MAT414 - Modern Algebra - Permutation Groups

Cycle Notation [1]

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Cycle Notation

Write the followings in the cyclic notations:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{bmatrix} \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{bmatrix}$$

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Find $\alpha\beta$.

Properties of Permutations

Theorem 5.1 - Products of Disjoint Cycles

Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

Theorem 5.2

Disjoint Cycles Commute

If the pair of cycles $\alpha = (a_1, a_2, \ldots, a_m)$ and $\beta = (b_1, b_2, \ldots, b_n)$ have no entries in common, then $\alpha\beta = \beta\alpha$.

Theorem 5.3

Order of a Permutation

The order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles.

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Find

- $|(132)(45)|$
- $|(1432)(56)|$
- $|(123)(456)(78)|$
- $|(123)(145)|$

Example 5

Determine the orders of the elements of S_7.

Example 6

Determine the number of elements in S_7 of order 12.

Example 7

Determine the number of elements in S_7 of order 3.

Theorem 5.4

Product of 2-Cycles

Every permutation of in S_n, $n > 1$, is a product of $2-$cycles.

Example

$$\begin{aligned} (1\ 2\ 3\ 4\ 5) &= \\ (1\ 6\ 3\ 2)(4\ 5\ 7) &= \end{aligned}$$

Lemma

In S_n, if $\epsilon = \beta_1\beta_2\beta_3\ldots\beta_r$, where the β_i's are 2-cycles, then r is even.

Theorem 5.5

Always Even or Always Odd

If a permutation α can be expressed as a product of an even (odd) number of 2-cycles, then every decomposition of α into a product of 2−cycles must have an an even (odd) number of 2−cycles.

In symbols, if

$$\alpha = \beta_1\beta_2\ldots\beta_r \text{ and } \alpha = \gamma_1\gamma_2\ldots\gamma_s,$$

where the β's and γ's are 2−cycles, then r and s are both even or both odd.

References

Joseph A. Gallian.
Contemporary Abstract Algebra.
Cengage Learning, 9th edition, 2017.