Tougaloo College
MAT414 - Modern Algebra
Howework 04 - Spring, 2025

Due Date : 03/28/2025

Finite Groups; Subgroups - Exercises

1. (Problem 6) In the group $\mathbb{Z}_{12}$, find $|a|$, $|b|$ and $|a+b|$ for each case.

   (a) $a = 6, b = 2$

   (10 Points)

   (b) $a = 3\ , b = 8$

   (10 Points)

   Total for Question 1: 20 Points

2. (Problem 7) If a, b, and c are group elements and $|a| = 6,\ |b| = 7$, express $(a^4c^{-2}b^4)^{-1}$ without using negative exponents.

   Total for Question 2: 20 Points

3. (Problem 13) For any group elements a and x, prove that $|xax^{-}1| = |a|$.

   Total for Question 3: 20 Points

4. (Problem 14) Prove that if a is the only element of order 2 in a group, then a lies in the center of the group.

   Total for Question 4: 20 Points

5. (Problem 34) If H and K are subgroups of a group G, prove that $H \cap K$ is a subgroup of G.

   Total for Question 5: 20 Points