MAT434: Theory of Mathematical Statistics

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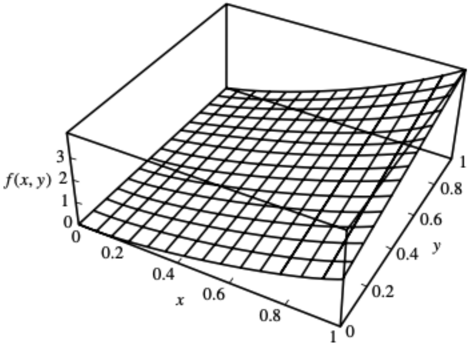
Continuous Random Variables

Example A

Consider the bivariate density function

$$f(x, y) = \frac{12}{7}(x^2 + xy), \ 0 \leq x \leq 1, \ 0 \leq y \leq 1.$$

Calculate $P(X > Y)$.



Continuous Random Variables

The **marginal cdf** of X, of F_X, is

$$\begin{aligned} F_X(x) &= P(X \leq x) \\ &= \lim_{y \to \infty} F(x, y) \\ &= \int_{-\infty}^{x} \int_{-\infty}^{\infty} f(u, y)\ dy\ du \end{aligned}$$

The marginal density of X is

$$f_X(x) = F'_X(x) = \int_{-\infty}^{\infty} f(x, y)\ dy$$

Example B

Consider the bivariate density function

$$f(x, y) = \frac{12}{7}(x^2 + xy), \ 0 \leq x \leq 1, \ 0 \leq y \leq 1.$$

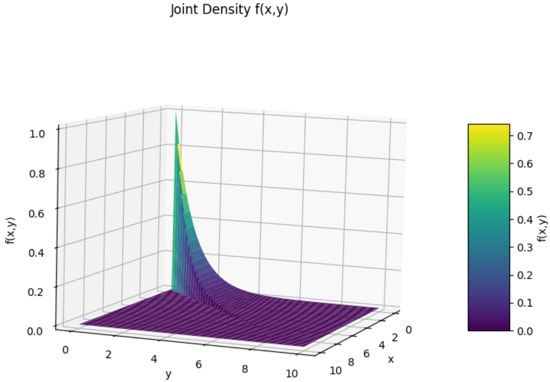
Find the marginal density of X.

Example D

Consider the following joint density:

$$f(x,y) = \begin{cases} \lambda^2 e^{-\lambda y}, & 0 \leq x \leq y, \ \ \lambda > 0 \\ 0, & \text{elsewhere} \end{cases}$$

Find the marginal densities.



Continuous Random Variables

- In some applications, it is useful to analyze distributions that are uniform over some region space.
- For example, in the plane, the random point (X, Y) is uniform over a region R, if for any $A \subset R$,

$$P((X, Y) \in A) = \frac{|A|}{|R|}$$

Example E

A point is chosen randomly in a disk of radius 1. Since the area of the disk is π,

$$f(x, y) = \begin{cases} \dfrac{1}{\pi} & \text{if } x^2 + y^2 \leq 1 \\ 0, & \text{otherwise} \end{cases}$$

Find $F_R(r)$, $f_R(r)$ $f_X(x)$ and $f_Y(y)$.

Independent Random Variables

Independent Random Variables

Definition

Random variables $X_1, X_2, X_3, \ldots, X_n$ are said to be *independent* if their joint cdf factors into the product of their marginal cdf's:

$$F(x_1, x_2, \ldots, x_n) = F_{X_1}(x_1) F_{X_2}(x_2) \ldots F_{X_n}(x_n)$$

for all $x_1, x_2, \ldots, x_n$.