

# MAT414 - Modern Algebra - Permutation Groups

## Cycle Notation

**Miraj Samarakkody**

Tougaloo College

Updated - April 6, 2025

# Cycle Notation

Write the followings in the cyclic notations:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{bmatrix} \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{bmatrix}$$

# Cycle Notation

Write the followings in the cyclic notations:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{bmatrix} \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{bmatrix}$$

Find  $\alpha\beta$ .

# Properties of Permutations

## Theorem 5.1 - Products of Disjoint Cycles

Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

## Theorem 5.2

### Disjoint Cycles Commute

If the pair of cycles  $\alpha = (a_1, a_2, \dots, a_m)$  and  $\beta = (b_1, b_2, \dots, b_n)$  have no entries in common, then  $\alpha\beta = \beta\alpha$ .

## Theorem 5.3

### Order of a Permutation

The order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles.

## Theorem 5.3

### Order of a Permutation

The order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles.

Find

- ▶  $| (132)(45) |$
- ▶  $| (1432)(56) |$
- ▶  $| (123)(456)(78) |$
- ▶  $| (123)(145) |$

## Example 5

Determine the orders of the elements of  $S_7$ .



## Example 6

Determine the number of elements in  $S_7$  of order 12.

## Example 7

Determine the number of elements in  $S_7$  of order 3.

# Theorem 5.4

Product of 2-Cycles