

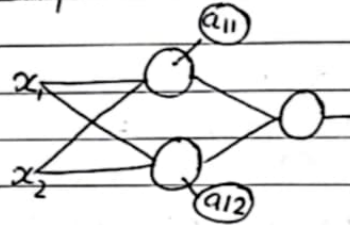
$$\text{normalize} = \left[\begin{array}{l} \text{mean} \rightarrow 0 \\ \text{sd} \rightarrow 1 \end{array} \right]$$



Batch Normalization - It is used to speed up neural network training

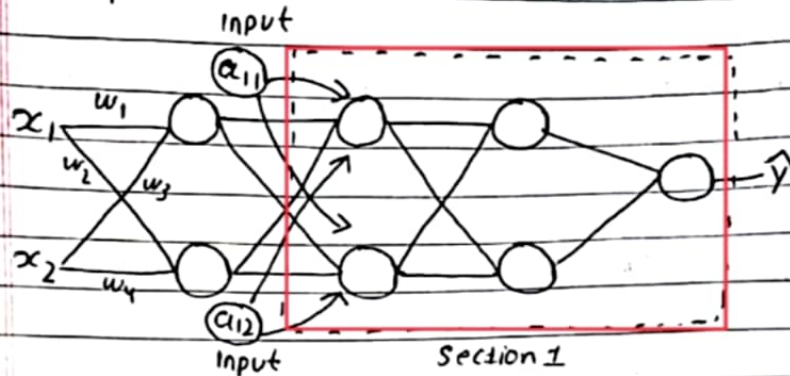
It normalizes the activation vectors from hidden layer using mean and variance of current batch

Example:



We normalize the input x_1, x_2 (It's normal case) but in Batch normalization process we also normalize the $[a_{11}, a_{12}]$.

Concept:

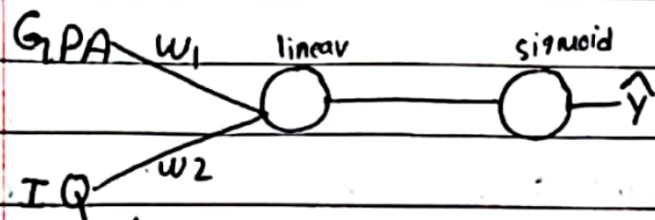


⇒ Section 1 hai individual neural network soham. Section 1 ko first layer ko node ley input a_{11} and a_{12} liya. a_{11} and a_{12} values after one batch of training change hain, kya ye ko distribution ne change hunge kya hame har batch ko training ke baad weight update henge. a_{11} and a_{12} (w_1, w_2, w_3, w_4) ke depend kare har batch ko training ke baad (w_1, w_2, w_3, w_4) ke change henge. So, Section 1 ko node ley different distribution input hoga. Toke training stable hoga ya model ley na chahi kare pani learn garga ya slow hoga. So, we use concept of batch normalization.

Jab a_{11} and a_{12} normalize hoga ye ko value between 0 and 1 ke range me hoga. So, Section 1 ko layer node ley stable data parur and training faster hoga.

[Internal covariate shift - change of distribution of activation due to change of network parameters during training]

GPA	IQ	Placement
8.9	100	1
6.2	25	0
8.3	2	0
9.2	70	1



$$z'_{11} = 8.9 \times w_1 + 100 \times w_2 + b_{11} \quad [z'_{11} \rightarrow \text{first row batch ko}]$$

$$= M^1$$

\downarrow first node
first layer

$[M^1 \rightarrow \text{linear output of first row}]$

Similarly we will have M^2, M^3, M^4

Our batch size is 4 so, in one batch we have 4 rows so, we will get 4 outputs $[M^1, M^2, M^3, M^4]$

Now, we will normalize them using formula

$$z'_{\text{nor}} = \frac{m^i - \mu}{\sigma} \quad [i \text{ is row no of batch}]$$

$$\therefore \mu = \frac{1}{n} \sum_{i=1}^n m^i$$

$$\Rightarrow \frac{[M^1 + M^2 + M^3 + M^4]}{4 (\text{batch size})}$$

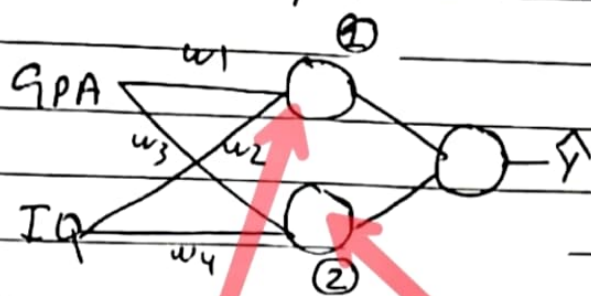
$$\sigma = \sqrt{\frac{1}{n} \sum_{i=1}^n (m^i - \mu)^2}$$

$$\text{Now, let's do for } z'_{\text{nor}} = \frac{m^1 - \mu}{\sigma + \epsilon}$$

(ϵ is small value if σ is 0 then ϵ will prevent it from undefined)

Similarly we will get $(z'_{\text{nor}}^1, z'_{\text{nor}}^2, z'_{\text{nor}}^3, z'_{\text{nor}}^4)$

If it has multiple nodes like below figure



GPA	IQ	Pluc
1	2	1
10	11	0

We have to calculate u and b separately for 1, 2 nodes and do normalize.

Node 1 Outputs:

$$\begin{aligned} &\rightarrow 1 \cdot w_1 + 2 \cdot w_3 + b_{11} \\ &\rightarrow 10 \cdot w_1 + 11 \cdot w_3 + b_{11} \\ &\text{(calculate separate } u \text{ and } b \text{ and normalize)} \end{aligned}$$

Node 2 Outputs:

$$\begin{aligned} &1 \cdot w_2 + 2 \cdot w_4 + b_{12} \\ &10 \cdot w_2 + 11 \cdot w_4 + b_{12} \\ &\text{(calculate separate } u \text{ and } b \text{ and normalize)} \end{aligned}$$

Now,

In depth Intuition

$$z'_{11} \rightarrow z'_{11, \text{nor}} \rightarrow p'_{11} \rightarrow g(p'_{11}) \rightarrow a'_{11}$$

(z'_{11} first layer ko first node ley batch ko first row ko output)

Here $p = \gamma \cdot z + \beta$ (γ and β are learnable parameters whose values can be find during training)

Taille normalize garda samro accuracy na aana sakxa tei varo γ and β learnable parameter sakheko yo pani w and b update vako jastai update hunxa during back propagation like

$$\gamma_{\text{new}} = \gamma_{\text{old}} - \frac{d \text{ Loss}}{d \gamma_{\text{old}}}$$

Generally $\gamma = 1$ and $\beta = 0$ during first time.