MAT434: Theory of Mathematical Statistics

Miraj Samarakkody

Tougaloo College

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2.3 Functions of a Random Variable

Proposition A

Proposition A

If $X \sim N(\mu, \sigma^2)$ and $Y = aX + b$, then $Y \sim N(a\mu + b, a^2\sigma^2)$

Applications of Proposition A

Consider the random variable

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- We can see that Z follows a standard normal distribution.
- Also we can see that probabilities for general normal random variables can be evaluated in terms of probabilities fro standard normal random variables.

Example B

Let $X \sim N(\mu, \sigma^2)$, and find the probability that X is less than σ away from μ; that is, find $P(|X - \mu| < \sigma)$.

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Thus, a normal random variable is within 1 standard deviation of its mean with probability 0.68

Example C

Find the density of $X = Z^2$, where $Z \sim N(0, 1)$.