Calculus I

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Limits and Derivatives

The Precise Definition of a Limit

Definition 1.

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that if $0 < |x - a\ | < \delta$ then $|f(x) - L| < \varepsilon$.

The Precise Definition of a Limit (ii)

Example: Prove that $\lim_{x \to 3} x^2 = 9$.

one-Sided Limits

Definition 2: Precise Definition of Left-Hand Limit.

$$\lim_{x \to a^-} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that
if $a - \delta < x < a$ then $|f(x) - L| < \varepsilon$.

Definition 3: Precise Definition of Right-Had limit.

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that
if $a < x < a + \delta$ then $|f(x) - L\ | < \varepsilon$.

Example: Use definition to prove that $\lim_{x \to 0^+} \sqrt{x} = 0$.

The limit Laws

The sum Laws

If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = M$ both exists, then

$$\lim_{x \to a} [f(x) + g(x)] = L + M$$

Proof: