Tougaloo College
MAT414 - Modern Algebra
Howework 04 - Spring, 2025

Due Date : 03/21/2025

Finite Groups; Subgroups - Exercises

1. (Problem 6) In the group $\mathbb{Z}_{12}$, find $|a|$, $|b|$ and $|a+b|$ for each case.
   (a) $a = 6, b = 2$

   Solution: $|a| = 2,\ |b| = 6,\ |a+b| = 3$

   (10 Points)

   (b) $a = 3\ , b = 8$

   Solution: $|a| = 4,\ |b| = 3,\ |a+b| = 12$

   (10 Points)

   Total for Question 1: 20 Points

2. (Problem 7) If a, b, and c are group elements and $|a| = 6,\ |b| = 7$, express $(a^4c^{-2}b^4)^{-1}$ without using negative exponents.

   Solution: $b^3c^2a^2$; Use Socks-Shoes properties.

   Total for Question 2: 20 Points

3. (Problem 13) For any group elements a and x, prove that $|xax^-1| = |a|$.

   Solution: Use method of contradiction.

   Total for Question 3: 20 Points

4. (Problem 14) Prove that if a is the only element of order 2 in a group, then a lies in the center of the group.

Solution:

- We can see that any $g \in G$, if $|g| = 2$ then $g = a$.
- Then prove $|xax^-1| = |a|$

Total for Question 4: 20 Points

5. (Problem 34) If H and K are subgroups of a group G, prove that $H \cap K$ is a subgroup of G.

Solution: Use the one-step subgroup test.

- $H \cap K$ is non-empty.
- If $x, y \in H \cap K$, then $xy^{-1} \in H$ and $xy^{-1} \in K$.

Total for Question 5: 20 Points

6. (Problem 35) Let G be a group. Show that $Z(G) = \cap_{a \in G} C(a)$.

Solution: Show that $Z(G) \subset \cap_{a \in G} C_a$ and $\cap_{a \in G} C(a) \subset Z(G)$.

Total for Question 6: 20 Points

7. (Problem 36)Let G be a group and let $a \in G$. Prove that $C(a) = C(a^{-1})$.