

MAT434: Theory of Mathematical Statistics

Conditional Distributions [1]

Miraj Samarakkody

Tougaloo College

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The Discrete Case

If X and Y are jointly distributed discrete random variables, the conditional probability that $X = x_i$ given that $Y = y_j$ is, if $p_Y(y_j) > 0$,

$$P(X = x_i | Y = y_j) = \frac{P(X = x_i, Y = y_j)}{P(Y = y_j)} = \frac{P_{XY}(x_i, y_j)}{p_Y(y_j)}$$

This probability is defined to be zero if $p_Y(y_j) = 0$.

Example

A fair coin is tossed three times: let X denote the number of heads on the first toss and Y the total number of heads.

x	y			
	0	1	2	3
0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$	0
1	0	$\frac{1}{8}$	$\frac{2}{8}$	$\frac{1}{8}$

Find the conditional frequency function of X given $Y = 1$.

Example B

Suppose that a particle counter is imperfect and independently detects each incoming particle with probability p . If the distribution of the number of incoming particles in a unit time is a Poisson distribution with parameter λ , what is the distribution of the number of counted particles.

References



John A. Rice.

Mathematical Statistics and Data Analysis.

Cengage Learning, 3rd edition, 2006.