Calculus I

Tougaloo College

Miraj Samarakkody

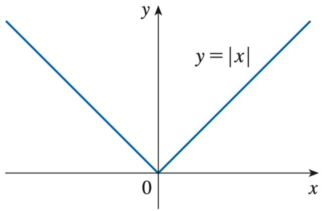
Calculating Limits Using the Limit Laws

Using one-sided limits

Theorem 1.

$$\lim_{x \to a} f(x) = L \text{ if and only if } \lim_{x \to a^-} f(x) = \lim_{x \to a^+} f(x) = L$$

Example: Show that $\lim_{x\to 0}|x| = 0$.

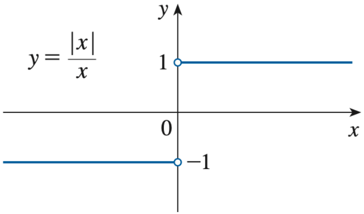


$$|x| = \begin{cases} x \text{ if } & x \geq 0 \\ -x \text{ if } & x < 0 \end{cases}$$

Take $\lim_{x\to 0^-}|x| = \lim_{x\to 0}(-x) = 0$. Then $\lim_{x\to 0^+}|x| = \lim_{x\to 0^+} x = 0$. Then limit is exist and and 0.

Using one-sided limits (ii)

Example: Prove that $\lim_{x\to 0} \frac{|x|}{x}$ does not exist.



Using one-sided limits (iii)

Example: If

$$f(x) = \begin{cases} \sqrt{x-4} & \text{if } x > 4 \\ 8 - 2x & \text{if } x < 4 \end{cases}$$

determine whether $\lim_{x \to 4} f(x)$ exists.

The Squeeze Theorem

Theorem 2.

If $f(x) \leq g(x)$ when x is near a (except possibly at a) and the limits of f and g both exist as x approaches a, then

$$\lim_{x \to a} f(x) \leq \lim_{x \to a} g(x)$$

The Squeeze Theorem (ii)

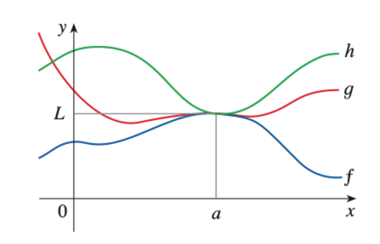
Theorem 3: The Squeeze Theorem.

If $f(x) \leq g(x) \leq h(x)$ when x is near a (except possibly at a) and

$$\lim_{x \to a} f(x) = \lim_{x \to a} h(x) = L$$

then

$$\lim_{x \to a} g(x) = L$$



The Squeeze Theorem (iii)

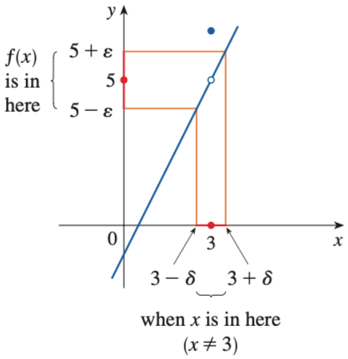
Example: Show that $\lim_{x\to 0} x^2 \sin\left(\frac{1}{x}\right) = 0$

Precise Definition of Limit

Definition 1.

$$\lim_{x \to a} f(x) = L$$

if for every number $\varepsilon > 0$ there is a number $\delta > 0$ such that if $0 < |x - a\,| < \delta$ then $|f(x) - L| < \varepsilon$.



Precise Definition of Limit (ii)

Example: Prove that $\lim_{x\to 3}(4x-5)=7$.

Precise Definition of Limit (iii)

Example: Prove that $\lim_{x \to 3} x^2 = 9$.