PreRequisites sheet for MTH 201
Edition 1

Hadi Al Hassan
hadi.alhassan@lau.edu
BS in Computer Science (2022-2025)
Lebanese American University
School of Arts and Sciences

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Abstract

I remember when i had Calculus 3, i had just come out of high-school having known nothing about integrals.
I spent a month unable to focus in class because i was scared of even starting. With some hard work, the resources i found, and the wonderful people i had help me. I got an A on Calculus 3. It is not an impossible task. Calc is just a measure of how much you practiced and understood a concept.
To help students just starting with the course, I have compiled this PDF consisting of all the rules and basics you need to understand coming into the course. No matter how late you start or how hard you think the course is, trust me. I literally studied For exam I from scratch 4 days before. You can do it.
A thank you to the following people:

- Akram Tabaa: The man who tutored me Calculus and helped me discover my love for math.
- Dr. May Hamdan, My Calc3 professor.
- Dr. Ahmad Makki, whose lectures were always a useful resource to study from. And the Doctor who changed my perspective and mentality towards studying.
- Dr. Leila issa, who was always there for guidance.
- Sara el Baba, whose Calc sheets were a fundamental resource.
- Izzidine Hnaini, My High school Math Teacher
- Mahabba, who was kind enough to point out mistakes in the sheet.

Best of luck and Enjoy MTH 201.
-Hadi

Contents

1 Derivative Rules:

The derivative of a function is the slope of a tangent to a function at a point.

Function F(x)	**Derivative F'(x)**
K (Constant)	0
x^n	$n \times x^{n-1}$
K.U (K constant)	K.U'
U + V	U' + V'
U.V	U'.V + V'.U
$\frac{U}{V}$	$\frac{U'V - V'U}{V^2}$
U^n	$n.U^{n-1}.U'$
$\frac{1}{x}$	$\frac{-1}{x^2}$
$\ln(x)$	$\frac{1}{x}$
$\ln(U)$	$\frac{U'}{U}$
e^x	e^x
e^U	$U'.e^U$

2 Integration Rules:

$$\int x^n \, dx = \frac{1}{n+1} \times x^{n+1} + c \qquad (n \neq -1)$$

$$\int U'.U^n \, dx = \frac{1}{n+1} \times U^{n+1} + c \qquad (n \neq -1)$$

$$\int \frac{1}{x} \, dx = \ln|x| + c$$

$$\int \frac{U'}{U} \, dx = \ln|U| + c$$

$$\int e^x \, dx = e^x + c$$

$$\int e^{ax+b} dx = \frac{1}{a}.e^{ax+b} + c$$

$$\int U'.e^U dx = e^U + c$$

3 Integration by Parts:

The General Rule Integration for By Parts is:

$$\int U dV = U.V - \int V dU$$

The trick is choosing U,
here is the trick Akram gave me to remember which term to choose for U.
It is a simple question:
What do **U** do?
The answer: U-***L.I.A.T.E***
Where:

- **L**: Logarithms
- **I**: Inverse ($\sin^{-1}$ / $\cos^{-1}$)
- **A**: Algebraic
- **T**: Trigonometric
- **E**: Exponential

Use this handy trick to know the order of priority of which term to set as U.

4 Integration by substitution:

Covered in depth in paul's online notes:
`https://tutorial.math.lamar.edu/Classes/CalcI/SubstitutionRuleIndefinite.aspx`

5 Partial Fractions:

Partial fractions is a handy technique of taking a rational expression and decomposing it into simpler rational expressions that we can add or subtract to get the original rational expression.

It is a very important technique to know for this course and is used quite often in the beginning.

the Condition to be able to use **partial fraction decomposition** is:

$\boxed{Deg_{numerator} \ < \ Deg_{denominator}}$

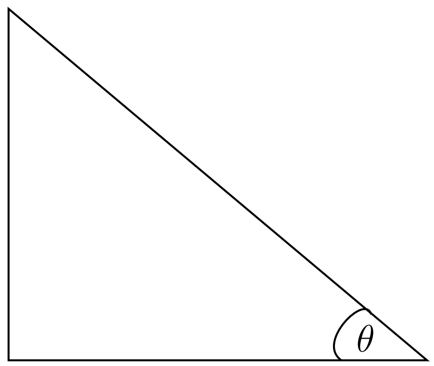
Recall: the degree of a polynomial is the value of the exponent.
Here is a handy table for determining how to proceed:

Factor in denominator	Term in partial fraction Decomposition
$ax+b$	$\frac{A}{ax+b}$
$(ax+b)^k$	$\frac{A_1}{ax+b} + \frac{A_2}{(ax+b)^2} + \ldots + \frac{A_k}{(ax+b)^k}$
ax^2+bx+c	$\frac{Ax+b}{ax^2+bx+c}$
$(ax^2+bx+c)^k$	$\frac{A_1x+B_1}{(ax^2+bx+c)^1} + \frac{A_2x+B_2}{(ax^2+bx+c)^2} + \ldots + \frac{A_kx+B_k}{(ax^2+bx+c)^k}$

Where k = 1,2,3...,

6 Trigonometric Functions and their Derivatives:

6.1 Recall of Trigonometry:



$$\sin(\theta) = \frac{opp}{hyp}$$

$$\cos(\theta) = \frac{adj}{hyp}$$

$$\tan(\theta) = \frac{opp}{adj}$$

6.2 Basic Trig Functions and their reciprocals:

$The\ reciprocal\ of:$

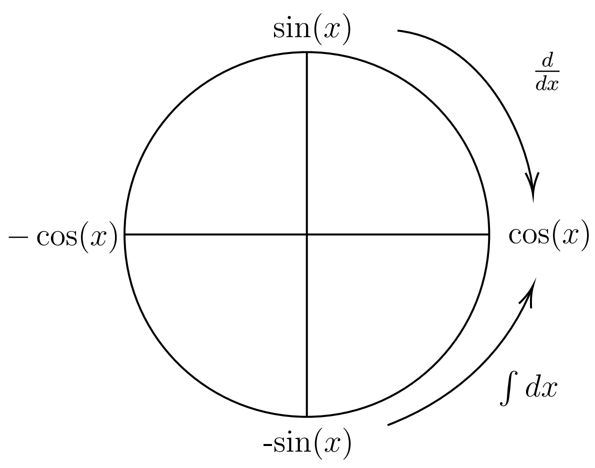
$\sin(\theta) \quad is\ cosec(\theta) = \csc(\theta)$

$\cos(\theta) \quad is \quad \sec(\theta)$

$\tan(\theta) \quad is \quad \cot(\theta)$

6.3 Derivatives integrals:

- $\frac{d}{dx}\sin(x) = \cos(x)$
- $\frac{d}{dx}\cos(x) = -\sin(x)$
- $\frac{d}{dx}\tan(x) = \sec^2(x)$
- $\frac{d}{dx}\sec(x) = \sec(x).\tan(x)$



7 Associated Arcs and Double angle formulas:

7.1 Remarkable angles:

Sine, Cosine, and Tangent of Some Remarkable Angles								
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
Degrees	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
Sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
Cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0	1
Tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	Undefined	0	Undefined	0

Table 1: Trigonometric values of some notable angles

7.2 Associated Arcs:

$$\sin(-\alpha) = -\sin(\alpha)$$

$$\cos(-\alpha) = \cos(\alpha)$$

7.3 Double Angle Formulas:

$$\cos(2\theta) = \cos^2(\theta) \; - \; \sin^2(\theta)$$

$$\cos(2\theta) = 2\cos^2(\theta) \; - \; 1$$

$$\cos(2\theta) = 1 \; - \; 2\sin^2(\theta)$$

$$\sin(2\theta) = 2\sin(\theta).\cos(\theta)$$

8 Inverse Trigonometric Functions:

$$\int \frac{1}{\sqrt{a^2 - u^2}} = \sin^{-1}(\frac{u}{a}) \ + \ c \quad (u^2 < a^2)$$

$$\int \frac{1}{a^2 + u^2} = \tan^{-1}(\frac{u}{a}) \ + \ c$$

$$\int \frac{1}{u\sqrt{u^2 - a^2}} = \frac{1}{a}\sec^{-1}|\frac{u}{a}| \ + \ c$$

Figure 1: Table 7.4 from Thomas Calculus

9 Hyperbolic Functions:

$$\sinh(x) = \frac{e^x - e^{-x}}{2} \qquad \frac{d}{dx}\sinh(x) = \cosh(x)$$

$$\cosh(x) = \frac{e^x + e^{-x}}{2} \qquad \frac{d}{dx}\cosh(x) = \sinh(x)$$

10 Limits:

Determinate forms	Indeterminate forms
1: $+\infty + \infty = +\infty$ 2: $-\infty - \infty = -\infty$	$+\infty - \infty$ **or** $-\infty + \infty$
3: $k + \infty = +\infty \longrightarrow 20^{25} + \infty = +\infty$ 4: $k - \infty = -\infty \longrightarrow 20^{25} - \infty = -\infty$	
5: $(+\infty)(+\infty) = +\infty$ 6: $(-\infty)(-\infty) = +\infty$ 7: $(+\infty)(-\infty) = -\infty$ 8: $(-\infty)(+\infty) = -\infty$	$\mathbf{0\,(\infty)} \longrightarrow (0)(+\infty)$ or $(0)(-\infty)$
9: $k(+\infty) = \begin{cases} +\infty \text{ if } k > 0 \longrightarrow +3(+\infty) = +\infty \\ -\infty \text{ if } k < 0 \longrightarrow -4(+\infty) = -\infty \end{cases}$	
10: $\frac{k}{+\infty} = \begin{cases} 0^+ \quad \text{if } k > 0 \quad \longrightarrow \frac{+5}{+\infty} = 0^+ \\ 0^- \quad \text{if } k < 0 \quad \longrightarrow \frac{-4}{+\infty} = 0^- \end{cases}$	
11: $\frac{k}{0} = \pm\infty \begin{cases} \longrightarrow \frac{+4}{0^+} = +\infty \ \& \ \frac{+4}{0^-} = -\infty \\ \longrightarrow \frac{-5}{0^+} = -\infty \ \& \ \frac{-5}{0^-} = +\infty \end{cases}$	$\frac{\infty}{\infty} \longrightarrow \frac{+\infty}{+\infty}$ or $\frac{+\infty}{-\infty}$ or $\frac{-\infty}{+\infty}$ or $\frac{-\infty}{-\infty}$ $\frac{0}{0}$
12: $\frac{+\infty}{0^+} \longrightarrow +\infty \ \& \ \frac{-\infty}{0^+} \longrightarrow -\infty$	
13: $\sqrt{+\infty} = +\infty$	$\sqrt{-\infty}$ ***undefined***.

Figure 2: Some limit rules (Credit A. Ashkar)

11 Limits of trigonometric Functions(and inverses):

$$-1 \leq \sin(x) \leq 1$$

$$\lim_{x \to -\infty} \tan^{-1}(x) = -\frac{\pi}{2}$$

$$\lim_{x \to \infty} \tan^{-1}(x) = \frac{\pi}{2}$$

12 Indeterminate Forms:

When performing Limits, Some may give us an expression that we cannot compute, or what we call an **Indeterminate form**.
Here are the Indeterminate form startegies my High school teacher gave me:

$$\frac{0}{0} \quad Or \quad \frac{\infty}{\infty} : L.Hopital's\ Rule\ (H.R)$$

$$0 \times (\infty) : \qquad reshape \qquad \frac{a}{b} = \quad \frac{a}{\frac{1}{b}} \qquad then\ H.R$$

$$\infty - \infty : \ Change\ the\ expression\ \ln(a) - \ln(b) = \ \ln(\frac{a}{b})$$

Other ways to deal with indeterminate forms:

- Taking Common factor
- Making same denominator

A review of L.Hopital's Rule: A rule to help simplify functions to compute their limit.

$$\lim_{x \to a} \frac{F(x)}{G(x)} \ = \lim_{x \to a} \frac{F'(x)}{G'(x)}$$

13 Rules/Techniques to Remember

13.1 Algebraic Expressions:

$$a^3 + b^3 = (a + b) \times (a^2 - ab + b^2)$$
$$a^3 - b^3 = (a - b) \times (a^2 + ab + b^2)$$

13.2 Completing the Square Method:

You've probably seen this while studying for the SAT's, it's just a neat way for you to reshape an expression to your convenience.
Example: Solve the quadratic equation $x^2 + 6x - 7 = 0$ by completing the square.

1. **Step 1:** Ensure that the coefficient of the x^2 term is 1. If it is not, divide the entire equation by the coefficient.

   In our example, the coefficient of x^2 is already 1, so we can proceed to the next step.

2. **Step 2:** Move the constant term to the other side of the equation.

   $x^2 + 6x = 7$

3. **Step 3:** Take half of the coefficient of the x term, square it, and add it to both sides of the equation.

   In this case, the coefficient of x is 6. Half of 6 is 3, and 3 squared is 9. Add 9 to both sides of the equation.

   $x^2 + 6x + 9 = 7 + 9$

   Simplifying the right side:

   $x^2 + 6x + 9 = 16$

4. **Step 4:** Express the left side of the equation as a perfect square trinomial.

   The left side of the equation can be factored as the square of a binomial: $(x + 3)^2$.

   $(x + 3)^2 = 16$

5. **Step 5:** Take the square root of both sides of the equation.

   $\sqrt{(x + 3)^2} = \sqrt{16}$

   $x + 3 = \pm 4$

6. **Step 6:** Solve for x.

   For the positive square root:

   $x + 3 = 4$ gives $x = 1$

   For the negative square root:

   $x + 3 = -4$ gives $x = -7$

Therefore, the solutions to the quadratic equation $x^2 + 6x - 7 = 0$ are $x = 1$ and $x = -7$.

14 Resources to use:

The following resources are Very useful for practice and properly grasping Calculus:

14.1 Dr Ahmad Makki's Lectures:

Those Lectures were one of the main reasons i got an A.

14.2 Dr Leila issa's Online recorded Lectures:

I used her lectures to understand double integrals in depth.

14.3 Paul's Online Notes:

Those lectures helped me understand substitution in depth, and get an 89 on Exam I.
Note that in the U.S this material is considered Calc 2.
https://tutorial.math.lamar.edu/classes/calcII/calcII.aspx

This is a link to my calculus archive, which contains all these resources and more: https://lauedu74602-my.sharepoint.com/:f:/g/personal/hadi_alhassan_lau_edu/EnSVpSawv-tPjqzjfv_xb50Bjtn15nlMkuVgu9MOO0v0PQ?e=xgbk1c

Type-setted by Hadi Al Hassan Using LaTeX
In reference to Thomas Calculus Textbook