MAT414 - Modern Algebra - Permutation Groups
Cycle Notation

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Updated - April 6, 2025

Cycle Notation

Write the followings in the cyclic notations:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{bmatrix} \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{bmatrix}$$

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Find $\alpha\beta$.

Properties of Permutations

Theorem 5.1 - Products of Disjoint Cycles

Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

Theorem 5.2

Disjoint Cycles Commute

If the pair of cycles $\alpha = (a_1, a_2, \ldots, a_m)$ and $\beta = (b_1, b_2, \ldots, b_n)$ have no entries in common, then $\alpha\beta = \beta\alpha$.

Theorem 5.3

Order of a Permutation
The order of a permutation of a finite set written in disjoint cycle form is the least common multiple of the lengths of the cycles.

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Find
- $|(132)(45)|$
- $|(1432)(56)|$
- $|(123)(456)(78)|$
- $|(123)(145)|$

Example 5

Determine the orders of the elements of S_7.

Example 6

Determine the number of elements in S_7 of order 12.

Example 7

Determine the number of elements in S_7 of order 3.

Theorem 5.4

Product of 2-Cycles