1 -

$L[n+2] = L[n+1] + L[n]$

$L[0] = 2$

$L[1] = 1$

$L[2] = L[1] + L[0]$

$= 1 + 2$

$= 3$

$L[2] = 3$

$L[3] = L[2] + L[1]$

$= 3 + 1$

$= 4$

$L[4] = L[3] + L[2]$

$= 4 + 3$

$= 7$

b) find const A & B such that

$$L[n] = A\left(\frac{1+\sqrt{5}}{2}\right)^n - B\left(\frac{1-\sqrt{5}}{2}\right)^n$$

satisfies diff + initial cond.

b)

$L[0] = A\Phi^n - B\psi^n = 2$

$L[1] = A\Phi^n - B\psi^n = 1$

$L[0] = A - B = 2 \qquad A = 2 + B$

$L[1] = A\Phi - B\psi = 1 \qquad B = -2 - A$

$A\Phi = 1 + B\psi$

$A = \dfrac{1 + B\psi}{\Phi}$

$A = \dfrac{1 + B\left(\frac{1-\sqrt{5}}{2}\right)}{\frac{1+\sqrt{5}}{2}}$

a)

$2A = \dfrac{2 + (1-\sqrt{5})}{(1+\sqrt{5})}$

$2A = \dfrac{2 + 3.23}{-1.236}$

$A = -2.11$

$-2.11 - B = 2$

$B = -(2 + 2.11)$

$= -4.11$

WRONG

$A\left(\frac{1+\sqrt{5}}{2}\right)^n - B\left(\frac{1-\sqrt{5}}{2}\right)^n = 1$

$A(1+\sqrt{5}) - B(1-\sqrt{5}) = 2$

$2 + B(1+\sqrt{5}) - B(1-\sqrt{5}) = 2$

$2(1+\sqrt{5}) + B(1+\sqrt{5}) - B(1-\sqrt{5}) = 2$

$2 + 2\sqrt{5} + \cancel{B} + B\sqrt{5} - \cancel{B} + B\sqrt{5} = 2$

$2 + 2\sqrt{5} + 2B\sqrt{5} = 2$

$2(1 + \sqrt{5} + B\sqrt{5}) = 2$

$2\sqrt{5} + 2B\sqrt{5} = 0$

$2\sqrt{5}(1 + B) = 0$

$1 + B = 0$

$B = -1$

$A = 2 + B$

$A = 2 - 1$

$= 1$

$L[0] = 1\left(\frac{1+\sqrt{5}}{2}\right)^0 - (-1)\left(\frac{1-\sqrt{5}}{2}\right)^0$

$= 1 + 1 = 2$ ✓

$L[1] = 1\left(\frac{1+\sqrt{5}}{2}\right)^1 - (-1)\left(\frac{1-\sqrt{5}}{2}\right)^1 = 1$ ✓

$L[2] = 3$

$L[3] = 4$

$L[4] = 7$

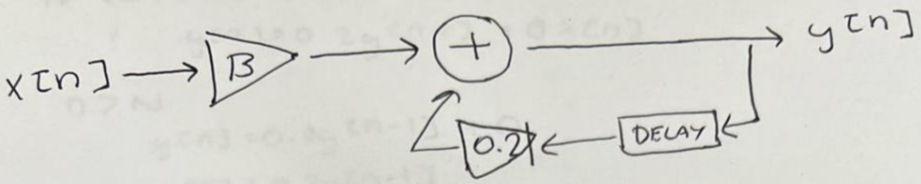
$n[1] = S[1]0.5 +$

$= 1$

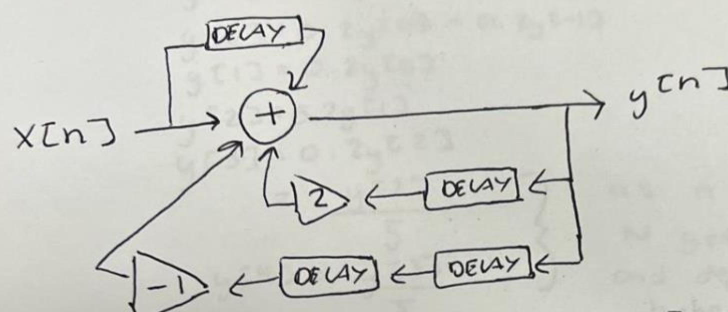
2a) $(x[n] + y[n-1])\alpha = y[n]$

2b) $(x[n] + x[n-1] + y[n-2])\alpha = y[n]$

3a) $y[n] = 0.2y[n-1] + \beta x[n]$



3b)



$y[n] = x[n] + x[n-1] + 2y[n-1] - y[n-2]$

4a) $y[n] = 0.2y[n-1] + \beta X[n]$

$X[n]$ for $n = 0, 1, 2 \ldots N$, able to $y[n]$ for $n = 1, 2 \ldots N$?

$X[n_0] = m$
$X[n_1] = f$
$X[n_2] = c$

$y[0] = 0.2y[-1] + \beta X[0]$
$y[1] = 0.2y[0] + \beta X[1]$
$y[1] = 0.2y[0] + \beta f$

$y[n] = 0.2y[n-1] + \beta X[n]$
$n=1$
$y[n_1] = 0.2y[0] + \beta f$
$y[n_2] = 0.2y[2-1] + \beta X[n_2]$
$y[n_2] = 0.2y[1] + \beta c$

YES BUT WE NEED TO KNOW $Y[0]$!

ACTUALLY NO BECAUSE YOU ARE NOT GIVEN $X[-1]$

4b) if $x[n] = 0$ for $n > N$, what's the behavior of $y[n]$ for $n > 0$

if $x[n] = 0$ when $n > N$

$y[n] = 0.2y[n-1] + \beta X[n]$

$n > N$

$y[n] = 0.2y[n-1] + 0$
$y[n] = 0.2y[n-1]$
$y[0] = 0.2y[0] = 0.2y[-1]$
$y[1] = 0.2y[0]$
$y[2] = 0.2y[1]$
$y[3] = 0.2y[2]$
$= \frac{y[2]}{5}$
$y[4] = \frac{y[3]}{5}$

as n increases, N gets smaller and smaller and depending on $y[0]$, behavior ca change.
converging to 0.

5)

$S[n+2] = 2S[n+1] + 2S[n]$

$S[n+1] = 2S[n] + 2S[n-1] \rightarrow S[n] = 2S[n-1] + 2[S][n-2]$

$S[0] = 0$

$S[1] = 1$

$S[2] = 2S[1] + 2S[0]$

$2S[0] = S[2] - 2S[1]$

$S[2] = 2(1) + 2(0)$

$S[2) = 2$

$S[1+2] = 2S(1+1) + 2S(1)$

$S[3] = 2S[2] + 2S[1]$

$S[3] = 2 \cdot 2 + 2(1)$

$S[3] = 4 + 2 = 6$

$S(2+2] = 2S[2+1] + 2S[2]$

$S[4] = 2S[3] + 2S[2]$

$S[4] = 2 \cdot 6 + 2 \cdot 2$

$= 12 + 4$

$S[4] = 16$

code

b) closed form $S[n] = Aa^n + Bb^n$

$S[n+2] = 2S[n+1] + 2S[n]$

$+ 2S[n]$

$Aa^{n+2} + Bb^{n+2} = 2(Aa^{n+1} + Bb^{n+1}) + 2(Aa^n + Bb^n)$

$S[0] = 0$

$S[1] = 1$

$Aa^{0+2} + Bb^{0+2} = 2(Aa^{0+1} + Bb^{0+1}) + 2(Aa^0 + Bb^0)$

$Aa^2 + Bb^2 = 2(Aa^1 + Bb^1) + 2(A + B)$

$Aa^2 + Bb^2 = 2Aa + 2Bb + 2A + 2B$

$0 = 2Aa + 2Bb + 2A + 2B - Aa^2 - Bb^2$

$$2Aa + 2Bb + 2A + 2B - Aa^2 - Bb^2 = 0$$

$$2(Aa + Bb + A + B) - Aa^2 - Bb^2 = 0$$

$$2Aa + 2A - Aa^2 + 2Bb + 2B - Bb^2 = 0$$

$$-Aa^2 + 2Aa + 2A - Bb^2 + 2Bb + 2B = 0$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = \frac{-2A \pm \sqrt{(2A)^2 - 4(-A)(2A)}}{2(-A)}$$

$$-Aa^2 + 2Aa + 2A = Bb^2 - 2Bb - 2B$$

$$b = \frac{+2B \pm \sqrt{(-2B)^2 - 4(B)(-2B)}}{2(B)}$$

$$a = \frac{-2A \pm \sqrt{4A^2 + 8A^2}}{-2A} \qquad b = \frac{2B \pm \sqrt{4B^2 + 8B^2}}{2B}$$

$$a = 1 \pm \sqrt{4A^2 + 8A^2} \qquad b = 1 \pm \sqrt{12B^2}$$

$$a = 1 \pm \sqrt{12A^2}$$

$$a = 1 \pm \sqrt{12A^2} = 1 \pm \sqrt{12}A$$

$$b = 1 \pm \sqrt{12B^2} = 1 \pm \sqrt{12}B$$

$$a - 1 = \sqrt{12A^2}$$

$$a - 1 = \sqrt{12}A$$

$$A = \frac{a-1}{\sqrt{12}}$$

$$B = \frac{b-1}{\sqrt{12}}$$

$$S[n] = 2S[n-1] + 2S[n-2]$$

$$a = 1 \pm \sqrt{12}\ A$$
$$b = 1 \pm \sqrt{12}\ B$$

$$A = \frac{a-1}{\sqrt{12}}$$

$$B = \frac{b-1}{\sqrt{12}}$$

$$S[n] = Aa^n + Bb^n$$

$$S(0) = 0$$
$$S(1) = 1$$

$$Aa^n + Bb^n = 2S(n-1) + 2S(n-2)$$

$$Aa^n + Bb^n = 2(Aa^{n-1} + Bb^{n-1}) + 2(Aa^{n-2} + Bb^{n-2})$$

$$Aa^n + Bb^n = 2Aa^{n-1} + 2Bb^{n-1} + 2Aa^{n-2} + 2Bb^{n-2}$$

$$Aa^n + Bb^n - 2a^{n-1} - 2Bb^{n-1} - 2Aa^{n-2} - 2Bb^{n-2} = 0$$

$$\cancel{Aa^n + Bb^n - 2a}$$

$$Aa^n + Bb^n - 2Aa^{n-1} - 2Aa^{n-2} - 2Bb^{n-1} - 2Bb^{n-2} = 0$$

$$Aa^n - 2Aa^{n-1} - 2Aa^{n-2} + Bb^n - 2Bb^{n-1} - 2Bb^{n-2} = 0$$

$$\cancel{Aa^n(1-2}$$

$$A(a^n - 2a^{n-1} - 2a^{n-2}) + B(b^n - 2b^{n-1} - 2b^{n-2}) = 0$$

$$\cancel{a(1)^n - 2^{n-1}}$$

$$\cancel{a = +2 \pm \sqrt{2^2 - 4(1)(}}$$

$$Aa^n(1 - 2a^{-1} - 2a^{-2}) + Bb^n(1 - 2b^{-1} - 2b^{-2})$$

$$Aan(-2a^{-2} - 2a^{-1} + 1) + Bb^n(-2b^{-2} - 2b^{-1} + 1) = 0$$

$$a = \frac{-(-2) \pm \sqrt{(-2)^2 - 4(-2)(1)}}{2(-2)}$$

$$= \frac{2 \pm \sqrt{(-4)+8}}{-4} = \frac{2 \pm \sqrt{4}}{-4} = \frac{2 \pm 2}{-4} =$$

$$a = -1$$

$$\frac{2+\cancel{4}2}{-4} = -1$$

$b=-1$ $S(2)=$

Part 1

$2Aa+2A-Aa^2+2Bb+2B-Bb^2=0$

$A(2a+2-a^2)+B(2b+2-b^2)=0$

$A(-a^2+2a+2)+B(-b^2+2b+2)=0$

$$a=\frac{-(2)\pm\sqrt{(2)^2-4(-1)(2)}}{2(-1)}$$

$$=\frac{-2\pm\sqrt{4+8}}{-2}$$

$$=+1\pm\sqrt{12}$$

$$b=\frac{-(2)\pm\sqrt{(2)^2-4(-1)(2)}}{2(-1)}$$

$$=\frac{-2\pm\sqrt{4-(-8)}}{-2}$$

$$b=1\pm\sqrt{12}$$

$A=-\frac{1}{\sqrt{12}}$ $B=\frac{-1}{\sqrt{12}}$? X

$S[n]=Aa^n+Bb^n$

$S[n+2]=2S[n+1]+2S[n]$

$Aa^{n+2}+Bb^{n+2}=2(Aa^{n+1}+Bb^{n+1})+2(Aa^n+Bb^n)$

$2(Aa^{n+1}+Bb^{n+1})+2(Aa^n+Bb^n)-Aa^{n+2}-Bb^{n+2}=0$

$2Aa^{n+1}+2Bb^{n+1}+2Aa^n+2Bb^n-Aa^{n+2}-Bb^{n+2}=0$

$2Aa^{n+1}+2Aa^n-Aa^{n+2}+2Bb^{n+1}+2Bb^n-Bb^{n+2}=0$

$Aa^n(2a+2-a^2)+Bb^n(2b+2-b^2)=0$

$Aa^n(-a^2+2a+2)+Bb^n(-b^2+2b+2)=0$

$$a = \frac{-2 \pm \sqrt{2^2 - 4(-1)(2)}}{2(-1)}$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2 \pm \sqrt{4+8}$$

$$= 1 \pm \sqrt{12} < \begin{matrix} \sqrt{4} \\ \sqrt{3} \end{matrix} = 1 \pm \sqrt{3}$$

$$a = 1 \pm \sqrt{3}$$
$$b = 1 \pm \sqrt{3}$$

$$Aa^n + Bb^n = S_n$$

$$S(0) = 0$$

$$Aa^0 + Bb^0 = 0$$

$$A + B = 0$$

$$A = -B$$

$$S_n(1) = 1$$

$$Aa^1 + Bb^1 = 1$$

$$-Ba^1 + Bb^1 = 1$$

$$-Ba + Bb = 1$$

$$B(-a + b) = 1$$

$$B = \frac{1}{-a + b}$$

$$B = \frac{1}{-(1 \pm \sqrt{3}) + \sqrt{1 \pm \sqrt{3}}} = -\frac{1}{2\sqrt{3}}$$

$$A = \frac{1}{2\sqrt{3}}$$

6)

Babylonian iterative method to find sqrts

$\sqrt{S}$ guess x_0

$$x_1 = \left(x_0 + \frac{S}{x_0}\right)\frac{1}{2}$$

iterate for convergence $x_{n+1} = \left(x_n + \frac{S}{x_n}\right)\frac{1}{2}$

$\downarrow$

$x_{n+1} \approx x_n$

a) if $S = 40$

$x_n = 5$

$$x_1 = \left(5 + \frac{40}{5}\right)\frac{1}{2} = 6.5$$

$$x_2 = \left(6.5 + \frac{40}{6.5}\right)\frac{1}{2} = 6.3269$$

CALCULATOR $\sqrt{40} = 6.3245$

b)

c, tol

$tol > x^2 - c$

FIX #8

$$v = \frac{dz}{dt} = 0$$

$E = \frac{1}{2}mv^2 + mgz$

$t = 0$

$E = E_0 = mgz_0$ $\quad$ @ $t=0$ $\begin{cases} v_0 = 0 \\ z_0 = 100m \end{cases}$

$$v = \pm\sqrt{2g(z_0 - z)}$$

#8 =

$z(\Delta t) = z(0) - \Delta t\sqrt{(2g(z(0) - z_0)}$

$= 100 - \Delta t\sqrt{2g(100-100)}$

$= 100 - 0$

$z(0) = 100$

$\cancel{z(0) = z(\Delta t)}$ $\quad z(\Delta t) = z_0$

$z(t + \Delta t) = z(t) - \Delta t\sqrt{2g(z - z_0)}$

$\rightarrow \quad z(0) = z_0 = 100$

$z(2\Delta t) = z(\Delta t) - \Delta t \quad 2g\sqrt{z(\Delta t) - z_0)}$

$z(2\Delta t) = z(\Delta t) = z_0$

NOT CHANGING!! = not falling?

b) $m\dfrac{d^2z}{dt^2} = -mg$

$\dfrac{dz}{dt}$ & $\dfrac{dv}{dt}$

① $m\dfrac{d^2z}{dt^2} = -mg$

② $\dfrac{dz}{dt} = v$

$m\dfrac{dv}{dt} = -mg$

$\dfrac{dz}{dt} = v$

$$\begin{bmatrix} m & 0 \\ 0 & 1 \end{bmatrix}\begin{bmatrix} \dfrac{dv}{dt} \\ \dfrac{dz}{dt} \end{bmatrix} = \begin{bmatrix} -mg \\ v \end{bmatrix}$$

$z(t(0)) = z(0) = 100$

$\frac{dz}{dt} = v$

$$\frac{z(t+\Delta t) - z(t)}{\Delta t} = v$$

$z(t+\Delta t) = z(t) + v\Delta t$

$z(\Delta t) = z(0) + v(0)\Delta t$

$z(\Delta t) = z(0)$

$$m\left(\frac{v(t+\Delta t) - v(t)}{\Delta t}\right) = -mg$$

$v(t+\Delta t) = v(t) - g\Delta t$

$v(\Delta t) = v(0) - g\Delta t$

$v(\Delta t) = -g\Delta t$

$z(2\Delta t) = z(\Delta t) + v(\Delta t)\Delta t$

$z(2\Delta t) = z_0 + (-g\Delta t)\Delta t$

$z(2\Delta t) = z_0 - g\Delta t^2$

a

$v(2\Delta t) = v(\Delta t) - g\Delta t$

$= -g\Delta t - g\Delta t$

$= -2g\Delta t$

$z(t(0)) = z_{(0)} = 100$

$\frac{dz}{dt} = v$

$\frac{z(t+\Delta t) - z(t)}{\Delta t} = v$

$z(t+\Delta t) = z(t) + v\Delta t$

$z(\Delta t) = z(0) + v(0)\Delta t$

$z(\Delta t) = z(0)$

$m\left(\frac{v(t+\Delta t) - v(t)}{\Delta t}\right) = -mg$

$v(t+\Delta t) = v(t) - g\Delta t$

$v(\Delta t) = v(0) - g\Delta t$

$v(\Delta t) = -g\Delta t$

$z(2\Delta t) = z(\Delta t) + v(\Delta t)\Delta t$

$z(2\Delta t) = z_0 + (-g\Delta t)\Delta t$

$z(2\Delta t) = z_0 - g\Delta t^2$

a

$v(2\Delta t) = v(\Delta t) - g\Delta t$

$= -g\Delta t - g\Delta t$

$= -2g\Delta t$

$$z(2\Delta t + \Delta t) = z(2\Delta t) + v(2\Delta t)\Delta t$$
$$= z_0 - g\Delta t^2 + (-2g\Delta t)\Delta t$$
$$= z_0 - 3g\Delta t^2 - 9$$