Scalable zkSNARKs for Matrix Computations
A Generic Framework for Verifiable Deep Learning

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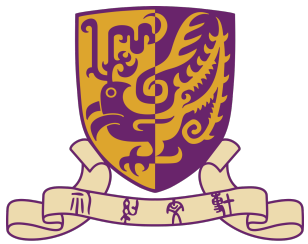


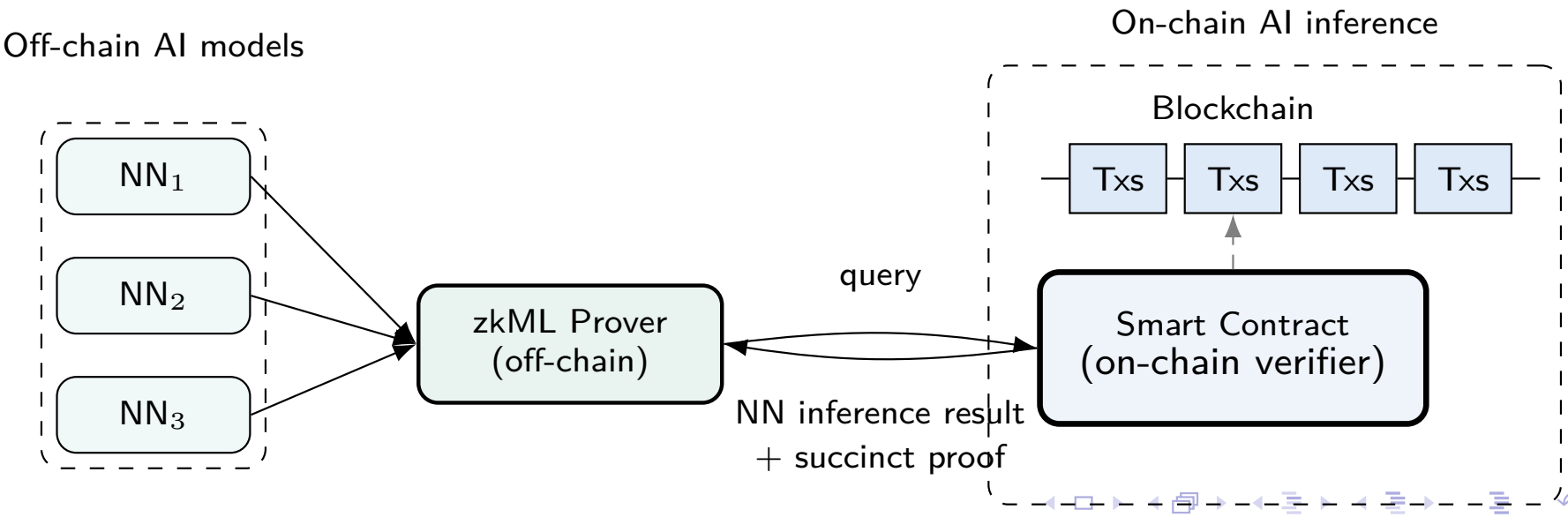

Succinct zkML: Enabling AI-Empowered Blockchain

Smart Contracts with Embedded AI Models

Future blockchain systems will require smart contracts that can:

- Enforce AML/KYC compliance on-chain
- Execute AI agent policies on-chain
- Perform autonomous risk assessment and monitoring

Succinct zkML: off-chain inference & proof generation + on-chain verification
Heavy ML computation is done by a single prover; all nodes verify a succinct proof.



The Problem We Address

Succinct Zero-Knowledge Machine Learning (zkML)

zkSNARKs (zero-knowledge Succinct Non-Interactive Arguments of Knowledge) for machine learning models (*e.g.*, neural networks, NNs): $Y = f(X)$, where both the NN f and the input data X may be secret, but the inference result Y is publicly verifiable.

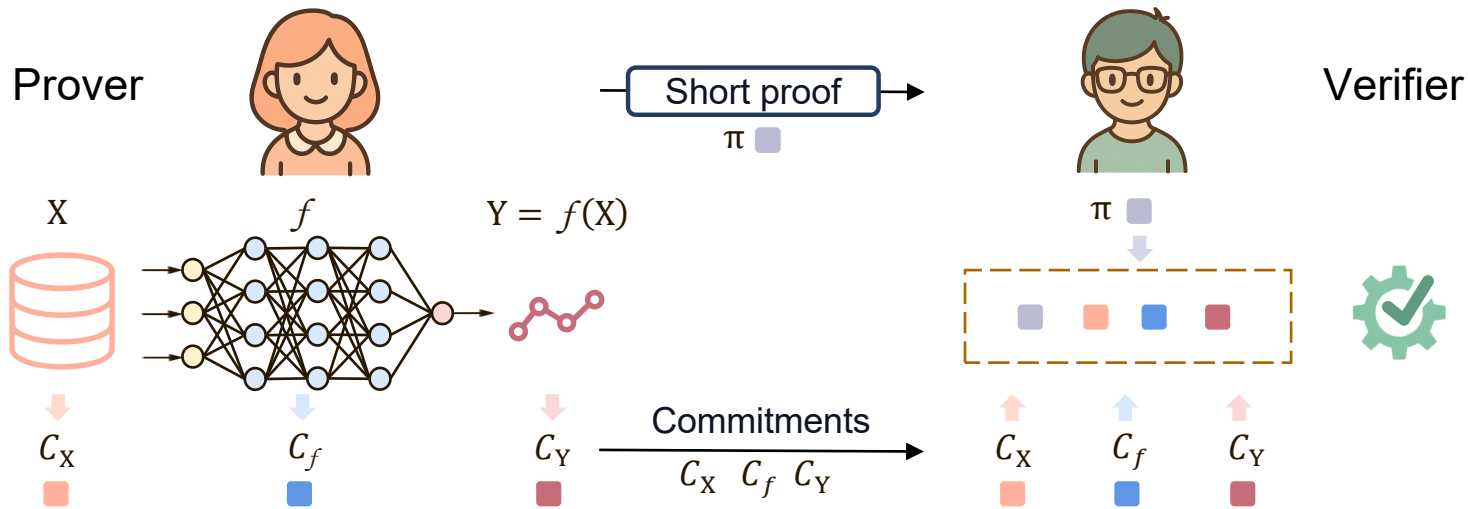


Figure: The prover produces a short proof to convince the verifier NN inference is correct.

Neural Networks as Matrix Expressions

A Generic View for zkML

NN computations can be represented as a directed acyclic graph (DAG) of matrix operations. If we design succinct proofs for DAG-structured matrix expressions, we obtain a generic zkML framework.

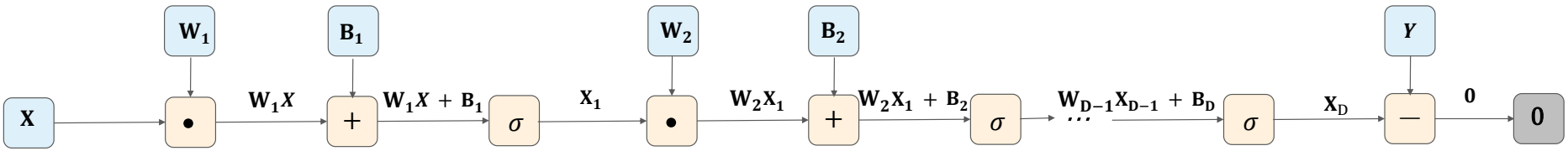


Figure: NN inference computation decomposes into a DAG of matrix operations.

- Fully connected layer: $\boldsymbol{Y} = \sigma(\boldsymbol{W}\boldsymbol{X} + \boldsymbol{b})$
- Convolutional layer: $\boldsymbol{Y} = \sigma(\text{conv}(\boldsymbol{W}, \boldsymbol{X}) + \boldsymbol{b})$
- Attention layer: $\boldsymbol{Y} = \sigma\left(\frac{\boldsymbol{Q}\boldsymbol{K}^\top}{\sqrt{d}}\right)\boldsymbol{V}$

Challenges of Succinct zkML

Efficiency

We aim to achieve all of the following:

- **Linear prover time**
- **Logarithmic** proof size and verifier time
- **Practical** RAM usage for billion-parameter NNs

NN Privacy

Hiding datasets and parameters via commitments is standard.
Hiding the NN architecture (the DAG topology) is much harder.

Flexibility

A unified framework that supports heterogeneous NN structures and matrix operations across diverse NN designs.

Building Block: LiteBullet via Folding

New Inner Product Proof

LiteBullet is a Bulletproof-style protocol for proving $c = \vec{a} \bullet \vec{b}$ using only field operations, without converting vectors into polynomials (polynomial-free).

- **Folding step.** Split vectors into left/right halves: $\vec{a} = (\vec{a}_L, \vec{a}_R)$, $\vec{b} = (\vec{b}_L, \vec{b}_R)$. For a random challenge x:

$$(\vec{a}_L + x^{-1}\vec{a}_R) \bullet (\vec{b}_L + x\vec{b}_R) = c + x^{-1}(\vec{a}_L \bullet \vec{b}_R) + x(\vec{a}_R \bullet \vec{b}_L).$$

- **Effect of folding.** One folding step reduces an n-dimensional check to a $(n/2)$-dimensional one.
- **Recursive reduction.** After $\log_2 n$ rounds, $c = \vec{a} \bullet \vec{b}$ reduces to a scalar product:

$$\hat{c} = \hat{a} \cdot \hat{b},$$

where $\hat{a} = \vec{a} \bullet \vec{\xi}^{-1}$, $\hat{b} = \vec{b} \bullet \vec{\xi}$ for tensor-structured random vectors $\vec{\xi}^{-1}$, $\vec{\xi}$ from the verifier's challenges.

- **Final step.** The prover employs additional protocols to prove:

$$\hat{a} = \vec{a} \bullet \vec{\xi}^{-1}, \quad \hat{b} = \vec{b} \bullet \vec{\xi}, \quad \text{for public tensor-structured } \vec{\xi}^{-1}, \vec{\xi}.$$

LiteBullet Applied to Matrix Multiplication

Freivalds' Algorithm (Review)

To check whether $\boldsymbol{ab} = \boldsymbol{c}$, Freivalds' algorithm verifies:

$$\boldsymbol{ab}\,\vec{\boldsymbol{r}} = \boldsymbol{c}\,\vec{\boldsymbol{r}},$$

for a random vector $\vec{\boldsymbol{r}} \in \mathbb{F}^n$.

- **Two-sided projection.** We apply Freivalds' Algorithm twice and prove:

$$\vec{\boldsymbol{l}}^{\top}\boldsymbol{ab}\,\vec{\boldsymbol{r}} \;=\; \vec{\boldsymbol{l}}^{\top}\boldsymbol{c}\,\vec{\boldsymbol{r}},$$

  where $\vec{\boldsymbol{l}} \in \mathbb{F}^m$ and $\vec{\boldsymbol{r}} \in \mathbb{F}^n$ are random vectors.
- **Reduction via LiteBullet.** Define:

$$v := \vec{\boldsymbol{l}}^{\top}\boldsymbol{c}\,\vec{\boldsymbol{r}} \;=\; \vec{\boldsymbol{l}}^{\top}\boldsymbol{ab}\vec{\boldsymbol{r}} \;=\; (\vec{\boldsymbol{l}}^{\top}\boldsymbol{a}) \bullet (\boldsymbol{b}\vec{\boldsymbol{r}}).$$

  Applying LiteBullet to this inner product reduces to checking:

$$\hat{a} = \vec{\boldsymbol{l}}^{\top}\boldsymbol{a}\,\vec{\boldsymbol{\xi}}^{-1}, \qquad \hat{b} = \vec{\boldsymbol{\xi}}^{\top}\boldsymbol{b}\,\vec{\boldsymbol{r}},$$

  where $\vec{\boldsymbol{\xi}}^{-1}$ and $\vec{\boldsymbol{\xi}}$ are public tensor-structured vectors.

Reducing Parent Projection to Child Projections

- **Structured randomness.** Use tensor-structured vectors $\vec{l}$ and $\vec{r}$ derived from the random challenges $x_{l,1}, \ldots, x_{l,\log_2 m}$ and $x_{r,1}, \ldots, x_{r,\log_2 n}$:

$$\vec{l} = (1, x_{l,1}) \otimes \cdots \otimes (1, x_{l,\log_2 m}), \qquad \vec{r} = (1, x_{r,1}) \otimes \cdots \otimes (1, x_{r,\log_2 n}).$$

- **Interpretation.** Using LiteBullet, projection $v = \vec{l}^{\top} \boldsymbol{c} \vec{r}$ reduces to proving:

$$\hat{a} = \vec{l}^{\top} \boldsymbol{a}\, \vec{\xi}^{-1} \quad \text{and} \quad \hat{b} = \vec{\xi}^{\top} \boldsymbol{b}\, \vec{r}.$$

  v: projection of the output matrix $\boldsymbol{c}$ onto $(\vec{l}, \vec{r})$.
  $\hat{a}$: projection of the left input matrix $\boldsymbol{a}$ onto $(\vec{l}, \vec{\xi}^{-1})$.
  $\hat{b}$: projection of the right input matrix $\boldsymbol{b}$ onto $(\vec{\xi}, \vec{r})$.

- **DAG View.** In the DAG of matrix expressions, proving a *parent* projection reduces to proving its *child* projections.

Proof Reduction for Various Matrix Operations

For all Type-1 operations $\circledast$, include matrix addition, multiplication, transpose, inner product, Hadamard product, Kronecker product, concatenation, and truncation, Evalyn applies the same reduction principle.

Atomic Evalyn Proof

Given random vectors $\vec{l}$ and $\vec{r}$, proving $v = \vec{l}^{\top}(\boldsymbol{a} \circledast \boldsymbol{b})\,\vec{r}$ reduces to proving:

$$v_a = \vec{l}_a^{\top}\,\boldsymbol{a}\,\vec{r}_a, \qquad v_b = \vec{l}_b^{\top}\,\boldsymbol{b}\,\vec{r}_b,$$

where $(\vec{l}_a, \vec{r}_a, \vec{l}_b, \vec{r}_b)$ are tensor-structured vectors related to $(\vec{l}, \vec{r})$.

$$\left\{\begin{array}{ll} \mathcal{P} \text{ produces a } \circledast\text{-specific proof } \pi \text{ for } v = \vec{l}^{\top}(\boldsymbol{a} \circledast \boldsymbol{b})\vec{r} & \text{(Prove)} \\ \mathcal{V} \text{ checks } \pi \text{ and:} & \text{(Verify)} \\ \{v_L = \vec{l}_L^{\top}\boldsymbol{a}\vec{r}_L \wedge v_R = \vec{l}_R^{\top}\boldsymbol{b}\vec{r}_R\} \text{ (recursively proven), where:} & \\ v_L \text{ and } v_R \text{ are the final elements of } \pi, \text{ and} & \text{(Eval)} \\ \vec{l}_L, \vec{r}_L, \vec{l}_R, \vec{r}_R \text{ are derived from } \vec{l}, \vec{r}, \text{ and the randomness from } \mathcal{V} & \text{(Link)} \end{array}\right\}.$$

Backward Proof Reduction

Post-Order Traversal for Node Evaluation

First, traverse the DAG in post-order to evaluate all nodes from leaves to the root.

Pre-Order Traversal for Proof Generation

Then traverse the DAG in pre-order to recursively reduce the root-node projection to leaf-node projections.

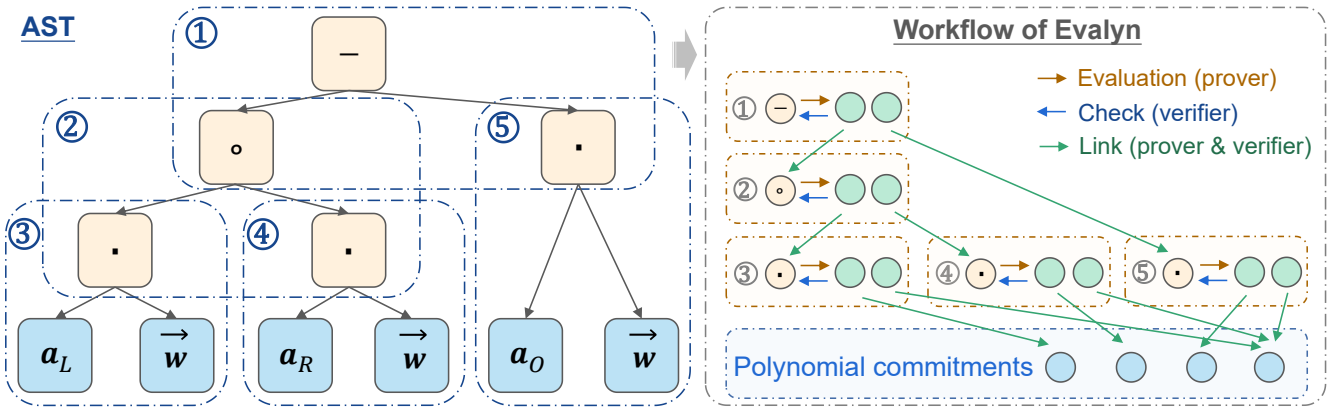


Figure: DAG for $\vec{\mathbf{0}} = (\boldsymbol{a}_L\vec{\boldsymbol{w}}) \circ (\boldsymbol{a}_R\vec{\boldsymbol{w}}) - \boldsymbol{a}_O\vec{\boldsymbol{w}}$ and the workflow for generating zkSNARKs through DAG traversal.

From Type-2 Operations to Type-1 Operations

Type-1 Operations

Operations that admit an efficient projection reduction proof (*e.g.*, matrix multiplication, addition, etc.).

Type-2 Operations

Operations for which no such efficient projection reduction proof is known (*e.g.*, nonlinear activations, convolution, etc.).

Our Strategy

Express Type-2 operations as equivalent Type-1 matrix expressions, so they can be handled by a unified reduction framework.

Example: Lookup Proof for Nonlinear Activation

- Nonlinear activations are Type-2 operations.
- For a monotone increasing σ, each output $c = \sigma(a)$ corresponds to an interval of valid preimages. The tables $\boldsymbol{k}_1$ and $\boldsymbol{k}_2$ store, for every possible function value c, the minimum and maximum preimages. The lookup table is:

$$(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{v}) = \{(a_1, a_2, c) : c = \sigma(a_1) = \sigma(a_2),\ a_1 \le b \le a_2 \quad \forall b \in \sigma^{-1}(c)\}.$$

- To prove $\boldsymbol{c} = \sigma(\boldsymbol{a})$, it suffices to show $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ such that:

$$\boldsymbol{a}_1 \le \boldsymbol{a} \le \boldsymbol{a}_2, \qquad \{(\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{c})\} \in \{(\boldsymbol{k}_1, \boldsymbol{k}_2, \boldsymbol{v})\}.$$

- Lasso [6] provides a lookup proof, which we express as vector equations to fit into our framework.

Committing to New Leaf Nodes

Transforming Type-2 operations into Type-1 expressions introduces new leaf nodes in the DAG. These nodes must be committed before applying proof reduction.

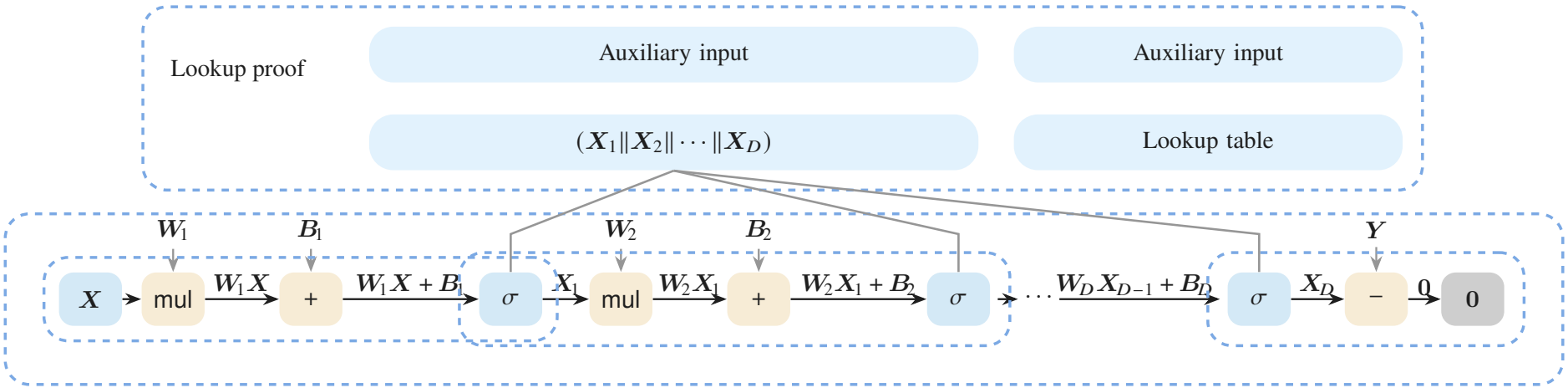


Figure: Commitments to new leaf nodes (blue rectangles) resulting from the Type-2 to Type-1 transformation.

Succinctness via PoP

Non-Succinct Proof from Proof Reduction

- Each DAG node yields at least a constant-size proof, so the total proof size grows with the number of nodes and breaks succinctness.
- For a network with D layers of $n \times n$ matrices, the proof size is $O(D \log n)$ —not succinct.

Proof of Proof (PoP)

We use an additional zkSNARK that proves the (non-succinct) verifier outputs true on the original reduction proof.

- This compresses the proof size and verifier time to $O(\log(Dn))$.
- It also enables hiding the NN structure by committing to the PoP circuit.

Hiding NN Architecture via PoP Commitments

Key Insight

The PoP circuit preserves the topology of the original matrix DAG.

- Represent the NN as a DAG of atomic matrix operations.
- Traverse the DAG to generate a sublinear proof. (Proof reduction)
- Commit to a smaller PoP circuit to hide the NN architecture.

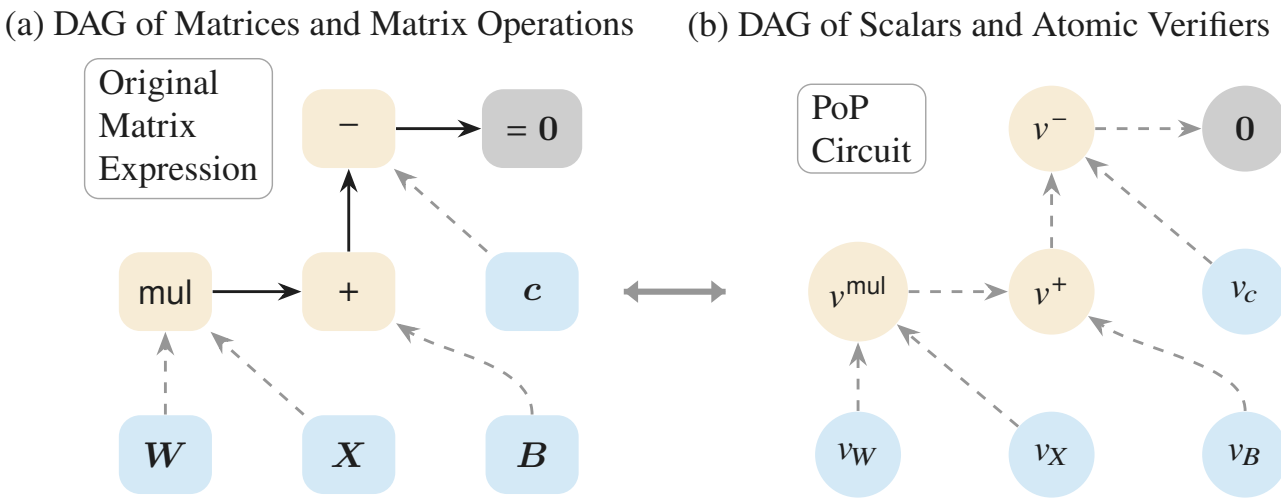


Figure: DAG for $\boldsymbol{c} = \boldsymbol{W}\boldsymbol{X} + \boldsymbol{B}$ and its PoP circuit. Each matrix is projected to a scalar, yielding a scalar DAG with the same topology, which becomes the PoP circuit. (Legend: root, leaf, operation)

Workflow of Evaly

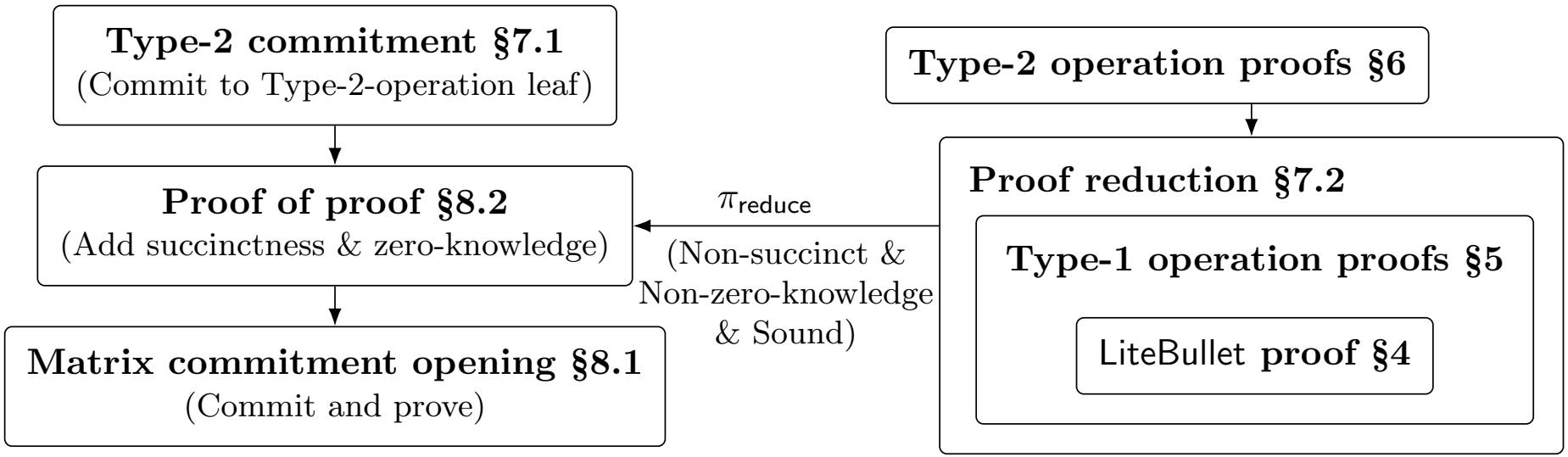


Figure: The Evalyn framework.

Comparison with IVC/PCD

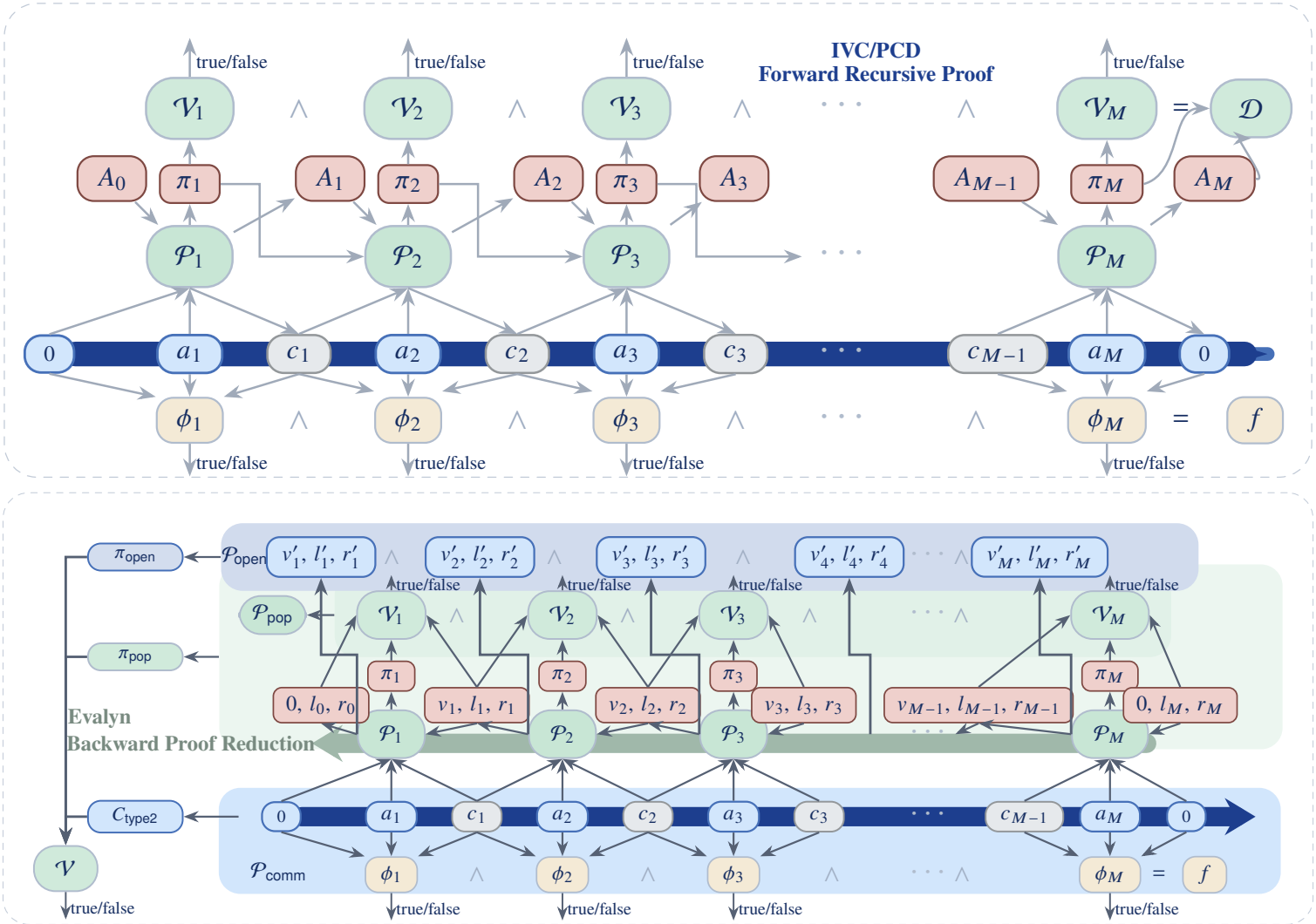


Figure: Comparison with IVC/PCD.

Asymptotic Efficiency

Table: Architecture/operation-specific proofs (D-layer NN inference, $n \times n$ matrices)

Protocol	Arch.[a]	Op.[a, b]	Prover Time	Proof Size	Verifier Time	Hiding[a]
Zen [1]	MLP/CNN	Conv.	$O(Dn^4 \log n)$[c]	$O(1)$	$O(1)$	Arch.&Par.
pvCNN [8]	CNN	Conv.	$O(Dn^4 \log n)$[c]	$O(1)$	$O(1)$	Arch.&Par.
vCNN [4]	CNN	Conv.	$O(Dn^2 \log n)$	$O(D)$	$O(D)$	Par.
KMC18 [3]	MLP	Basics	$O(Dn^2 \log n)$	$O(D \log n)$	$O(D \log n)$	Par.
Trustless [2]	MLP	Basics	$O(Dn^2)$	$O(D \log n)$	$O(D \log n)$	Par.
zkCNN [5]	CNN	Conv.	$O(Dn^2)$	$O(D \log^2 n)$	$O(Dn)$[d]	Par.
zkLLM [7]	LLM	Softmax	$O(Dn^2)$	$O(D \log n)$	$O(Dn)$[d]	Par.
Evalyn	General	General	$O(Dn^2)$	$O(\log(Dn))$	$O(\log(Dn))$	Arch.&Par.

[a] Shorthand: Arch.: Architecture; Op.: Operation; Par.: Parameter.
[b] Matrix multiplication and ReLU are the basics and are omitted. Conv.: Convolution.
[c] Complexities for $n \times n$ matrix multiplication/convolution itself are $O(n^3)/O(n^4)$.
[d] zkCNN/zkLLM use Hyrax PCS; verifier time complexity larger than its proof size.

Concrete Efficiency

We benchmark an MLP with 2^{10} layers, each using a $2^{10} \times 2^{10}$ weight matrix ($\sim 10^9$ parameters total). The prover times were 34.8 min/36.8 min.

Table: Performance on a 2^{10}-layer NN (~ 1 billion parameters)

Layer i: $\boldsymbol{X}_i \leftarrow \text{sigmoid}(\boldsymbol{W}_i \boldsymbol{X}_{i-1} + \boldsymbol{B}_i)$	8-Bit	16-Bit
Unverified NN inference (no proof)	3.53 s	4.64 s
Setup time	2.44 s	2.43 s
Structure reference string size	22.02 MB	22.02 MB
NN commitment time	2,023.87 s	2,097.80 s
Structure commitment (single thread)	747.07 s	737.30 s
Parameter commitment	1,276.80 s	1,360.50 s
NN commitment size	42.27 KB	42.27 KB
Prover time	2,090.71 s	2,210.88 s
Auxiliary input commitment	733.08 s	755.61 s
Proof reduction ($\mathbb{F}$ operations)	218.80 s	293.16 s
PoP proving (single thread)	775.36 s	765.05 s
Leaf node opening	25.46 s	31.23 s
Fiat–Shamir transform proving	338.01 s	356.83 s
Proof size	98.84 KB	98.84 KB
Verifier time (single thread)	173.19 ms	172.72 ms
Peak RAM	87.42 GB	103.63 GB

Parallelized across 64 CPU threads unless noted "single thread"

Summary of Contributions

- A generic and succinct zkML framework
- Hides both NN parameters and NN structures
- Competitive efficiency with practical prover time
- Flexibility across diverse NN structures and operations
- A polynomial-free inner product proof (LiteBullet)

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Thank you!
Questions are welcome.