

MAT434: Theory of Mathematical Statistics - Joint Distributions Independent Random Variables [1]

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Independent Random Variables

Definition

Random variables $X_1, X_2, X_3, \dots, X_n$ are said to be *independent* if their joint cdf factors into the product of their marginal cdf's:

$$F(x_1, x_2, \dots, x_n) = F_{X_1}(x_1)F_{X_2}(x_2) \dots F_{X_n}(x_n)$$

for all x_1, x_2, \dots, x_n .

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This definition holds for both continuous and discrete random variables:

- ▶ For discrete random variables, it is equivalent to their joint frequency function factors.
- ▶ For continuous random variables, it is equivalent to their joint density function factors.

If X and Y are independent, then

$$P(X \in A, Y \in B) = P(X \in A)P(Y \in B)$$

Example A

Suppose that the point (X, Y) is uniformly distributed on the square

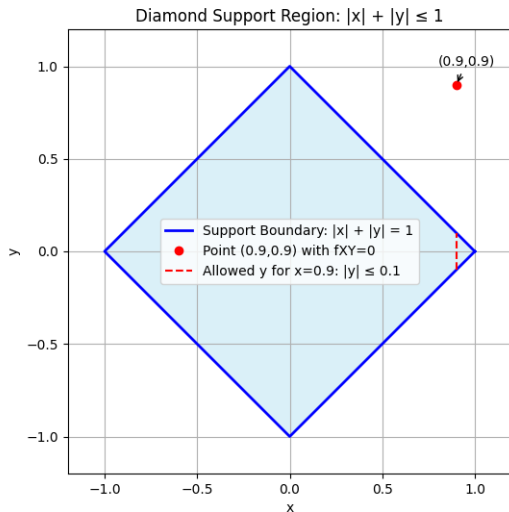
$$S = \{(x, y) \mid -1/2 \leq x \leq 1/2, -1/2 \leq y \leq 1/2\}$$

$$f_{XY}(x, y) = \begin{cases} 1 & \text{for } (x, y) \in S \\ 0 & \text{otherwise} \end{cases}$$

Prove that X and Y are independent.

Example B

Now consider rotating the square by 45° to obtain the diamond shape.



References



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