1 \leq X \leq x_2, y_1 \leq Y \leq y_2) = F(x_2, y_2) - F(x_2, y_1) \\ - F(x_1, y_2) + F(x_1, y_1) \end{aligned}$$



Discrete Random Variables

Suppose that X and Y are discrete random variables defined on the same sample space and that they take on values $x_1, x_2, \ldots$, and $y_1, y_2, \ldots$ respectively.

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Their **joint frequency function** $p(x, y)$ is

$$p(x_i, y_j) = P(X = x_i, Y = y_j)$$

Example

A fair coin is tossed three times and let X denote the number of heads on the first toss and Y the total number of heads. Find joint frequency function of X and Y.

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Find the frequency function of Y (p_Y) from the joint frequency function.

The p_Y is called the **marginal frequency function** of Y.

Marginal Frequency Function

Let X and Y are discrete random variables defined on the same sample space. The marginal function of X can be written as

$$p_X(x) = \sum_i p(x, y_i)$$

In the similary way, we can write the marginal frequency function for Y.