Tougaloo College
MAT414 - Modern Algebra
Howework 05 - Spring, 2025

Finite Groups; Subgroups - Exercises

1. (Problem 35) Let G be a group. Show that $Z(G) = \cap_{a \in G} C(a)$.

> **Solution:** Show that $Z(G) \subset \cap_{a \in G} C_a$ and $\cap_{a \in G} C(a) \subset Z(G)$.

Total for Question 1: 20 Points

2. (Problem 36) Let G be a group and let $a \in G$. Prove that $C(a) = C(a^{-1})$.

Cyclic Groups

3. List the elements of subgroups $\langle 3 \rangle$ and $\langle 7 \rangle$ in $U(20)$.

> **Solution:**
> $$\langle 3 \rangle = \langle 7 \rangle = \{1, 3, 9, 7\}$$

4. Find an example of a non-cyclic group, all of whose proper subgroups are cyclic.

> **Solution:** Q_8

5. In Z_{24}, find a generator for $\langle 21 \rangle \cap \langle 10 \rangle$. Suppose that $|a| = 24$. Find a generator for $\langle a^{21} \rangle \cap \langle a^{10} \rangle$. In general, what is a generator for the subgroup $\langle a^m \rangle \cap \langle a^n \rangle$

> **Solution:** $\langle a^m \rangle \cap \langle a^n \rangle = \langle a^{\mathrm{lcm}(m,n)} \rangle$

6. Suppose that a cyclic group G has exactly three subgroups: G itself, $\{e\}$, and a subgroup of order 7. What is $|G|$?

Solution: $|G| = 49$

7. Determine the subgroup lattice for Z_{12}. Generalize to Z_{p^2q}, where p and q are distinct primes.

Solution: