Calculus I

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Calculating Limits Using the Limit Laws

Limit Laws

Suppose that c is a constant and the limits

$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x \to a}[f(x) + g(x)] = \lim_{x \to a} f(x) + \lim_{x \to a} g(x)$
2. $\lim_{x \to a}[f(x) - g(x)] = \lim_{x \to a} f(x) - \lim_{x \to a} g(x)$
3. $\lim_{x \to a}[cf(x)] = c \lim_{x \to a}[f(x)]$
4. $\lim_{x \to a}[f(x)g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x)$
5. $\lim_{x \to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)}$
6. **Power Law**

$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n,$$

Limit Laws (ii)

where n is a positive number.

7. **Root Law**

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)},$$

where n is a positive integer. If n is even we assume that $f(x) > 0$.

8. $\lim_{x \to a} c = c$
9. $\lim_{x \to a} x = a$
10. $\lim_{x \to a} x^n = a^n$, where n is a positive integer.
11. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$, where n is a positive integer.

Limit Laws (iii)

2. $\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x}$

Evaluating Limits by Direct Substitution

Direct Substitution Property

If f is a polynomial or a rational function and a is in the domain f, then

$$\lim_{x \to a} f(x) = f(a)$$

Example: Find $\lim_{x \to 1} \frac{x^2 - 1}{x - 1}$.

Evaluating Limits by Direct Substitution (ii)

If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \to a} f(x) = \lim_{x \to a} g(x)$, provided that the limits exists.

Example: Find $\lim_{x \to 1} g(x)$, where

$$g(x) = \begin{cases} x + 1 & \text{if } x \neq 1 \\ \pi & \text{if } x = 1 \end{cases}$$

Evaluating Limits by Direct Substitution (iii)

Example Evaluate $\lim_{h\to 0} \frac{(3+h^2)-9}{h}$.

Evaluating Limits by Direct Substitution (iv)

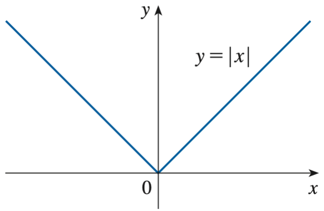
Example: Find $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$.

Using one-sided limits

Theorem 1.

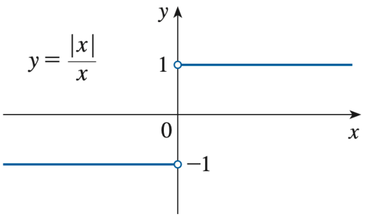
$$\lim_{x\to a} f(x) = L \text{ if and only if } \lim_{x\to a^-} f(x) = \lim_{x\to a^+} f(x) = L$$

Example: Show that $\lim_{x\to 0}|x| = 0$.



Using one-sided limits (ii)

Example: Prove that $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.



Using one-sided limits (iii)

Example: If

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8 - 2x & \text{if } x < 4 \end{cases}$$

determine whether $\lim_{x \to 4} f(x)$ exists.