

MAT414 - Modern Algebra

Miraj Samarakkody

Tougaloo College

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Example

Find all generators of the cyclic group $U(50)$.

Fundamental Theorem of Cyclic Groups

Theorem 4.3

Fundamental Theorem of Cyclic Group

Every subgroup of a cyclic group is cyclic. Moreover, if $|\langle a \rangle| = n$, then the order of any subgroup of $\langle a \rangle$ is a divisor of n ; and, for each positive divisor k of n , the group $\langle a \rangle$ has exactly one subgroup of order k —namely, $\langle a^{n/k} \rangle$.

Example

Suppose $G = \langle a \rangle$ and G has order 30. Find all the subgroups of G .

Corollary

Subgroups of \mathbb{Z}_n

For each positive divisor k of n , the set $\langle n/k \rangle$ is the unique subgroup of \mathbb{Z}_n of order k ; moreover, these are the only subgroups of \mathbb{Z}_n .

Example 7

Write the list of subgroups of \mathbb{Z}_{30} .

Example 8

Find the generators of the subgroup of order 9 in \mathbb{Z}_{36} .