Calculus I

Tougaloo College

Miraj Samarakkody

Calculating Limits Using the Limit Laws

Limit Laws

Suppose that c is a constant and the limits

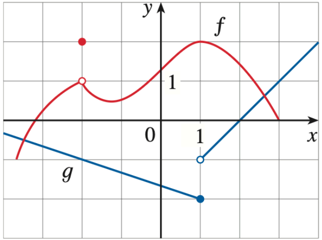
$$\lim_{x \to a} f(x) \quad \text{and} \quad \lim_{x \to a} g(x)$$

exist. Then

1. $\lim_{x\to a}[f(x) + g(x)] = \lim_{x\to a} f(x) + \lim_{x\to a} g(x)$
2. $\lim_{x\to a}[f(x) - g(x)] = \lim_{x\to a} f(x) - \lim_{x\to a} g(x)$
3. $\lim_{x\to a}[cf(x)] = c\lim_{x\to a}[f(x)]$
4. $\lim_{x\to a}[f(x)g(x)] = \lim_{x\to a} f(x) \cdot \lim_{x\to a} g(x)$
5. $\lim_{x\to a}\left[\frac{f(x)}{g(x)}\right] = \frac{\lim_{x\to a} f(x)}{\lim_{x\to a} g(x)}$

Limit Laws (ii)

Example: Use the limit laws and the graph of f and g in the figure to evaluate the following limits.

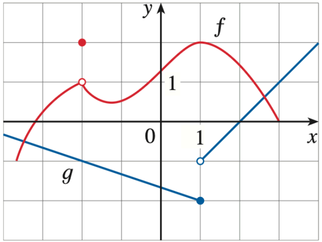


1. $\lim_{x \to -2} [f(x) + 5g(x)]$

Limit Laws (iii)

$$\begin{aligned}
\lim_{x\to -2}[f(x)+5g(x)] &= \lim_{x\to -2} f(x) + \lim_{x\to -2} 5g(x) \quad \text{by law } (1) \\
&= \lim_{x\to -2} f(x) + 5 \lim_{x\to -2} g(x) \text{ by law } (3) \\
&= 1 + 5 \times (-1) \\
&= 1 - 5 = -4
\end{aligned}$$

Limit Laws (iv)

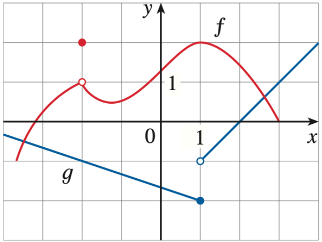


2. $\lim_{x\to 1}[f(x)g(x)]$

$$\lim_{x\to 1}[f(x)g(x)] = \lim_{x\to 1} f(x) \cdot \lim_{x\to 1} g(x) \text{ By law (4)}$$
$$= 2 \times \lim_{x\to 1} g(x) \quad \text{does not exist}$$

because $\lim_{x\to 1^-} g(x) = -2$ and $\lim_{x\to 1^+} g(x) = -1$. They are not equal.

Limit Laws (v)



3. $\lim_{x\to 2} \frac{f(x)}{g(x)}$

$$\lim_{x\to 2} \frac{f(x)}{g(x)} = \frac{\lim_{x\to 2} f(x)}{\lim_{x\to 2} g(x)}$$
$$= \frac{1.5 \approx}{0}$$

Limit does not exist.

Limit Laws (vi)

6. **Power Law**

$$\lim_{x \to a} [f(x)]^n = \left[\lim_{x \to a} f(x)\right]^n,$$

where n is a positive number.

7. **Root Law**

$$\lim_{x \to a} \sqrt[n]{f(x)} = \sqrt[n]{\lim_{x \to a} f(x)},$$

where n is a positive integer. If n is even we assume that $f(x) > 0$.

8. $\lim_{x \to a} c = c$
9. $\lim_{x \to a} x = a$
10. $\lim_{x \to a} x^n = a^n$, where n is a positive integer.
11. $\lim_{x \to a} \sqrt[n]{x} = \sqrt[n]{a}$, where n is a positive integer.

Limit Laws (vii)

Example

Evaluate the following limits and justify each steps.

1. $\lim_{x\to 5}(2x^2 - 3x + 4)$

$$\begin{aligned}
\lim_{x\to 5}(2x^2 - 3x + 4) &= \lim_{x\to 5}(2x^2) - \lim_{x\to 5} 3x + \lim_{x\to 5} 4 (\text{By the laws (1) and (2)}) \\
&= 2\lim_{x\to 5} x^2 - 3\lim_{x\to 5} x + \lim_{x\to 5} 4 (\text{By law (3)}) \\
&= 2\left(\lim_{x\to 5} x\right)^2 - 3\lim_{x\to 5} x + \lim_{x\to 5} 4 (\text{By law 10}) \\
&= 2\left(\lim_{x\to 5} x\right)^2 - 3\lim_{x\to 5} x + 4 (\text{By law (8)}) \\
&= 2\times(5)^2 - 3\times 5 + 4 = 50 - 15 + 4 = 39 \ (\text{By the law (9)})
\end{aligned}$$

Limit Laws (viii)

2. $\lim_{x \to -2} \frac{x^3+2x^2-1}{5-3x}$

Evaluating Limits by Direct Substitution

Direct Substitution Property

If f is a polynomial or a rational function and a is in the domain f, then

$$\lim_{x \to a} f(x) = f(a)$$

Example: Find $\lim_{x\to 1} \frac{x^2-1}{x-1}$.

Evaluating Limits by Direct Substitution (ii)

If $f(x) = g(x)$ when $x \neq a$, then $\lim_{x \to a} f(x) = \lim_{x \to a} f(x)$, provided that the limits exists.

Example: Find $\lim_{x \to 1} g(x)$, where

$$g(x) = \begin{cases} x + 1 & \text{if } x \neq 1 \\ \pi & \text{if } x = 1 \end{cases}$$

Evaluating Limits by Direct Substitution (iii)

Example Evaluate $\lim_{h \to 0} \frac{(3+h^2)-9}{h}$.

Evaluating Limits by Direct Substitution (iv)

Example: Find $\lim_{t\to 0} \frac{\sqrt{t^2+9}-3}{t^2}$.