

Calculus I

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Calculating Limits Using the Limit Laws

Limit Laws

Suppose that c is a constant and the limits

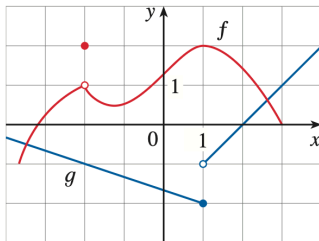
$$\lim_{x \rightarrow a} f(x) \quad \text{and} \quad \lim_{x \rightarrow a} g(x)$$

exist. Then

1. $\lim_{x \rightarrow a} [f(x) + g(x)] = \lim_{x \rightarrow a} f(x) + \lim_{x \rightarrow a} g(x)$
2. $\lim_{x \rightarrow a} [f(x) - g(x)] = \lim_{x \rightarrow a} f(x) - \lim_{x \rightarrow a} g(x)$
3. $\lim_{x \rightarrow a} [cf(x)] = c \lim_{x \rightarrow a} [f(x)]$
4. $\lim_{x \rightarrow a} [f(x)g(x)] = \lim_{x \rightarrow a} f(x) \cdot \lim_{x \rightarrow a} g(x)$
5. $\lim_{x \rightarrow a} \left[\frac{f(x)}{g(x)} \right] = \frac{\lim_{x \rightarrow a} f(x)}{\lim_{x \rightarrow a} g(x)}$

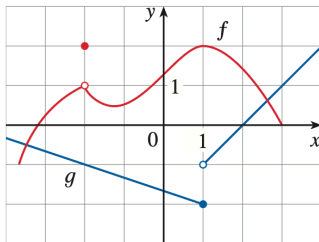
Limit Laws (ii)

Example: Use the limit laws and the graph of f and g in the figure to evaluate the following limits.



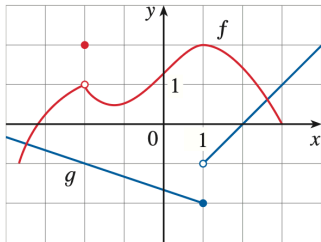
1. $\lim_{x \rightarrow -2} [f(x) + 5g(x)]$

Limit Laws (iii)



2. $\lim_{x \rightarrow 1} [f(x)g(x)]$

Limit Laws (iv)



3. $\lim_{x \rightarrow 2} \frac{f(x)}{g(x)}$

Limit Laws (v)

6. Power Law

$$\lim_{x \rightarrow a} [f(x)]^n = \left[\lim_{x \rightarrow a} f(x) \right]^n,$$

where n is a positive number.

7. Root Law

$$\lim_{x \rightarrow a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \rightarrow a} f(x)},$$

where n is a positive integer. If n is even we assume that $f(x) > 0$.

8. $\lim_{x \rightarrow a} c = c$

9. $\lim_{x \rightarrow a} x = a$

10. $\lim_{x \rightarrow a} x^n = a^n$, where n is a positive integer.

11. $\lim_{x \rightarrow a} \sqrt[n]{x} = \sqrt[n]{a}$, where n is a positive integer.

Limit Laws (vi)

Example

Evaluate the following limits and justify each steps.

1. $\lim_{x \rightarrow 5} (2x^2 - 3x + 4)$

Limit Laws (vii)

2. $\lim_{x \rightarrow -2} \frac{x^3 + 2x^2 - 1}{5 - 3x}$

Evaluating Limits by Direct Substitution

Direct Substitution Property

If f is a polynomial or a rational function and a is in the domain f , then

$$\lim_{x \rightarrow a} f(x) = f(a)$$

Example: Find $\lim_{x \rightarrow 1} \frac{x^2 - 1}{x - 1}$.

Evaluating Limits by Direct Substitution (ii)

If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x)$, provided that the limits exists.

Example: Find $\lim_{x \rightarrow 1} g(x)$, where

$$g(x) = \begin{cases} x + 1 & \text{if } x \neq 1 \\ \pi & \text{if } x = 1 \end{cases}$$

Evaluating Limits by Direct Substitution (iii)

Example Evaluate $\lim_{h \rightarrow 0} \frac{(3+h^2)-9}{h}$.

Evaluating Limits by Direct Substitution (iv)

Example: Find $\lim_{t \rightarrow 0} \frac{\sqrt{t^2+9}-3}{t^2}$.