Tougaloo College
MAT434 - Theory of Mathematical Statistics
Howework 04 - Spring, 2025

2.5 Problems

1. Let X be a normal random variable with $\mu = 5$ and $\sigma = 10$. Find:
   (a) $P(X > 10)$

   Solution: $P(X > 10) = 1 - P(X \leq 10) = 1 - \Phi(0.5) = 0.3085$

   (b) $P(-20 < X < 15)$

   Solution: $\Phi(1) - \Phi(-2.5) = \Phi(1) - 1 + \Phi(2.5) = 0.8351$

   (c) the value of x such that $P(X > x) = 0.065$

   Solution: $z = 1.52$. Therefore $x = 20.2$

2. If $X \sim N(\mu, \sigma^2)$, show that $P(|X - \mu| \leq 0.675\sigma) = 0.5$

   Solution: Convert into the standard normal distribution gives the equality.

3. If $X \sim N(0, \sigma^2)$, find the density of $Y = |X|$.

   Solution:
   $$\frac{2}{\sigma\sqrt{2\pi}} exp\left(-\frac{y^2}{2\sigma^2}\right)$$