MAT414 - Modern Algebra

Miraj Samarakkody

Tougaloo College

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Cyclic Groups

Theorem 4.2 - $\langle a^k \rangle = \langle a^{\gcd(n,k)} \rangle$ and $|a^k| = n/\gcd(n,k)$

Let a be an element of order n is a group and let k be a positive integer. Then $< a^k > = < a^{gcd(n,k)} >$ and $|a^k| = n/gcd(n,k)$

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Proof steps:

We let $d = \mathrm{gcd}(n,k)$

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  - Write $d = ns + kt$ for some integers s, t
- Here we show that $|a^d| \leq n/d$ and then $|a^k| = n/d$.