MAT102 - College Algebra - Polynomial and Rational Functions

3.2 Introduction to Polynomial Functions [1]

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Updated - June 4, 2025

Determine the End Behavior of a Polynomial Function

Definition of a Polynomial Function

Let n be a natural number and $a_n, a_{n-1}, \ldots, a_1, a_0$ be real numbers, where $a_n \neq 0$. Then a function defined by

$$f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0$$

is called a **Polynomial function of degree** n.

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Examples for non-polynomial functions.

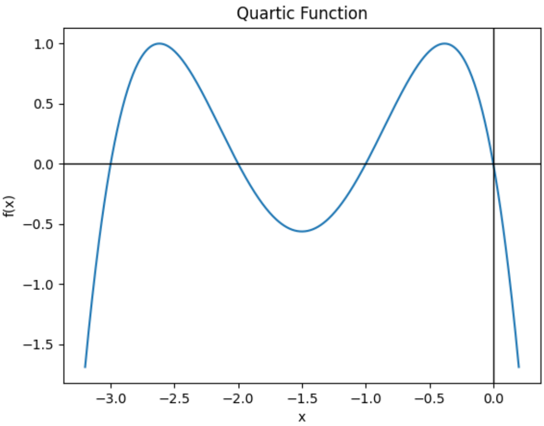
Several Special Cases of Polynomial Functions

Let $a \neq 0$.

$f(x) = c$	constant function	degree 0
$g(x) = ax + b$	linear function	degree 1
$h(x) = ax^2 + bx + c$	quadratic function	degree 2
$j(x) = ax^3 + bx^2 + cx + d$	cubic function	degree 3
$k(x) = ax^4 + bx^3 + cx^2 + dx + e$	quartic function	degree 4

Smoothness and Continuity

Smooth and Continuous



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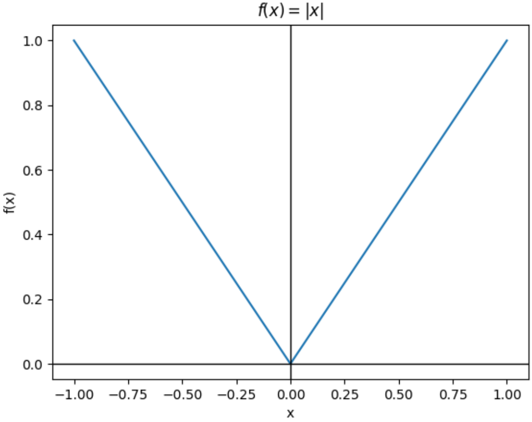
Not Smooth



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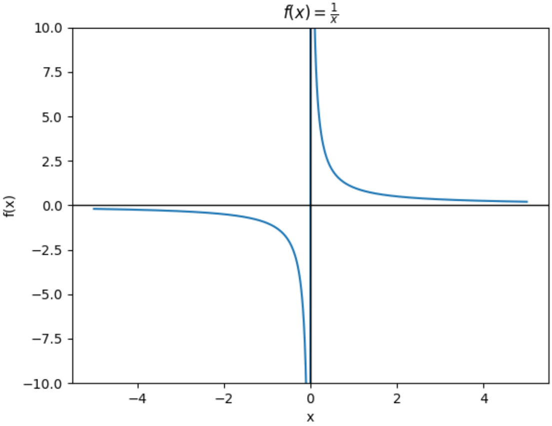

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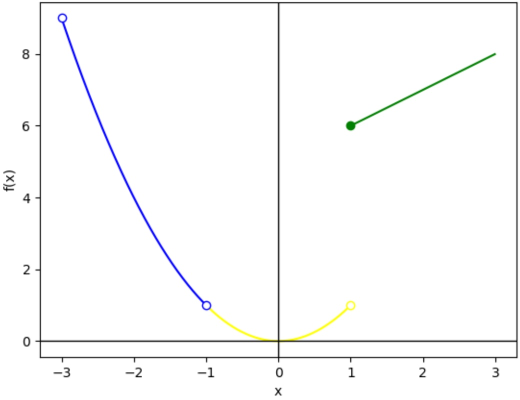

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Consider the function defnined by

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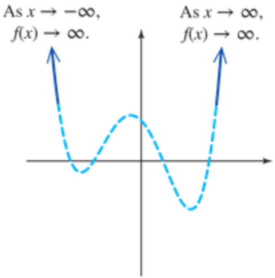
The leading term has the greatest exponent on x.

The Leading Term Test

Consider a polynomial function given by

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$$

n is even and a_n positive



End behavior: up left/up right

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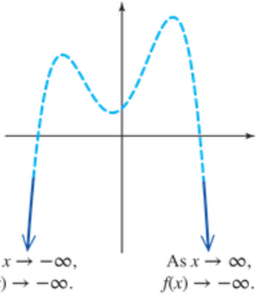

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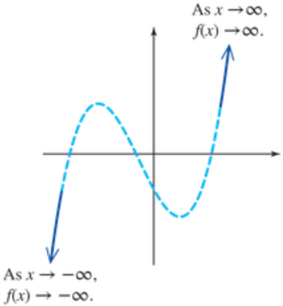

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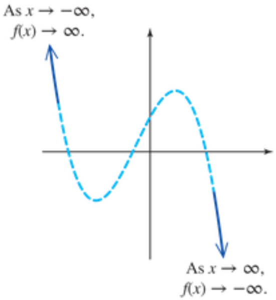

End behavior: up left/down right

Example - Determining End Behavior

Use the leading term to determine the end behavior of the graph of the function.

1. $f(x) = -4x^5 + 6x^4 + 2x$
   pause
2. $g(x) = \frac{1}{4}x(2x-3)^3(x+4)^2$

Example - Determining the Zeros of a Polynomial Function

Find the zeros of the function defined by

$$f(x) = x^3 + x^2 - 9x - 9.$$

Example - Determine the Zeros of a Polynomial Function

Find the zeros of the function defined by $f(x) = -x^3 + 8x^2 - 16x$.

Touch Points and Cross Points

Let f be a polynomial function and let c be a real zero of f. Then the point $(c, 0)$ is an $x-$intercept of the graph of f. Furethermore,

- If c is a zero of odd multiplicity, then the graph crosses the $x-$axis at c. The point $(c, 0)$ is called a **cross point**.

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- If c is a zero of odd multiplicity, then the graph crosses the $x-$axis at c. The point $(c, 0)$ is called a **cross point**.
- If c is a zero of even multiplicity, then the graph touches the $x-$axis at c and turns back around. The point $(c, 0)$ is called a **touch point**.

Determining Zeros and Multiplicities

Determine the zeros and their multiplicities for the given functions.

- $m(x) = \dfrac{1}{10}(x-4)^2(2x+5)^3$

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- $m(x) = \dfrac{1}{10}(x-4)^2(2x+5)^3$
- $n(x) = x^4 - 2x^2$

Intermediate Value Theorem

Let f be a polynomial function. For $a < b$, if $f(a)$ and $f(b)$ have opposite signs, then f has at least one zero on the interval $[a, b]$.

Example - Applying the Intermediate Value Theorem

Show that $f(x) = x^4 + 6x^3 - 26x + 15$ has a zero on the interval $[1, 2]$.

Number of Turning Points of a Polynomial Function

Let f represent a polynomial function of degree n. Then the graph of f has at most $n-1$ turning points.

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Why at most?

References

Julie Miller and Donna Gerken.
College Algebra.
McGraw-Hill Education, New York, 2nd edition, 2016.