Lab Manual for MTH 112

Precalculus II: Trigonometry

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Precalculus II: Trigonometry

Mathematics Faculty
Portland Community College

February 13, 2026

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Preface

This lab manual was designed with the intent of being used in the following manner. In each section:

- Students will complete the Preparation Exercises before receiving instruction on the content. Some instructors will have students do this before coming to class, while others might do it as a warmup in class.
- There will be some instructor-led presentation of the content. This may be a formal class lecture, a discovery activity, a video lecture, or something else.
- Students will then engage in Practice Exercises in a group setting to reinforce their initial understanding of the foundational concepts.
- An instructor will then assign one or more group activities. Because many of these activities are webbased and instructors can choose to use different activities, we have not included any of the possible activities in this document.
- The Definitions are meant to provide a single location for all definitions and some key concepts. Students can use this as a resource after having covered the topics in class.
- Students will complete some or all of the Exit Exercises, as decided by their instructor, to summarize their understanding of key concepts.
- In general, not every section of this lab manual will take the same amount of class time. Some sectionsmight be half-day topics, while others could be two-day topics.
- Each instructor will identify for their class which exercises will be submitted as part of the course grade.

Course Resource Links:

Algebra and Trigonometry 2e	`tiny.cc/111Z-Textbook`
MTH 111 Supplement	`tiny.cc/111Z-Supplement`

Contents

Chapter 1

Functions

1.1 Functions and Function Notation

In this section, we'll develop our understanding of functions and function notation, whether the function is presented as a set of ordered pairs, a table of values, a graph, or an equation. We'll also learn how to evaluate a function given an input value and to solve for an input given a function's output value.

Textbook Reference

This relates to content in §3.1 and §3.2 of Algebra and Trigonometry 2e[1].

Preparation Exercises

1. Many of us have at least one restaurant we would love to go to for a meal. Go online and find a menu for a restaurant you'd choose to eat at.

- **(a)** What is the name of the restaurant and what type of food do they serve?
- **(b)** What is a dish you would like to get and how much does it cost?
- **(c)** Are there any other dishes on the menu that cost the same as your dish? If so, what are they?
- **(d)** If I ask you the price of any specific dish from the menu, do you know how much it costs?
- **(e)** If you know how much I paid for a dish, do you know exactly what I ordered based just on the price? Explain your answer.

Practice Exercises

1. Determine if each of the following relations show y as a function of x. Explain your reasoning by referencing the definition of a function. If a relation is not a function provide a specific example why the definition was not satisfied. Assume all ordered pairs are of the form (x, y).

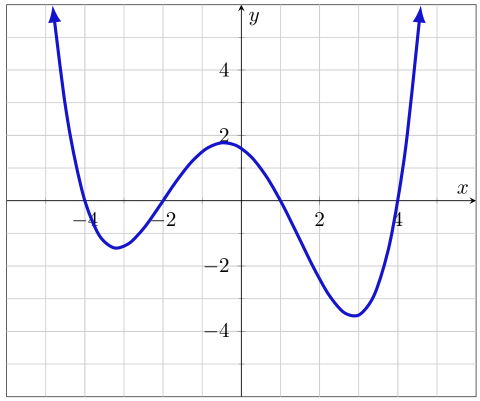
- **(a)** $\{(\text{red}, \text{pepper}), (\text{green}, \text{pear}), (\text{purple}, \text{grape}), (\text{orange}, \text{orange}), (\text{yellow}, \text{pepper}), (\text{red}, \text{onion})\}$

[1] `http://tiny.cc/111Z-Textbook`

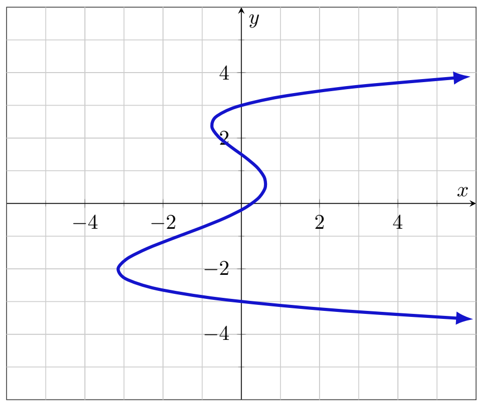
(b)

x	-5	3	-1	2	4	6
y	9	-4	2	-4	-5	-6

(c)



(d)



2.

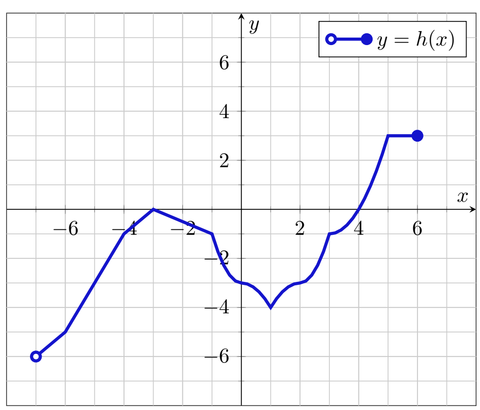
(a) Let $y = g(x)$ be defined as the set of (x, y) ordered pairs:

$$\{(-60, 5), (-23, 4), (-4, 3), (3, 2), (4, 1), (5, 0), (12, -1)\}$$

(i) Find $g(5)$.

(ii) Solve $g(x) = 3$.

(b) Let $y = h(x)$ be defined by the graph below.



(i) Find $h(-4)$.

(ii) Solve $h(x) = -3$.

(c) Let $f(x) = x^2 - 3$.

(i) Find $f(-4)$.

(ii) Solve $f(x) = 46$.

Definitions

Definition 1.1.1 Relation. A **relation** is a set of (x, y) ordered pairs.

The variable of x is called the **independent variable** or **input variable**. Each individual x-value is referred to as a **input** or **input value**.

The variable of y is called the **dependent variable** or **output variable**. Each individual y-value is referred to as a **output** or **output value**. ◊

Example 1.1.2 Relation. The set

$$\{(0, -2), (1, -1), (2, 0), (1, -3), (3, -4)\}$$

is a relation. □

Definition 1.1.3 Function. A **function** is a relation where each possible input value is paired with *exactly one* output value. We say, "The output is a function of the input," and often write this algebraically as $y = f(x)$. ◊

Example 1.1.4 Function. The set

$$\{(0, -2), (1, -1), (4, 0), (9, 1), (16, 2)\}$$

is a function. □

Example 1.1.5 Not a function. The set

$$\{(0, -2), (1, -1), (2, 0), (1, -3), (3, -4)\}$$

is a relation, but *not* a function. □

Definition 1.1.6 Domain and Range. The **Domain** of a relation or function is the set of all possible input values. The **Range** of a relation or function is the set of all possible output values. ◊

Example 1.1.7 Domain and Range. Given the function

$$\{(0, -2), (1, -1), (4, 0), (9, 1), (16, 2)\}$$

the domain of the function is $\{0, 4, 1, 9, 16\}$ and the range of the function is $\{-2, 0, -1, 1, 2\}$. □

Definition 1.1.8 Vertical Line Test. If a vertical line can be drawn that intersects the graph more than once, the graph is not the graph of a function with x as the independent variable and y as the dependent variable. ◊

Example 1.1.9

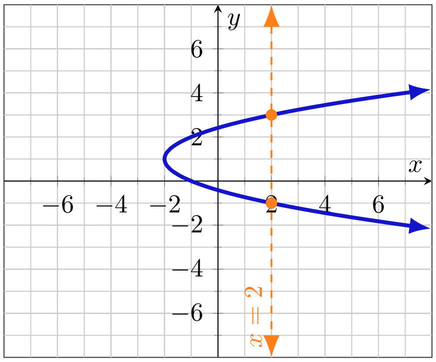


Figure 1.1.10 Does Not Pass

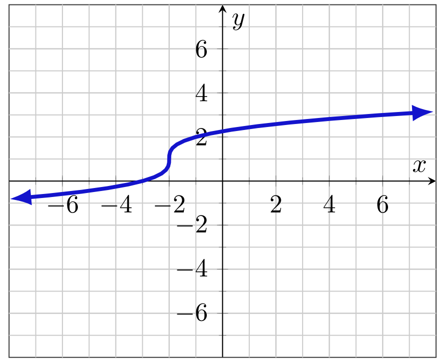


Figure 1.1.11 Passes

□

Exit Exercises

1.

(a) What is the definition of a function?

(b) What are two examples of functions that you use in your daily life outside of school? Explain how these are functions, referencing the definition of a function.

(c) What do you look for in the graph of a relation to determine if the graph is the graph of a function or not? Fully explain your answer.

2. If $f(x) = 2x^2 - 7x$, evaluate $f(-3)$.

3. If $g(x) = x^2 + 6x$, solve $g(x) = 16$.

Reflection

1. On a scale of 1–5, how are you feeling with the concepts related to the graphical behaviors of functions?

1.2 Domain, Range, and Behaviors of Functions

In this section, we'll learn to identify the domain and range of functions given in various forms, as well as determine when a function exhibits important behaviors.

Textbook Reference

This relates to content in §3.2 and §3.3 of Algebra and Trigonometry 2e[1].

Preparation Exercises

1.

(a) For any real number k other than 0, what is $\dfrac{0}{k}$ and why?

(b) For any real number k other than 0, what is $\dfrac{k}{0}$ and why?

[1] `http://tiny.cc/111Z-Textbook`

(c) Given $f(x) = \sqrt{x+2}$, evaluate $f(23)$ and $f(-18)$.

2.

(a) Draw $x > -2$ on a number line and write the set of values in both interval and set-builder notations.

Hint. Here is a review video[2] of these two notations.

(b) Draw $x \leq 6$ on a number line and write the set of values in both interval and set-builder notations.

Practice Exercises

1. Algebraically find the domain of the following functions. State the domains in both interval notation and set-builder notation.

(a) $f(x) = \sqrt{-2x+18}$

(b) $g(t) = \sqrt[3]{3t-24}$

(c) $h(k) = \dfrac{k+3}{k-9}$

2. Find the domain and range of the function p graphed in Figure 1.2.1. State both in interval notation and set-builder notation.

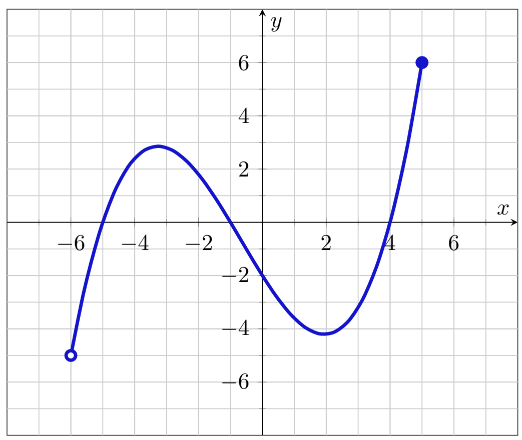


Figure 1.2.1 $y = p(x)$

3. Below are the graphs of $y = r(t)$ in Figure 1.2.2 and $y = s(t)$ in Figure 1.2.3.

[2] https://youtu.be/aLvRu8Int4M

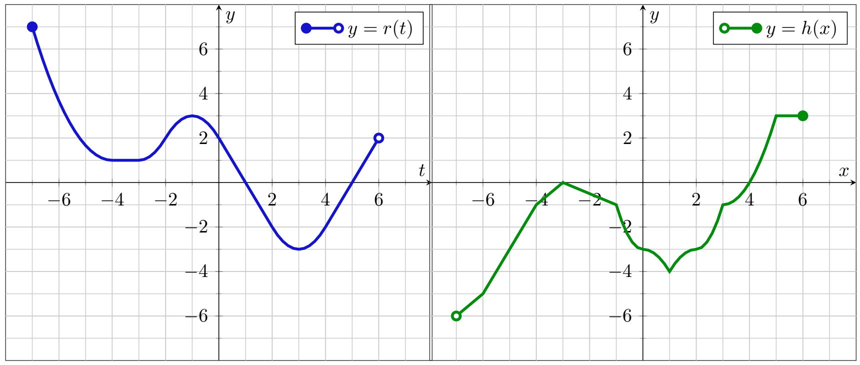


Figure 1.2.2 $y = r(t)$ **Figure 1.2.3** $y = s(t))$

(a) Over what intervals is r increasing?

(b) Over what interval is s negative?

(c) What is the absolute maximum value of r?

(d) State any local minimum points of s.

(e) Over what intervals is r constant?

(f) Over what intervals is s decreasing?

(g) Over what interval is r positive?

(h) What is the absolute minimum value of s?

Definitions

Definition 1.2.4 Domain and Range. The **domain** of a function is the set of all possible input values for the function.

The **range** of a function is the set of all possible output values for the function.

The domain and range are commonly stated using interval notation or set-builder notation. ◊

Example 1.2.5 Domain and Range. View this Desmos graph[3] to see an interactive example of these definitions. □

Definition 1.2.6 Positive and Negative. A function f is **positive** if the output values are greater than 0. f is positive when $f(x) > 0$.

A function f is **negative** if the output values are less than 0. f is negative when $f(x) < 0$. ◊

Example 1.2.7 Positive and Negative. View this Desmos graph[4] to see an interactive example of these definitions. □

Definition 1.2.8 Increasing, Decreasing, and Constant. Let f be a function that is defined on an open interval I, with a and b in I and $b > a$.

f is **increasing** on I if $f(b) > f(a)$ for all a and b in I. In other words, as you move left-to-right on the interval I, your y-values increase.

f is **decreasing** on I if $f(b) < f(a)$ for all a and b in I. In other words, as you move left-to-right on the interval I, your y-values decrease.

f is **constant** on I if $f(b) = f(a)$ for all a and b in I. In other words, as

[3] `https://tiny.cc/111Z-DomRang`
[4] `https://tiny.cc/111Z-PosNeg`

you move left-to-right on the interval I, your y-values do not change. ◊

Example 1.2.9 Positive and Negative. View this Desmos graph[5] to see an interactive example of these definitions. □

Definition 1.2.10 Local Minimum or Maximum. Given a function f that is defined on an open interval I, with c in I.

f has a **local maximum** at $x = c$ if $f(c) \geq f(x)$ for all x in I. The **local maximum value** of f is the output $f(c)$.

f has a **local minimum** at $x = c$ if $f(c) \leq f(x)$ for all x in I. The **local minimum value** of f is the output $f(c)$. ◊

Example 1.2.11

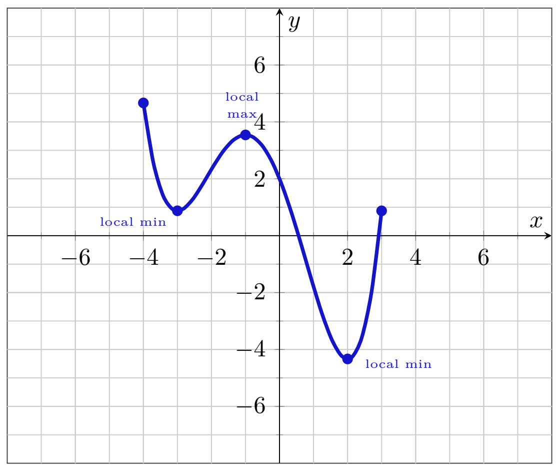


Figure 1.2.12 Local Extrema

In Figure 1.2.12, the function has two local minimum points and one local maximum point.

- The local minimum value of about 0.9 occurs at $x = -3$.
- The local minimum value of about -4.3 occurs at $x = 2$.
- The local maximum value of about 3.5 occurs at $x = -1$.

□

Definition 1.2.13 Absolute Minimum or Maximum. f has an **absolute maximum** at $x = c$ if $f(c) \geq f(x)$ for all x in the domain of f. The **absolute maximum value** of f is the output $f(c)$.

f has an **absolute minimum** at $x = c$ if $f(c) \leq f(x)$ for all x in the domain of f. The **absolute minimum value** of f is the output $f(c)$. ◊

Example 1.2.14

[5] `https://tiny.cc/111Z-IncDec`

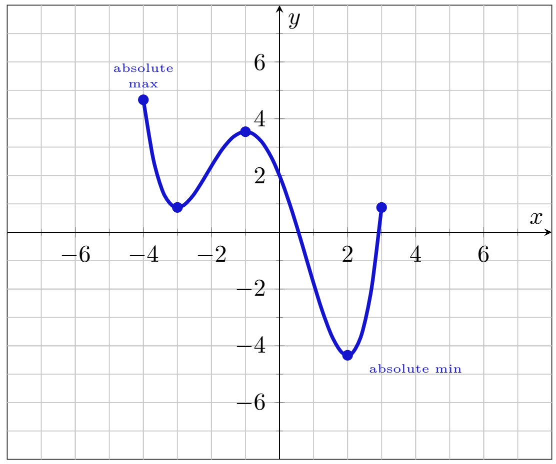


Figure 1.2.15 Absolute Extrema

In Figure 1.2.15, the function has an absolute minimum point and an absolute maximum point.

- The absolute minimum value of about -4.3 occurs at $x = 2$.
- The absolute maximum value of about 4.8 occurs at $x = -4$.

□

Exit Exercises

1. What is the domain of $f(x) = \sqrt{x}$? What is the domain of $g(x) = \sqrt[3]{x}$? Why are these domains different?

2. Graphically speaking, what is the difference between a function being negative and a function decreasing?

3. For the function F graphed in Figure 1.2.16, answer the following.

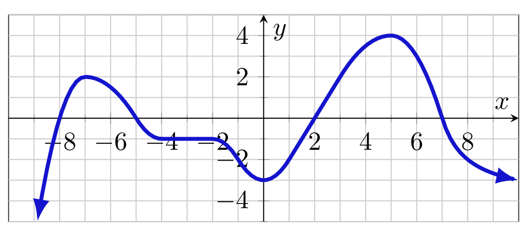


Figure 1.2.16 $y = F(x)$

(a) Over what intervals is F increasing?

(b) What is the range of F?

(c) Over what intervals is F negative?

(d) What are any local minimum points on F?

(e) Over what intervals is F constant?

(f) What is the absolute maximum value of F?

Reflection

1. On a scale of 1-5, how are you feeling with the concepts related to the graphical behaviors of functions?

1.3 Average Rates of Change and the Difference Quotient

In this section, we'll learn to identify the domain and range of functions given in various forms, as well as determine when a function exhibits important behaviors.

Textbook Reference

This relates to content in §3.3 of Algebra and Trigonometry 2e[1].

Preparation Exercises

1. Suppose you're driving south on I-5 in Oregon and you pass mile marker 294 in Portland at 1:35 PM. Later, you pass mile marker 194 in Eugene at 3:05 PM.

(a) What was your average speed (in miles per hour) of the trip from Portland to Eugene?

(b) What was your speed at any particular moment, say as you drove past mile marker 256 in Salem?

2. Let $f(x) = x^2 - 2x$. Evaluate and simplify the following.

(a) $f(4)$

(b) $f(-6)$

(c) $f(a)$

(d) $f(a + b)$

Practice Exercises

1. The function in Table 1.3.1 below shows the cost of movie tickets in the U.S. in the year t.

Costs obtained from `https://www.natoonline.org/data/ticket-price`

[1] `http://tiny.cc/111Z-Textbook`

Table 1.3.1 Price of Movie Tickets in the U.S.

t (year)	$m(t)$ (in dollars)
1995	4.35
1999	5.06
2003	6.03
2009	7.50
2013	8.13
2017	8.97
2021	10.17

(a) What is the unit of the average rate of change in the price of a movie over any time period?

(b) What is the average rate of change in the price of a movie ticket from 2003 to 2021?

2. Given $f(n) = \frac{1}{3}n^2 - 1$, find the average rate of change of f on the interval $[3, 9]$.

3. Find and simplify the difference quotient for each of the following functions.

(a) $f(x) = -6x + 8$

(b) $g(x) = -2x^2 - 5x$

Definitions

Definition 1.3.2 Rate of Change. A **rate of change** describes how the output values change in relation to a change in the input values. The unit for the rate of change is "output unit(s) per input unit." ♢

Definition 1.3.3 Average Rate of Change. The **average rate of change** for a function f between two input values x_1 and x_2 is the difference in their output values divided by the difference in the two input values. The average rate of change is calculated using the formula

$$\text{average rate of change } = \frac{f(x_2) - f(x_1)}{x_2 - x_1}, x_1 \neq x_2$$

The average rate of change is the slope of the line between the two points $(x_1, f(x_1))$ and $(x_2, f(x_2))$. ♢

Example 1.3.4 The function $E(x)$ gives the cost of a dozen eggs (in dollars) x years after 2010. If we know $E(19) = 1.362$ and $E(23) = 2.666$, we can find the average rate of change as

$$\begin{aligned}\frac{E(23) - E(19)}{23 \text{ years } - 19 \text{ years}} &= \frac{\$2.666 - \$1.362}{4 \text{ years}} \\ &= \frac{\$1.304}{4 \text{ years}} \\ &\approx \$0.33/\text{year}\end{aligned}$$

This shows that between 2019 and 2023, the cost of a dozen eggs increased on average by about \$0.33/year. □

Definition 1.3.5 Difference Quotient. The **difference quotient** for a function f is given by the formula

$$\frac{f(x+h)-f(x)}{h}, h \neq 0$$

The difference quotient is the average rate of change between the two points $(x, f(x))$ and $(x+h, f(x+h))$. ◊

Example 1.3.6 Given the function $f(x) = 3x^2 - 4x$, the difference quotient would be evaluated as

$$\begin{aligned}\frac{f(x+h)-f(x)}{h} &= \frac{(3(x+h)^2 - 4(x+h)) - (3x^2 - 4x)}{h} \\ &= \frac{3x^2 + 6xh + 3h^2 - 4x - 4h - 3x^2 + 4x}{h} \\ &= \frac{6xh + 3h^2 - 4h}{h} \\ &= \frac{h(6x+3h-4)}{h} \\ &= 6x + 3h - 4, h \neq 0\end{aligned}$$

□

Exit Exercises

1.

(a) What are two situations in your daily life that involve an average rate of change? What are the units for these rates of change?

(b) What is the formula for the difference quotient for the function k that has an input variable p?

(c) If you have a function m that gives the price of a gallon of milk in the year t, what would be the unit for the average rate of change of m?

2. Find and simplify the difference quotient for the function $f(t) = \dfrac{3}{t-6}$.

Reflection

1. On a scale of 1-5, how are you feeling with the concepts related to the graphical behaviors of functions?

1.4 Piecewise Defined Functions

In this section, we'll explore piecewise-defined functions, which are functions constructed from pieces of several other functions. We'll find the domain and range of these types of functions, as well as graph, evaluate, and solve them. And given the graph of a piecewise-defined function, we'll construct the formula for the graph's function.

Textbook Reference

This relates to content in §3.2 of Algebra and Trigonometry 2e[1].

Preparation Exercises

1. A family event charges \$4/person, with a maximum of \$20 for any single family.

   (a) How much will a family of three pay?

   (b) How much will a family of seven pay?

   (c) At what number of people does the calculation change from being per person to a single charge for the whole family?

2. In November 2022, Portland General Electric set the rates for basic residential service as a function of the number of kilowatt-hours (kWh) of energy used. The rates in November 2022 were \$0.0642/kWh when up to 1000 kWh (kilowatt-hours) are used and if greater than 1000 kWh are used, then the first 1000kWh are billed at the \$0.0642/kWh rate and \$0.07002/kWh is charged for the energy usage greater than the initial 1000 kWh.

   (a) What is the cost for using 740 kWh?

   (b) What is the cost for using 1320 kWh?

   (c) What is the formula to find the cost for using x kWh if x is greater than 1000 kWh?

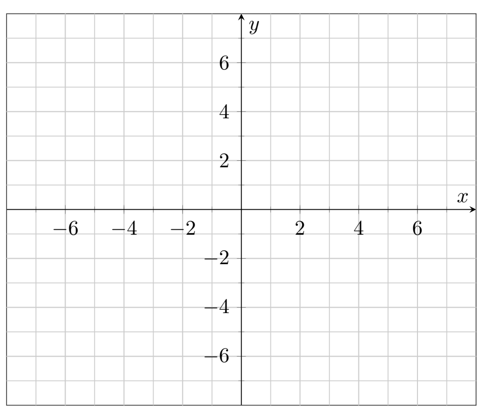
Practice Exercises

1. Let $f(x) = \begin{cases} x^2 - 4 & \text{if } -2 \leq x < 3 \\ \frac{2}{3}x - 1 & \text{if } x \geq 3 \end{cases}$

   (a) Evaluate $f(5)$.

   (b) What is the domain of f?

   (c) Graph $y = f(x)$ below.



2. The graph of $y = g(x)$ is in Figure **??**.

[1] `http://tiny.cc/111Z-Textbook`