MAT102 - College Algebra - Polynomial and Rational Functions

3.1 Quadratic Functions and Applications [1]

Miraj Samarakkody

Tougaloo College

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Graph a Quadratic Function Written in Vertex Form

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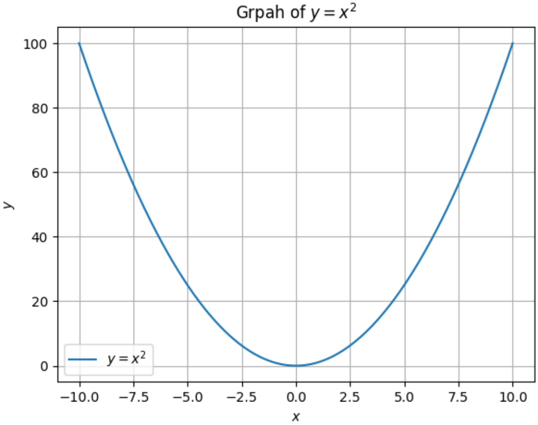

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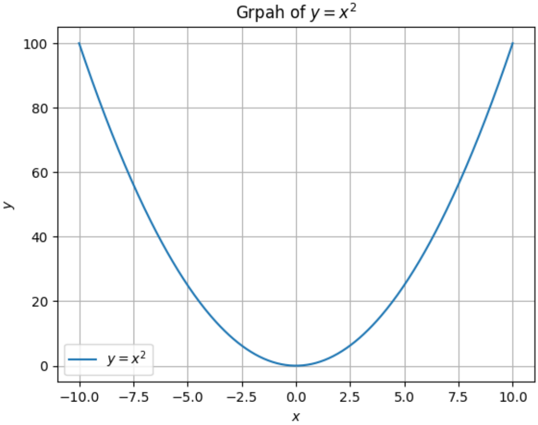

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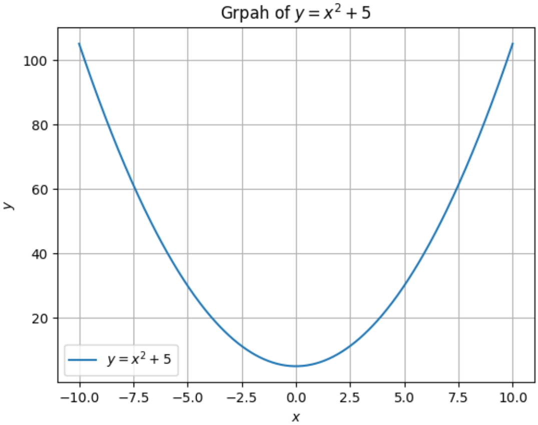

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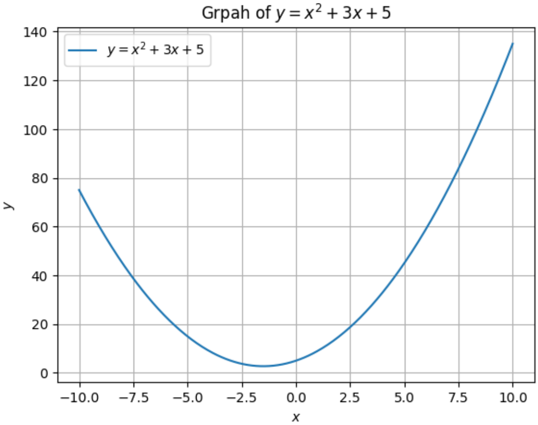

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- The axis of symmetry is $x = h$. This is the vertical line that passes through the vertex.

Example - Analyzing and Graphing a Quadratic Function

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8. Write the domain and range in interval notation.

Vertex Formula to Find the Vertex of a Parabola

For $f(x) = ax^2 + bx + c$ $(a \neq 0)$, the vertex is given by

$$\left(\frac{-b}{2a}, f\left(\frac{-b}{2a} \right) \right)$$

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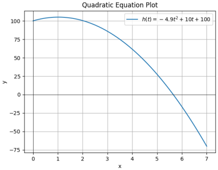
- If $b^2 - 4ac < 0$, the graph of $y = f(x)$ has no $x-$intercept.

Example - Using a Quadratic Function for Projectile Motion

A stone is thrown from a 100-m cliff at an initial speed of 20 m/sec at an angle of 30^0 from the horizontal. The height of the stone can be modeled by $h(t) = -4.9t^2 + 10t + 100$, where $h(t)$ is the height in meters and t is the time in seconds after the stone is released.



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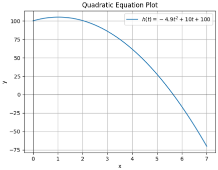

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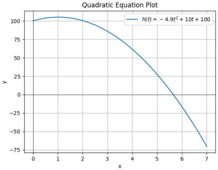

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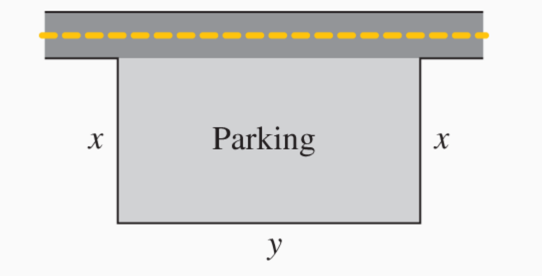
1. Determine the time at which the stone will be at its maximum height.
2. Determine the maximum height.
3. Determine the time at which the stone will hit the ground.

Example - Applying a Quadratic Function to Geometry

A parking area is to be constructed adjacent to a road. The develper has purchased 340 ft of fencing. Determine dimesions for the parking lot that would maximize the area. Then find the maximum area.



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