QUANTITATIVE ANALYSIS

MULTIPLE REGRESSION

AGENDA

- 1. Follow-up
- 2. Project Checklist
- 3. Multiple Regression
- 4. Assessing Model Fit
- 5. Additional Assumptions

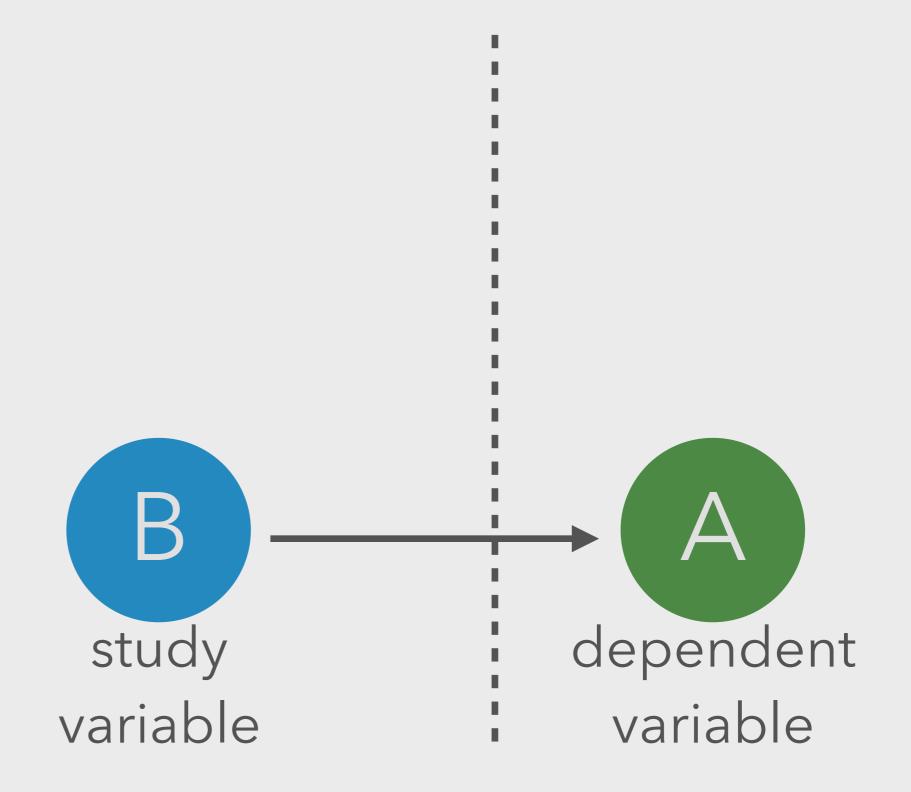
1 FOLLOW-UP

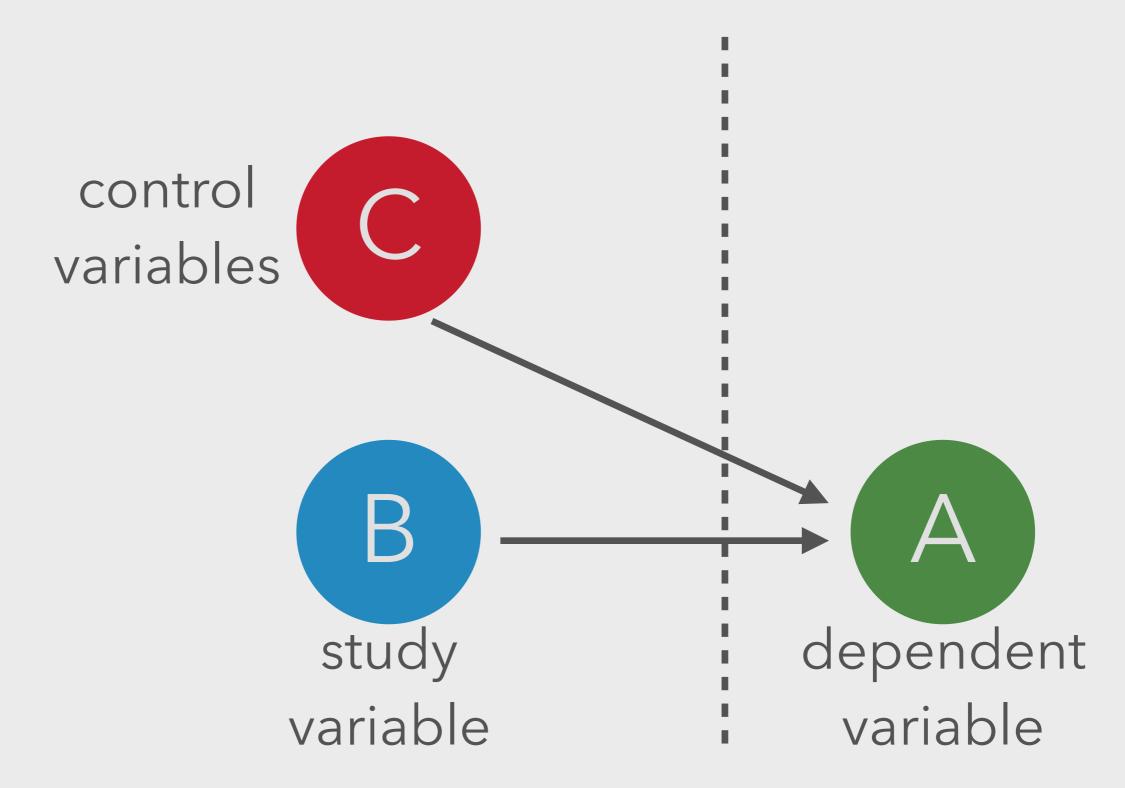
2 PROJECT CHECKLIST

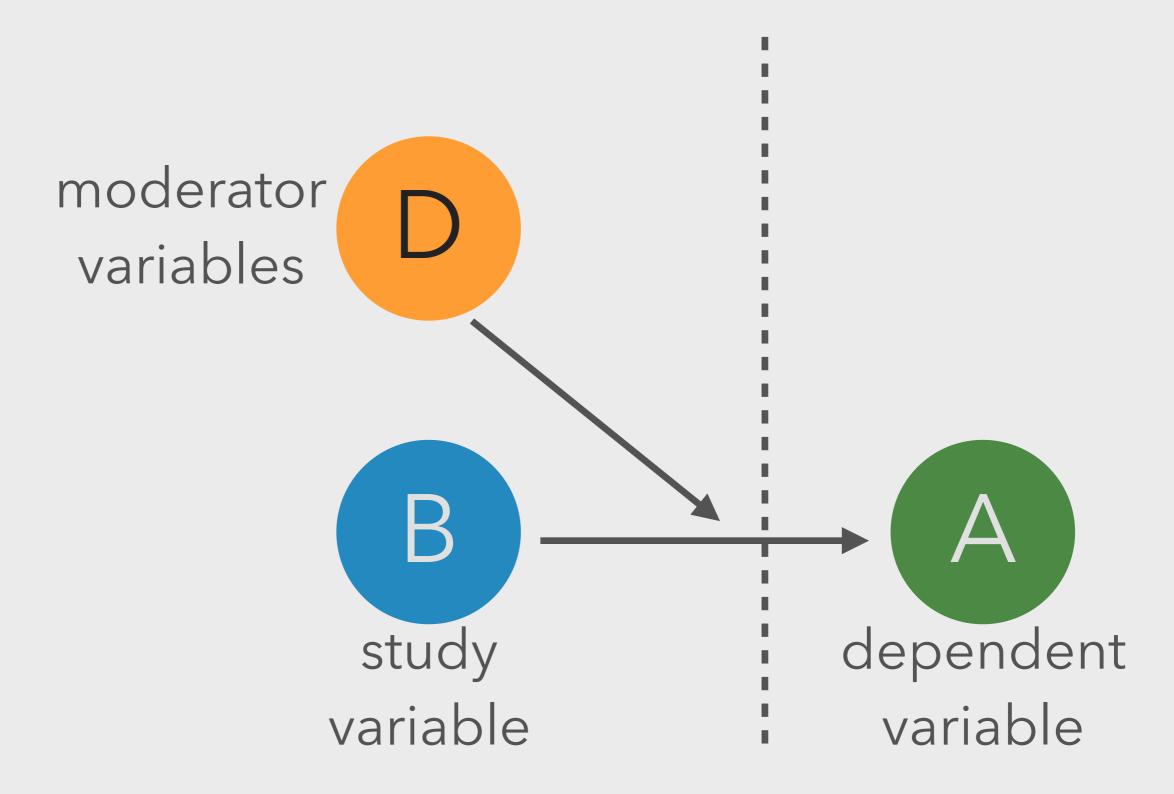
LONG'S CHECKLIST

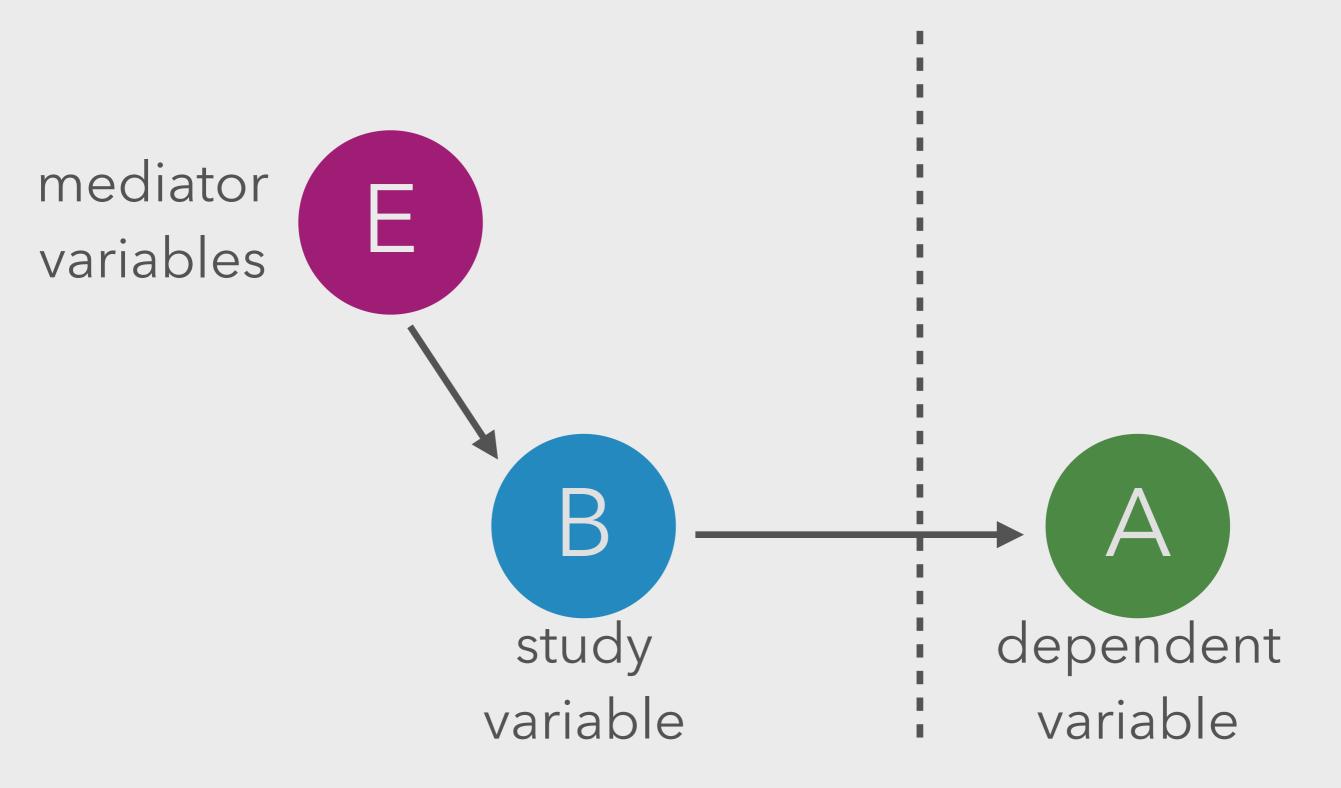
VERIFY THAT YOUR FULL CLEAN UP VERIFY YOUR STACK EXECUTES CLEANLY AND **STRAY FILES RESEARCH LOG CONSISTENTLY** ARCHIVE ALL FILES, DOCUMENTATION, AND DRAFTS LINK RESULTS IN PAPER MAKE YOUR CODE OF YOUR PAPER TO OUTPUT AND CODE **AND ANALYSES AVAILABLE**

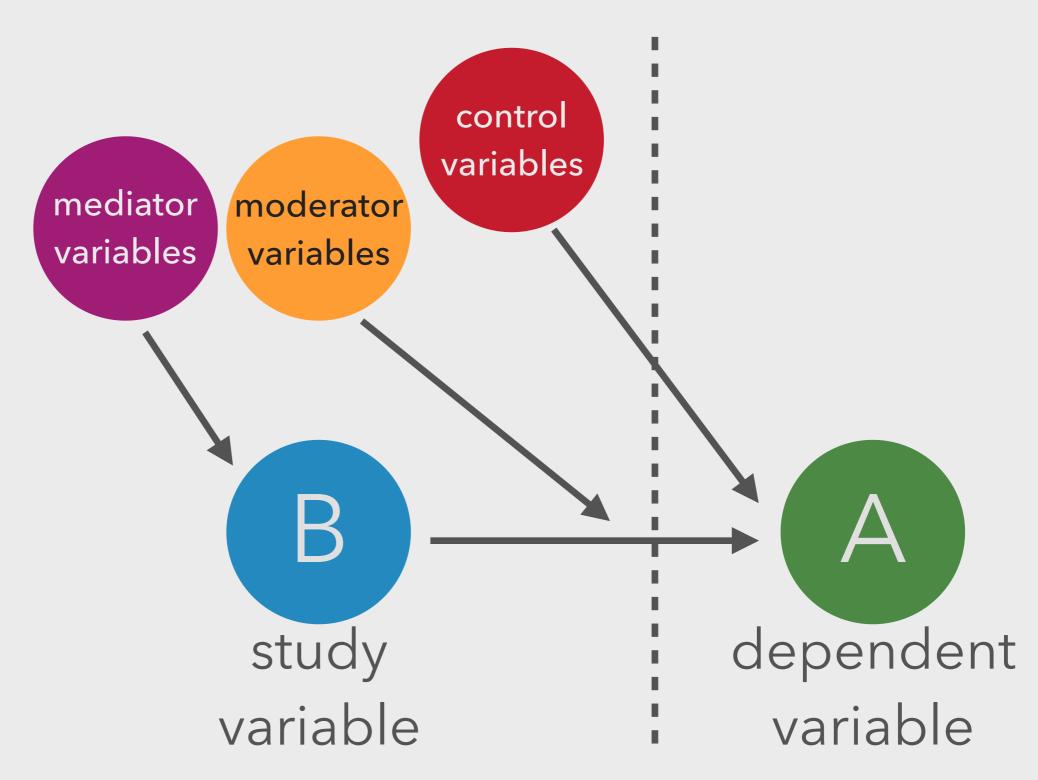
3 MULTIPLE REGRESSION











What is the effect of vehicle weight on engine efficiency, controlling for differences in length, displacement, and gear ratio, and the effect of being produced outside of the U.S.?

```
y = b_0 + b_i x_i + \varepsilon

y = \text{dependent variable}

b_0 = \text{constant}

x_i = \text{independent variable } i
```

 b_i = beta value of IV i

DV = mpg IV = weight, length, displacement, gear_ratio, and foreign (0 = domestic, I = foreign)

$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$

$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$

$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$



$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$
 constant

$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$
 constant

A hypothetical car with a weight, length, displacement, and gear_ratio of 0 that was produced in the United States.

$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$
 study variable

$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$

control* variables - size

$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$

control*
variable place of
manufacture

$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$



$$y_{mpg} = b_0 + b_1 x_{weight} + b_2 x_{length} + b_3 x_{displacement} + b_4 x_{gear_ratio} + b_5 x_{foreign} + \epsilon$$

10-15 obs per predictor

regress mpg weight

Source	55	dŤ	MS	Num	ber of obs	=	/4
+				- F(1	, 72)	=	134.62
Model	1591.9902	1	1591.9902	2 Pro	b > F	=	0.0000
Residual	851.469256	72	11.8259619) R-s	quared	=	0.6515
+				- Adj	R-squared	=	0.6467
Total	2443.45946	73	33.4720474	l Roo	t MSE	=	3.4389
mpg	Coef.	Std. Err.	t	P> t	[95% Cor	nf.	Interval]
weight _cons	0060087 39.44028	.0005179 1.614003	-11.60 24.44	0.000 0.000	0070411 36.22283		0049763 42.65774

regress mpg weight length displacement gear_ratio

Source	SS	df	MS		ber of obs	=	74
+·				- F(4	, 69)	=	34.02
Model	1621.3296	4	405.3324	1 Pro	b > F	=	0.0000
Residual	822.12986	69	11.9149255	5 R-s	quared	=	0.6635
+				- Adj	R-squared	=	0.6440
Total	2443.45946	73	33.4720474	1 Roo	t MSE	=	3.4518
·							
							Tn+onvol1
mpg	Coef. 	Std. Err.	t 	P> t	[95% C	oni .	Interval]
weight	0042442	.0019929	-2.13	0.037	00821	99	0002684
length	0798266	.0563051	-1.42	0.161	- . 19215	21	.032499
displacement	.0071945	.0115007	0.63	0.534	01574	88	.0301377
gear_ratio	.8116129	1.59094	0.51	0.612	-2.3622	25	3.985451
_cons	45.24802	8.476583	5.34	0.000	28.33	77	62.15834

regress mpg weight length displacement gear_ratio foreign

Source	SS	df	MS	Number of obs	=	74
+				F(5, 68)	=	29.17
Model	1666.45626	5	333.291252	Prob > F	=	0.0000
Residual	777.0032	68	11.4265176	R-squared	=	0.6820
+				Adj R-squared	=	0.6586
Total	2443.45946	73	33.4720474	Root MSE	=	3.3803

mpg	 Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
weight length displacement gear_ratio foreign _cons	0041683	.001952	-2.14	0.036	0080634	0002731
	0907552	.0554125	-1.64	0.106	2013292	.0198188
	.0077379	.0112658	0.69	0.495	0147428	.0302185
	2.372516	1.744781	1.36	0.178	-1.109141	5.854172
	-2.446102	1.230878	-1.99	0.051	-4.902281	.0100763
	42.98684	8.378649	5.13	0.000	26.26751	59.70618

	model 1	model 2	model 3
weight	-0.006***	-0.004*	-0.004*
length		-0.80	-0.91
displacement		0.007	0.007
gear_ratio		0.812	2.373
foreign			-2.446
constant	39.440***	45.248***	42.987***
adjusted r ²	0.644	0.644	0.657
anova	34.02***	34.02***	29.17***

EFFECT SIZES - ETA SQUARED

. estat esize

Effect sizes for linear models

Source	Eta-Squared	df	[95% Conf.	Interval]
Model	.6820069	5	.5242855	.7457006
weight length displacement gear_ratio foreign	.0628428 .0379503 .0068897 .0264713 .0548899	1 1 1 1		.1951824 .1570285 .090549 .1366094 .1836488

ETA-SQUARED CAN BE INTERPRETED AS: 0.01 (SMALL), 0.06 (MODERATE), AND 0.14 (LARGE)

4 ASSESSING MODEL FIT

ROOT MEAN SQUARED ERROR

regress mpg weight length displacement gear_ratio foreign

Source	SS	df	MS	Numl	per of obs	=	74
+				F(5)	, 68)	=	29.17
Model	1666.45626	5	333.291252	Prol	o > F	=	0.0000
Residual	777.0032	68	11.4265176	R-s	quared	=	0.6820
+				Adj	R-squared	=	0.6586
Total	2443.45946	73	33.4720474	Roo	t MSE	=	3.3803
mpg	Coef.	Std. Err.	t	P> t	[95% Cor	nf.	Interval]
+							
weight	0041683	.001952	-2.14	0.036	0080634	4	0002731
length	0907552	.0554125	-1.64	0.106	2013292	2	.0198188
displacement	.0077379	.0112658	0.69	0.495	0147428	3	.0302185
gear_ratio	2.372516	1.744781	1.36	0.178	-1.109141	1	5.854172
foreign	-2.446102	1.230878	-1.99	0.051	-4.902281	1	.0100763
_cons	42.98684	8.378649	5.13	0.000	26.26751	1	59.70618

AIC AND BIC

estat ic

Akaike's information criterion and Bayesian information criterion

Model	0bs	ll(null)	ll(model)	df	AIC	BIC
.	74	-234.3943	-192.0025	6	396.005	409.8294

Note: N=Obs used in calculating BIC; see [R] BIC note.

	model 1	model 2	model 3
weight	-0.006***	-0.004*	-0.004*
length		-0.80	-0.91
displacement		0.007	0.007
gear_ratio		0.812	2.373
foreign			-2.446
constant	39.440***	45.248***	42.987***
adjusted r ²	0.644	0.644	0.657
anova	34.02***	34.02***	29.17***
rmse	3.439	3.452	3.380
aic	394.777	398.183	396.005
bic	399.386	409.703	409.829
anova rmse aic	34.02*** 3.439 394.777	34.02*** 3.452 398.183	29.17*** 3.380 396.005

5 ADDITIONAL ASSUMPTIONS

BASIC ASSUMPTIONS REVIEW

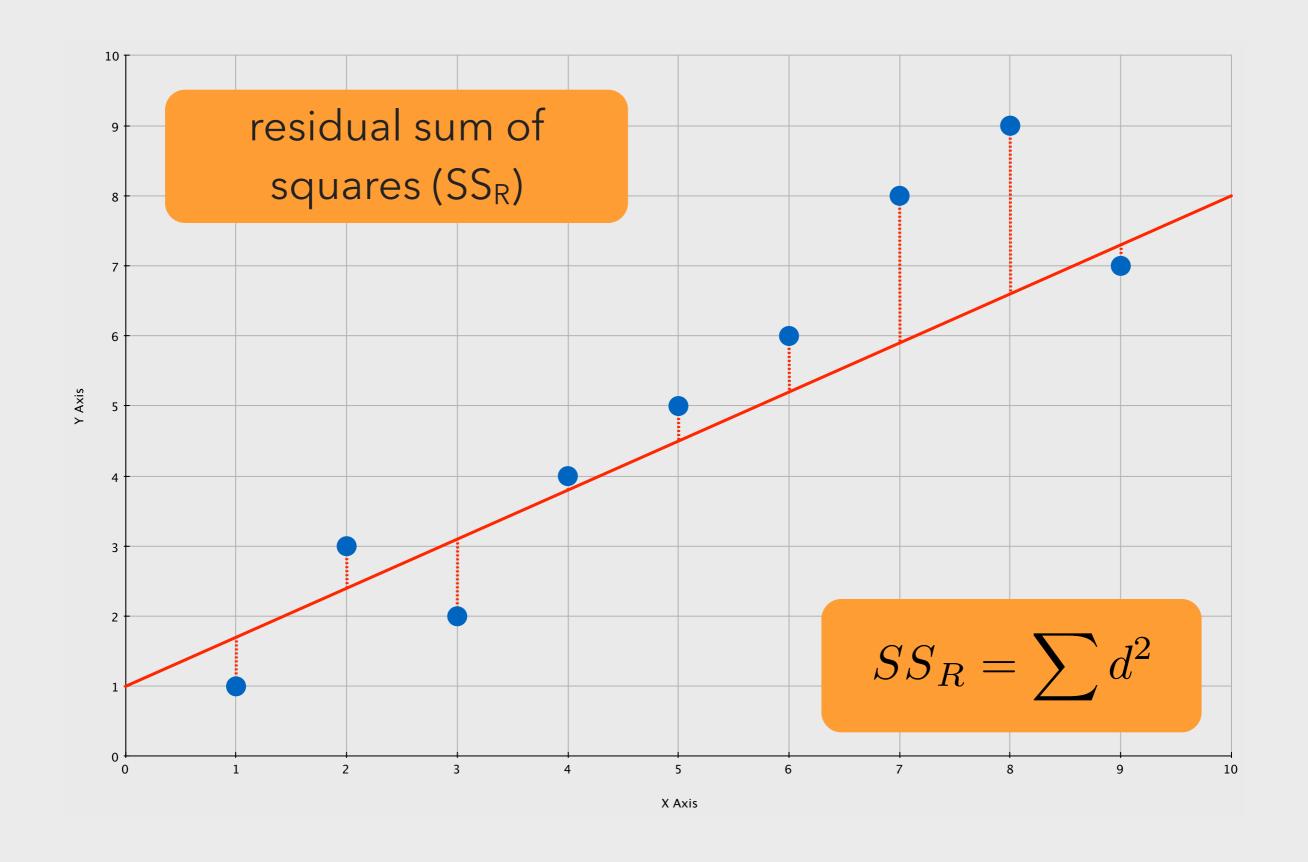
- DV must be continuous
- IV may be:
 - Dummy
 - Ordinal
 - Continuous
- IV's must have a variance > 0
- Relationship is linear
- DV should be normal
- No significant outliers

AUTOCORRELATION

- DV must be continuous
- IV may be:
 - Dummy
 - Ordinal
 - Continuous
- IV's must have a variance > 0
- Relationship is linear
- DV should be normal

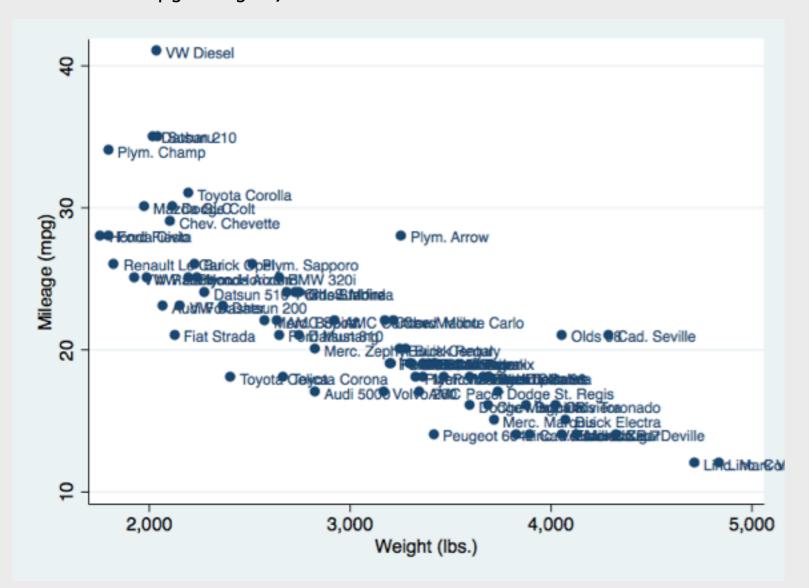
Regression models should be free of outliers and influential observations.

THE MECHANICS OF OLS REGRESSION



DETECTING OUTLIERS

. scatter mpg weight, mlabel(make)



. list make mpg if mpg > 32

	+	+
	make	mpg
67.	Plym. Champ	34
72.	Datsun 210	35
73.	Subaru	35
74.	VW Diesel	41 j
	+	+

```
predict varName [, val]
```

- . predict yhat
- predict rstandard, rstandard
- . predict residual, residual

"STANDARDIZED"
RESIDUALS ARE
STANDARDIZED USING A
Z-SCORE TO MAKE
COMPARISONS EASY

predict varName [, val]

- . predict yhat
- predict rstandard, rstandard
- predict residual, residual
- . list make mpg yhat residual rstandard if abs(rstandard) > 1.96

					
	make	mpg	yhat	residual	rstand~d
13.	Cad. Seville	21	14.61354	6.386456	2.045138
35.	Olds 98	21	14.52349	6.476507	1.978079
57.	Datsun 210	35	26.58223	8.417767	2.563203
66.	Subaru	35	26.90177	8.098228	2.480626
71.	VW Diesel	41	27.63491	13.36509	4.103542
	1				

"STANDARDIZED"
RESIDUALS ARE
STANDARDIZED USING A
Z-SCORE TO MAKE
COMPARISONS EASY

```
predict varName [, val]
```

- . predict yhat
- predict rstandard, rstandard
- . predict residual, residual
- . list make mpg yhat residual rstandard if abs(rstandard) > 2.58

	make	mpg	yhat	residual	 rstand~d
71.	VW Diesel	41	27.63491	13.36509	4.103542

"STANDARDIZED"
RESIDUALS ARE
STANDARDIZED USING A
Z-SCORE TO MAKE
COMPARISONS EASY

```
predict varName [, val]
```

predict leverage, leverage

LEVERAGE IS A
MEASURE OF HOW
INFLUENTIAL A
PARTICULAR
OBSERVATION IS

```
predict varName [, val]
```

- predict leverage, leverage
- list make leverage if leverage > (2*5+2)/74

LEVERAGE IS A
MEASURE OF HOW
INFLUENTIAL A
PARTICULAR
OBSERVATION IS

$$leverage > \frac{2k+2}{n}$$

dfbeta

```
dfbeta
```

```
_dfbeta_1: dfbeta(weight)
_dfbeta_2: dfbeta(length)
_dfbeta_3: dfbeta(displacement)
_dfbeta_4: dfbeta(gear_ratio)
_dfbeta_5: dfbeta(foreign)
```

DFBETA IS A
MEASURE OF
DIFFERENCE BETWEEN
BETA WHEN OBS IS
INCLUDED AND
EXCLUDED

dfbeta

```
. dfbeta
 _dfbeta_1: dfbeta(weight)
 _dfbeta_2: dfbeta(length)
 _dfbeta_3: dfbeta(displacement)
 _dfbeta_4: dfbeta(gear_ratio)
 _dfbeta_5: dfbeta(foreign)
. list make _dfbeta_1 if abs(_dfbeta_1) > 2/sqrt(74)
      make __dfbeta_1 |
 2. | AMC Pacer -.4294335
 7. | Buick Opel .2481316
13. | Cad. Seville .5760965
42. | Plym. Arrow .9853229
64. | Peugeot 604 - 4080613
71. | VW Diesel .3212889
```

DFBETA IS A
MEASURE OF
DIFFERENCE BETWEEN
BETA WHEN OBS IS
INCLUDED AND
EXCLUDED

 $DFBeta > 2/\sqrt{n}$

IDENTIFYING PROBLEMATIC OBSERVATIONS

Outliers

Leverage

	4.				
	•	make		leverage	:
	٠.				'
2.		AMC Pacer		. 1756483	
7.		Buick Opel		.3513954	
14.		Chev. Chev	ette	.1852391	
42.		Plym. Arrow	W	.2429746	
	+				-+

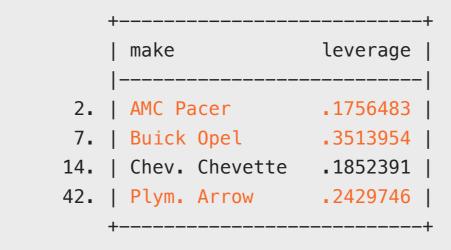
Influence

	+		
	1	make	_dfbeta_1
	- -		
2.		AMC Pacer	4294335
7.		Buick Opel	.2481316
13.		Cad. Seville	. 5760965
42.		Plym. Arrow	.9853229
64.	-	Peugeot 604	4080613
	-		I
71.	٠.	VW Diesel	.3212889
	+		+

IDENTIFYING PROBLEMATIC OBSERVATIONS

Outliers

Leverage

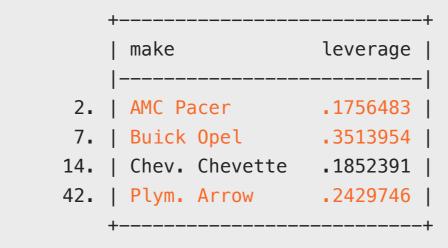


Influence

IDENTIFYING PROBLEMATIC OBSERVATIONS

Outliers

Leverage



Influence

- generate insample = 1
- replace insample = 0 if make == "AMC Pacer" | ///
 make == "Buick Opel" | make == "Cad. Seville" | ///
 make == "Plym. Arrow" | make == "VW Diesel"

REMOVING PROBLEMATIC OBSERVATIONS

regress mpg weight length displacement gear_ratio foreign if insample == 1

Source	SS	df	MS	Number of obs	=	69
	t			F(5, 63)	=	41.80
Model	1505.58963	5	301.117926	Prob > F	=	0.0000
Residual	453.888632	63	7.20458147	R-squared	=	0.7684
	t			Adj R-squared	=	0.7500
Total	1959.47826	68	28.8158568	Root MSE	=	2.6841

mpg	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
weight length displacement gear_ratio foreign cons	0093139	.0022109	-4.21	0.000	013732	0048958
	.0462243	.0599751	0.77	0.444	0736264	.1660749
	.0179895	.0111534	1.61	0.112	0042988	.0402779
	1.771099	1.421635	1.25	0.217	-1.069811	4.61201
	-2.544551	.9992889	-2.55	0.013	-4.541471	5476319
	32.20851	7.94102	4.06	0.000	16.33965	48.07737

REMOVING PROBLEMATIC OBSERVATIONS

	model 3	model 4
weight	-0.004*	-0.009***
length	-0.91	0.046
displacement	0.007	0.017
gear_ratio	2.373	1.771
foreign	-2.446	-2.545*
constant	42.987***	32.209***
n	74	69
adjusted r ²	0.657	0.750
anova	29.17***	41.80***
rmse	3.380	2.684
aic	396.005	337.792
bic	409.829	351.197

AUTOCORRELATION

- DV must be continuous
- IV may be:
 - Dummy
 - Ordinal
 - Continuous
- IV's must have a variance > 0
- Relationship is linear
- DV should be normal
- No significant outliers

Autocorrelation refers to the normality of the residuals estimated during the regression analysis.

CHECKING FOR NORMALITY

```
predict varName [, val]
```

- predict residualIS if insample == 1, residual
- swilk residualIS

Shapiro-Wilk W test for normal data

Variable	0bs	W	V	Z	Prob>z
residual	69	0.91591	5 . 116	3.547	

HOMOSKEDASTICITY

- DV must be continuous
- IV may be:
 - Dummy
 - Ordinal
 - Continuous
- IV's must have a variance > 0
- Relationship is linear
- DV should be normal
- No significant outliers
- Residuals are normally distributed

Homoskedasticity refers to the residuals having a constant variance.

CHECKING FOR HOMOSKEDASTICITY

. estat hettest

```
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
Ho: Constant variance
Variables: fitted values of mpg
```

```
chi2(1) = 11.48

Prob > chi2 = 0.0007
```

A STATISTICALLY
SIGNIFICANT TEST MEANS
THAT THE ERRORS ARE
HETEROSKEDASTIC (NOT
CONSTANT)

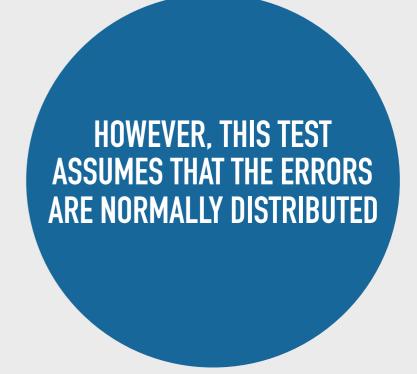
CHECKING FOR HOMOSKEDASTICITY

. estat hettest

```
Breusch-Pagan / Cook-Weisberg test for heteroskedasticity
Ho: Constant variance
Variables: fitted values of mpg
```

```
chi2(1) = 11.48

Prob > chi2 = 0.0007
```



CHECKING FOR HOMOSKEDASTICITY

. estat imtest, white

White's test for Ho: homoskedasticity

against Ha: unrestricted heteroskedasticity

chi2(19) = 28.70Prob > chi2 = 0.0709

Cameron & Trivedi's decomposition of IM-test

 THE WHITE'S TEST RELAXES THE ASSUMPTION OF NORMALLY DISTRIBUTED RESIDUALS BUT HAS LESS POWER, PARTICULARLY AS THE NUMBER OF PREDICTORS INCREASES

FIXING POSSIBLE HETEROSKEDASTIC ERRORS

- Respecify the model have predictors been left out?
- Transform variables are some relationships non-linear? Transformation (outside scope of course) can help fix this.
- Use robust standard errors.

REMOVING PROBLEMATIC OBSERVATIONS

regress mpg weight length displacement gear_ratio foreign if insample == 1, robust

Robust [95% Conf. Interval] Std. Err. P>|t| Coef. mpg t weight | -.0093139 -.0133238 -.005304 **.**0020066 -4**.**64 0.000 length | .0462243 .0516243 0.90 0.374 -.0569388 .1493873 0.052 -.0001804 .0361594 displacement | .0179895 .0090925 1.98 0.227 -1.13266 4.674859 gear_ratio | 1.771099 1.22 1.453085 foreign | -2.5445510.017 1.041033 -2,44 -4.624889 -.4642131 32.20851 7.474281 4.31 0.000 17.27235 47.14467 _cons |

ADJUSTING FOR HETEROSKEDASTICITY

	model 4	model 5
weight	-0.009***	-0.009***
length	0.046	0.046
displacement	0.017	0.017^
gear_ratio	1.771	1.771
foreign	-2.545*	-2.545*
constant	32.209***	32.209***
n	69	69
adjusted r ²	0.750	0.750
anova	41.80***	41.80***
rmse	2.684	2.684
aic	337.792	337.792
bic	351.197	351.197

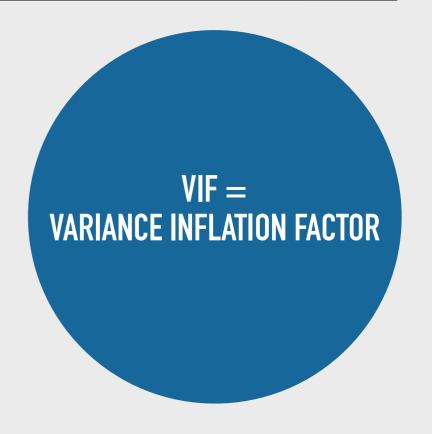
HOMOSKEDASTICITY

- DV must be continuous
- IV may be:
 - Dummy
 - Ordinal
 - Continuous
- IV's must have a variance > 0
- Relationship is linear
- DV should be normal
- No significant outliers
- Residuals are normally distributed
- Residuals are homoskedastic

Predictors must not have multicollinearity (must not be highly correlated with each other)

estat vif

Variable	VIF	1/VIF
		
weight	27.63	0.036188
length	16.93	0.059053
displacement	9.73	0.102749
gear_ratio	3.86	0.259257
foreign	2.02	0.493873
		
Mean VIF	12.04	



estat vif

Variable	VIF	1/VIF
weight length displacement gear_ratio foreign	27.63 16.93 9.73 3.86 2.02	0.036188 0.059053 0.102749 0.259257 0.493873
+ Mean VIF	12.04	



estat vif

Variable	VIF	1/VIF
weight length displacement gear_ratio foreign	27.63 16.93 9.73 3.86 2.02	0.036188 0.059053 0.102749 0.259257 0.493873
Mean VIF	12.04	

IF VIF VALUES ARE PROBLEMATIC, RE-SPECIFY THE MODEL

DOCUMENT DETAILS

Document produced by <u>Christopher Prener, Ph.D</u> for the Saint Louis University course SOC 5050: QUANTITATIVE ANALYSIS - APPLIED INFERENTIAL STATISTICS. See the <u>course wiki</u> and the repository <u>README.md</u> file for additional details.



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