CHRISTOPHER PRENER, PH.D.
FALL, 2017

QUANTITATIVE ANALYSIS

ANOVA

AGENDA

1. Front Matter
2. ANOVA Theory
3. One-way ANOVA in `R`
4. ANOVA Assumptions
5. Back Matter

1 FRONT MATTER

ANNOUNCEMENTS

WP-16, Lab-14, Lab-15, and PS-10 are due next Monday

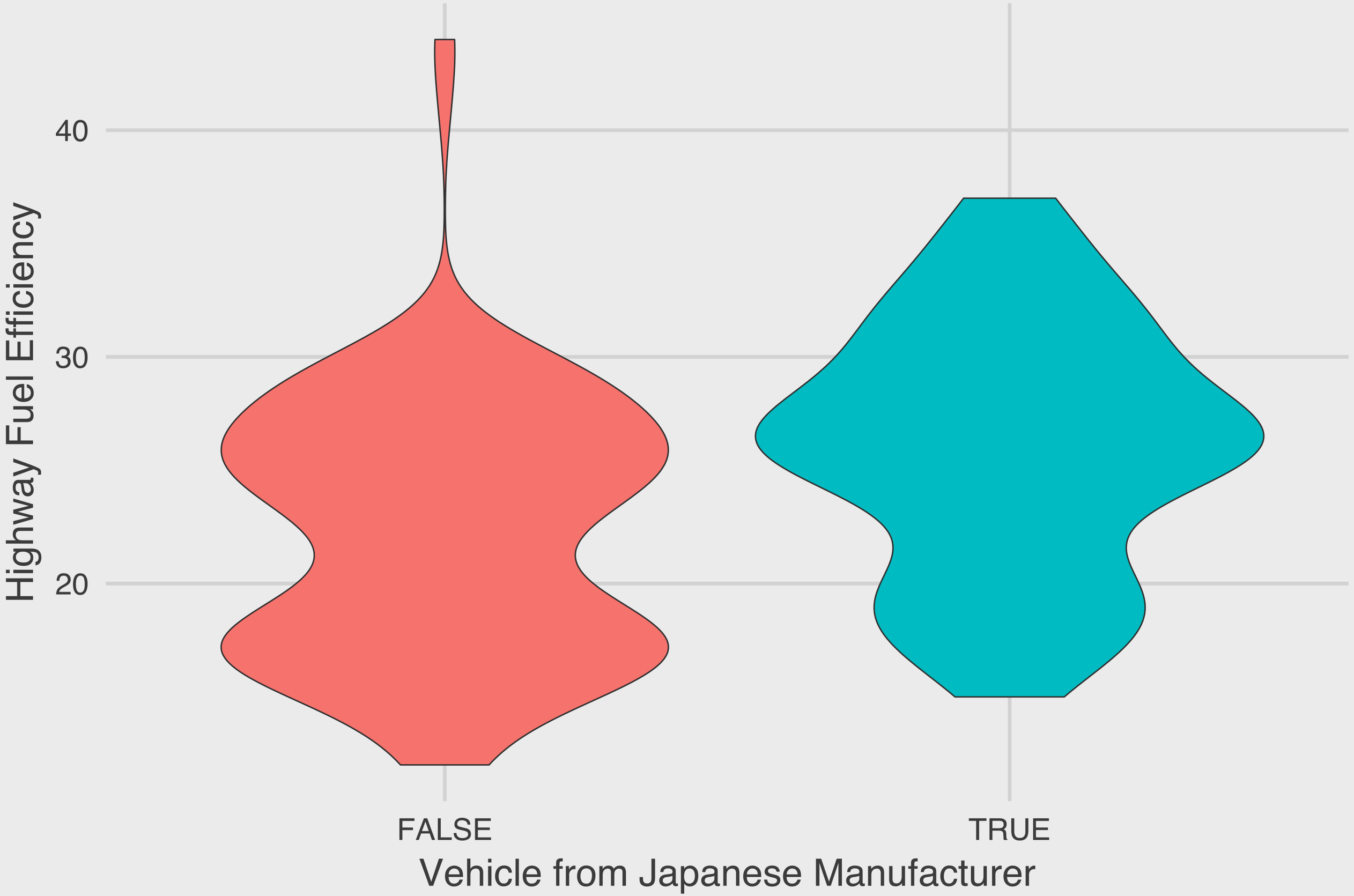
Final project presentations begin at *4pm* on Monday, December 18th in 2718 Morrissey. I will have a laptop there, or you can present using your own device.

Final project rubrics will be posted tomorrow. Please check them as you work on your projects!

2 ANOVA THEORY

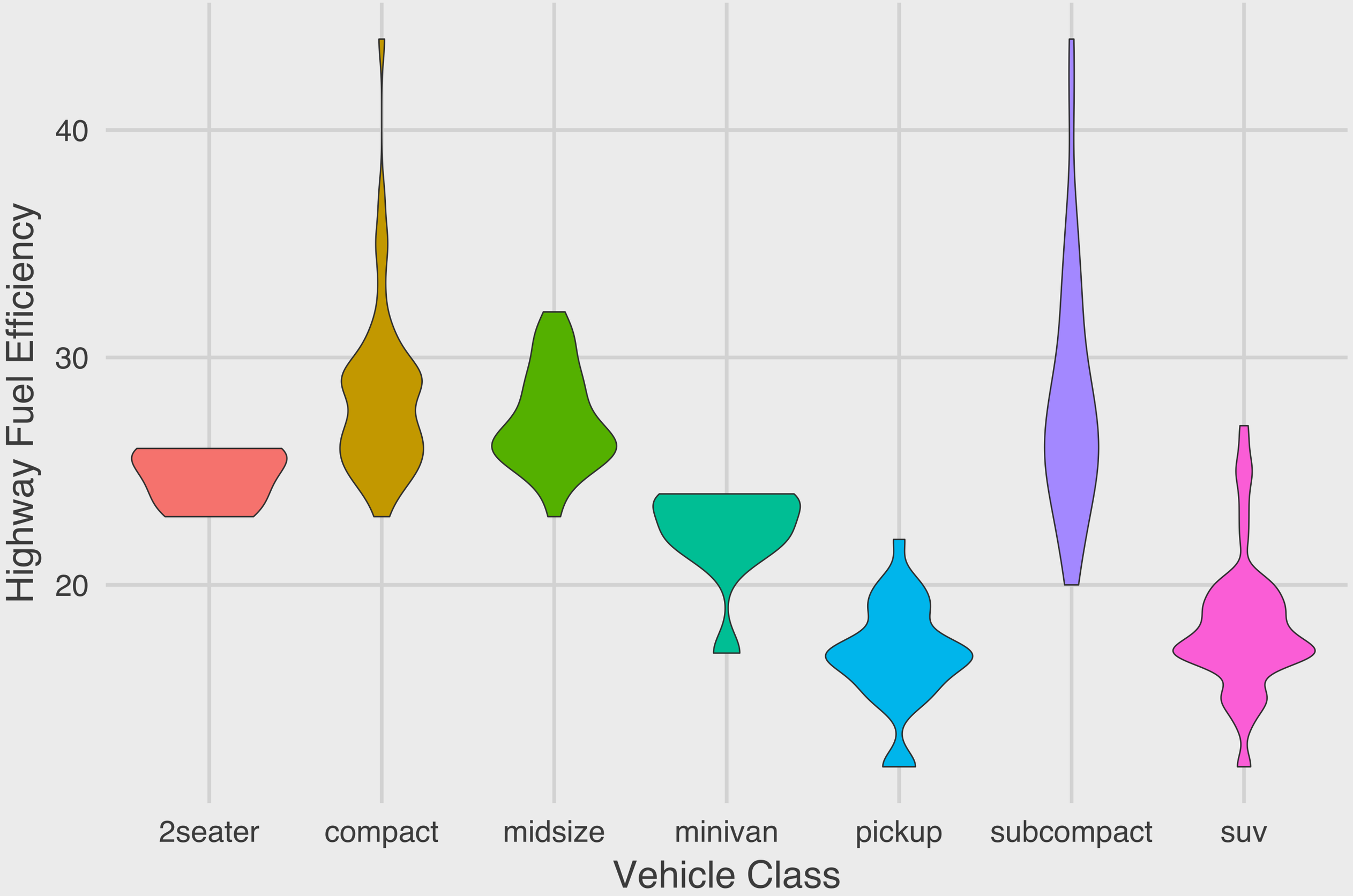
Fuel Efficiency for Japanese Vehicles

Data from ggplot2's mpg Data Set



Fuel Efficiency by Vehicle Type

Data from ggplot2's mpg Data Set

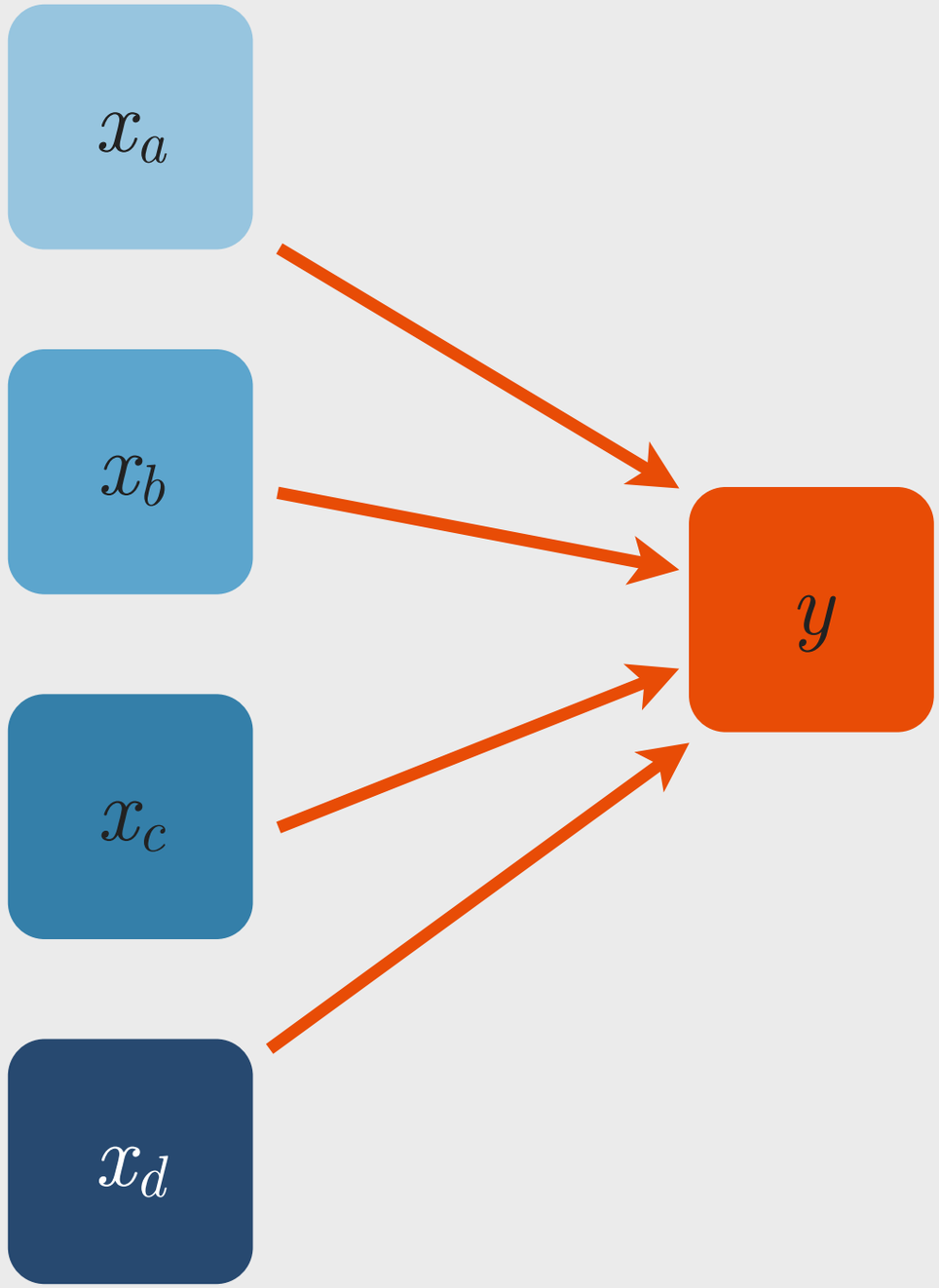


ANOVA

- Both ANOVA and regression are special cases of the generalized linear model

- ANOVAs are primarily used in experimental settings

- ANOVAs share some characteristics with t-tests in that mean comparisons are being made



ANOVA

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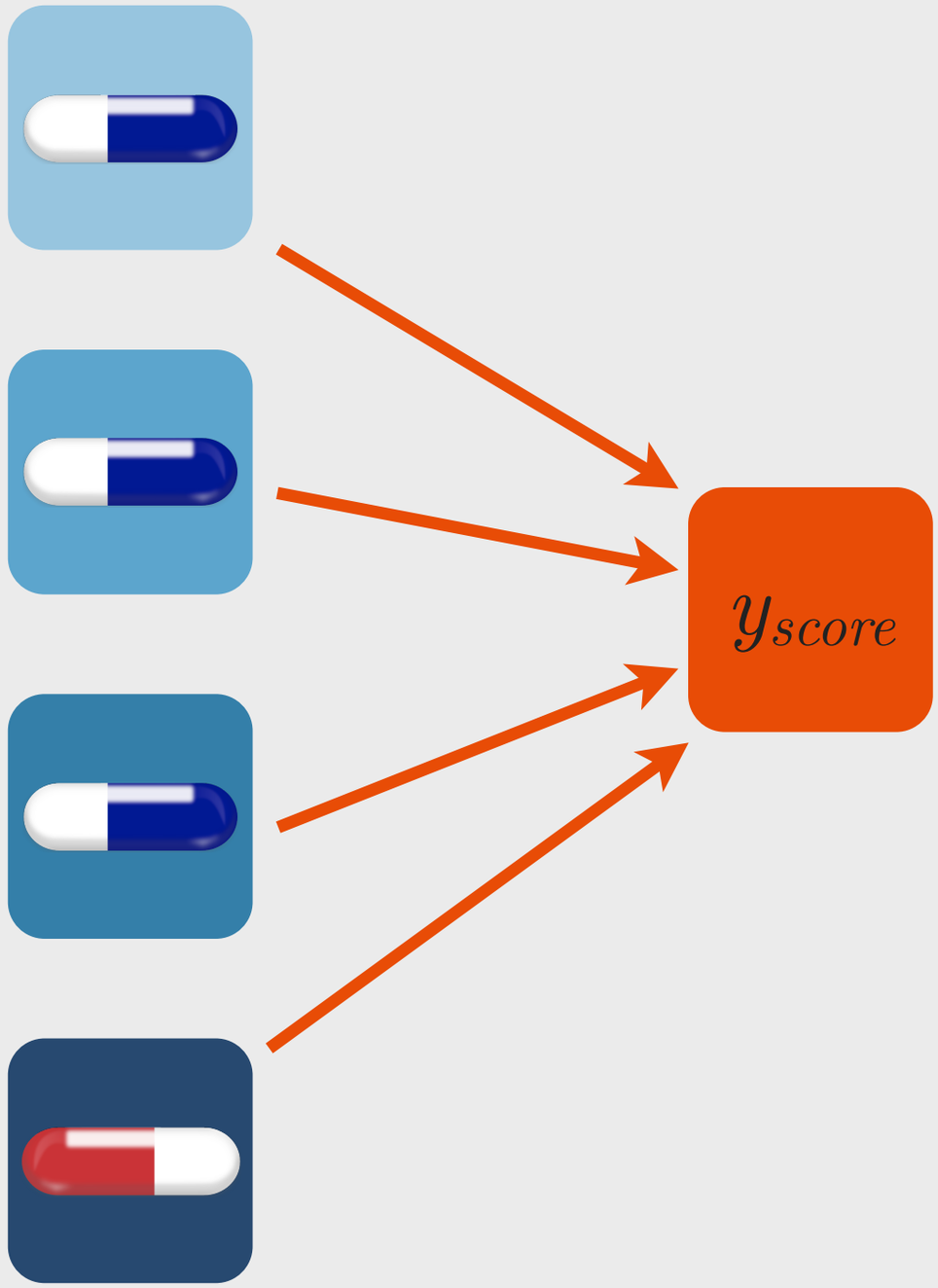

GROUPING VALUES

f(x) `group_by(`*dataFrame, varName*`)`

Parameters:

- *data*
- *vari* eted

"by

Both functions in section available in `dplyr`
Download via CRAN alone or as part of `tidyverse`

GROUPING OBSERVATIONS

`group_by(`*`dataFrame`*`, `*`varName`*`)`

Parameters:

- *`dataFrame`* is the data frame or tibble to be modified
- *`varName`* is the grouping variable that you want operations completed "by group"

GROUPING OBSERVATIONS

f(x) `group_by(`*`dataFrame, varName`*`)`

Using the `class` variable from `ggplot2`'s `mpg` data:

```
> group_by(mpg, class)
```

Needs a second function to perform "grouped by" operations; can be used in a pipe with the *`dataFrame`* omitted

GROUPING OBSERVATIONS

f(x) `group_by(dataFrame, varName)`

Using the `class` variable from `ggplot2`'s `mpg` data:

```
> group_by(mpg, class)
```

Data can also be un-grouped using `group_by()`'s compliment, `ungroup(dataFrame)`

SUMMARIZING OBSERVATIONS

f(x)

`summarize(`*`dataFrame, newVar = sumFun`*`)`

Parameters:

- *`dataFrame`* is the data frame or tibble to be modified *that has grouped data*
- *`newVar`* is the new variable to be created that stores the results of the operation performed
- *`sumFun`* is one of the available summary functions, including `first()`, `last()`, `nth()`, `n()`, `IQR()`, `min()`, `max()`, `median()`, `mean()`, `var()`, and `sd()`

SUMMARIZING OBSERVATIONS

f(x)

`summarize(dataFrame, newVar = sumFun)`

Using the `ggplot2`'s `mpg` data:

```
> summarize(mpg, count = n())
```

Will give you a count of the number of observations in `mpg`

SUMMARIZING OBSERVATIONS

f(x)

`summarize(dataFrame, newVar = sumFun)`

Using the `hwy` variable from `ggplot2`'s `mpg` data:

```
> summarize(mpg, meanHwy = mean(hwy))
```

Will give you the mean of the variable `hwy`, but it will not be grouped unless `group_by()` has already be used!

SUMMARIZING OBSERVATIONS

f(x)

`summarize(dataFrame, newVar = sumFun)`

Using multiple arguments from `ggplot2`'s `mpg` data:

```
> summarize(mpg, count = n(), meanHwy = mean(hwy))
```

Will give you the mean of the variable `hwy`, but it will not be grouped unless `group_by()` has already be used!

SUMMARIZING OBSERVATIONS

```
> mpg %>%
      group_by(class) %>%
      summarise(count = n(), meanHwy = mean(hwy))
# A tibble: 7 x 3
        class count   meanHwy
        <chr> <int>     <dbl>
1    2seater     5 24.80000
2    compact    47 28.29787
3    midsize    41 27.29268
4    minivan    11 22.36364
5     pickup    33 16.87879
6 subcompact    35 28.14286
7        suv    62 18.12903
```

3 ONE-WAY ANOVA IN R

ANOVA

f(x) `aov(yvar ~ xvar, data = dataFrame)`

Parameters:

- *yvar*
- *xvar*
- *dataFrame* is a data frame or tibble

Both functions in section available in `stats`
Included in standard distributions of R

ANOVA

f(x) `aov(yvar ~ xvar, data = dataFrame)`

Parameters:

- *yvar* is the dependent variable
- *xvar* is the factor-formatted independent variable
- *dataFrame* is a data frame or tibble

ANOVA

f(x) `aov(yvar ~ xvar, data = dataFrame)`

Using the `hwy` and `class` variables from `ggplot2`'s `mpg` data:

```
> aov(hwy ~ class, data = mpg)

<<<<< OUTPUT OMITTED >>>>>
```

Save the model output to an object for reference later!

ANOVA

```
> model <- aov(hwy ~ class, data = mpg)

> summary(model)
             Df Sum Sq Mean Sq F value Pr(>F)
class         6   5683   947.2   83.39 <2e-16 ***
Residuals   227   2578    11.4
---
Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
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How would you interpret this result?

ANOVA

```
> model <- aov(hwy ~ class, data = mpg)

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```

The model's results ($f = 83.39,\ df = 6,\ p < .001$) suggest that there is meaningful variation between the mean highway fuel efficiency of vehicles from different classes.

TUKEY HONEST SIGNIFICANT DIFFERENCES

`TukeyHSD(`*`model`*`)`

Parameters:

- *`model`* is an ANOVA model object

TUKEY HONEST SIGNIFICANT DIFFERENCES

`TukeyHSD(model)`

Using the `model` object created from `ggplot2`'s `mpg` data:

```
> TukeyHSD(model)
```

<<<<< OUTPUT OMITTED >>>>>>

Will calculate ever permutation of combinations and test them to see if the mean difference for each is statistically significant.

TUKEY HONEST SIGNIFICANT DIFFERENCES

```
> TukeyHSD(model)
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = hwy ~ class, data = mpg)

$class
                         diff         lwr       upr     p adj
compact-2seater      3.4978723  -1.2185908  8.214335 0.2962191
midsize-2seater      2.4926829  -2.2568476  7.242213 0.7070356
minivan-2seater     -2.4363636  -7.8442474  2.971520 0.8321849
pickup-2seater      -7.9212121 -12.7329120 -3.109512 0.0000377
subcompact-2seater   3.3428571  -1.4507195  8.136434 0.3713580

<<<<< OUTPUT TRUNCATED >>>>>>
```

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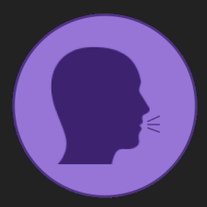

Of the comparisons with “two-seater” sports cars, the only mean difference that was statistically significant based on the Tukey post-hoc test was the relationship with pickup trucks ($p < .001$).

4 ANOVA ASSUMPTIONS

ASSUMPTIONS

- y should be normally distributed
  - Use standard techniques to evaluate normality
- the categories within x should have equal (homogeneous) variance
- There should be no significant outliers
  - Use the Bonferonni test (`car::outlierTest()`) discussed in Week-14

HOMOGENEITY OF VARIANCE

f(x) `bartlett.test(`*yvar* ~ *xvar*, data = *dataFrame*`)`

Parameters:

- *yvar*
- *xvar*
- *dataFrame* is a data frame or tibble

Available in `stats`
Included in standard distributions of `R`

HOMOGENEITY OF VARIANCE

f(x) `bartlett.test(`*yvar* ~ *xvar*, data = *dataFrame*`)`

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HOMOGENEITY OF VARIANCE

f(x) `bartlett.test(`*yvar* `~` *xvar*`, data =` *dataFrame*`)`

Using the `hwy` and `class` variables from `ggplot2`'s `mpg` data:

```
> bartlett.test(hwy ~ class, data = mpg)
```

<<<<< OUTPUT OMITTED >>>>>

The null and alternative hypotheses are the same as the Levene's test (see Week-07 and Week-08)

HOMOGENEITY OF VARIANCE

```
> bartlett.test(hwy ~ class, data = mpg)

    Bartlett test of homogeneity of variances

data:  hwy by class
Bartlett's K-squared = 50.523, df = 6, p-value = 3.692e-09
```

HOMOGENEITY OF VARIANCE

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Bartlett's K-squared = 50.523, df = 6, p-value = 3.692e-09
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How would you interpret this result?

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> bartlett.test(hwy ~ class, data = mpg)

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Bartlett's K-squared = 50.523, df = 6, p-value = 3.692e-09
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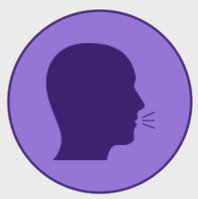

The results of the Bartlett Test ($k^2 = 50.523,\ df = 6,\ p < .001$) indicate that these data do not meet the homogeneity of variance assumption for ANOVA.

5 BACK MATTER

WHAT WE COVERED TODAY

2. ANOVA Theory

3. One-way ANOVA in `R`

4. ANOVA Assumptions

REMINDERS

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