

MAT414 - Modern Algebra

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Permutation Groups

Cycle Notation

Write the followings in the cyclic notations:

$$\alpha = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 1 & 4 & 6 & 5 & 3 \end{bmatrix} \quad \beta = \begin{bmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 3 & 1 & 6 & 2 & 4 \end{bmatrix}$$

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Find $\alpha\beta$.

Properties of Permutations

Theorem 5.1 - Products of Disjoint Cycles

Every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.

Theorem 5.2

Disjoint Cycles Commute

If the pair of cycles $\alpha = (a_1, a_2, \dots, a_m)$ and $\beta = (b_1, b_2, \dots, b_n)$ have no entries in common, then $\alpha\beta = \beta\alpha$.

Theorem 5.3

Order of a Permutation

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Find

- ▶ $| (132)(45) |$
- ▶ $| (1432)(56) |$
- ▶ $| (123)(456)(78) |$
- ▶ $| (123)(145) |$

Example 5

Determine the orders of the elements of S_7 .

Example 6

Determine the number of elements in S_7 of order 12.

Example 7

Determine the number of elements in S_7 of order 3.