Tougaloo College
MAT414 - Modern Algebra
Howework 04 - Spring, 2025

Duration : 03/21/2025

Finite Groups; Subgroups - Exercises

1. (Problem 6) In the group $\mathbb{Z}_{12}$, find $|a|$, $|b|$ and $|a + b|$ for each case.

   (a) $a = 6, b = 2$

   > **Solution:** $|a| = 2$, $|b| = 6$, $|a + b| = 3$

   (b) $a = 3$, $b = 8$

   > **Solution:** $|a| = 4$, $|b| = 3$, $|a + b| = 12$

2. (Problem 7) If a, b, and c are group elements and $|a| = 6$, $|b| = 7$, express $(a^4 c^{-2} b^4)^{-1}$ without using negative exponents.

   > **Solution:** $b^3 c^2 a^2$; Use Socks-Shoes properties.

3. (Problem 13) For any group elements a and x, prove that $|xax^-1| = |a|$.

   > **Solution:** Use method of contradiction.

4. (Prroblem 14) Prove that if a is the only element of order 2 in a group, then a lies in the center of the group.

   > **Solution:**
   >
   > - We can see that any $g \in G$, if $|g| = 2$ then $g = a$.
   >
   > - Then prove $|xax^-1| = |a|$

5. (Problem 34) If H and K are subgroups of a group G, prove that $H \cap K$ is a subgroup of G.

Solution: Use the one-step subgroup test.

- $H \cap K$ is non-empty.

- If $x, y \in H \cap K$, then $xy^{-1} \in H$ and $xy^{-1} \in K$.

<div align="right">Total for Question 5: 0 Points</div>

Solution: ksjfjsj

6. Evaluate the integrals:

(a) $\int_0^1 (x^e + e^x)\, dx$ (10 Points)

(b) $\int_{-2}^1 \frac{1}{x^4}\, dx$ (10 Points)

(c) $\int 4x^3 e^{x^4}\, dx$ (10 Points)

(d) $\displaystyle\int_1^2 \frac{e^{1/x}}{x^2}\,dx$ (10 Points)

(e) $\displaystyle\int \sin^3\theta \cos^4\theta\,d\theta$ (10 Points)

Total for Question 6: 50 Points

7. Sketch the region enclosed by the given curves, then find the area of the region.

$$y = \sin x, \ y = x, \ x = \pi/2, \ x = \pi.$$

Total for Question 7: 20 Points

8. Use the washer or cylindrical shell method to find the volume of the solid obtained by rotating the region bounded by the curves $y^2 = x$ and $x = 2y$ about the $y-$axis.

Total for Question 8: 20 Points

9. Find the average value of the following function on the interval $[-1, 1]$.

$$f(x) = \frac{x^2}{(x^3 + 3)^2}$$

Total for Question 9: 10 Points

10. Evaluate the following integral using integration by parts.

$$\int t^2 \sin \beta t \ dt,$$

where β is a constant.

Total for Question 10: 15 Points