Study Nuclear Winter with a radiative-convective climate model

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Abstract

1 Introduction

Planet Earth's climate dynamics can be studied with models of varying complexity. Radiative-convective models (RCMs) provide an intermediate complexity approach to the simulation of climate, evaluating an average atmospheric temperature profile. Temperature is averaged over all latitudes and longitudes and is a function of time and altitude. Since only one spatial dimension is used, it is addressed as 1-dimensional model. The atmosphere is modelled as a vertical column divided in parallel layers, each containing specific gases, hence presenting specific optical and physical properties.

Physical processes driving the model are radiation absorption, transmission, reflection and air convection which determine the energy exchange between layers. More complex mechanisms such horizontal heat transport are neglected.

The present work intends to study the atmospheric temperature profile averaged on the Northern Hemisphere in presence of conditions ...

2 Methods

The RCM developed for the present work approximates the atmosphere as a vertical column divided in plane-parallel layers. A bijective relation exists between altitude z and pressure P (cfr. section B.1), hence they can be used interchangeably as vertical coordinates. The origin of coordinate z is set at ground level and it is identified by value z_{g} to keep generality. On the contrary, the Top-Of-Atmosphere (TOA) altitude z_{TOA} is chosen arbitrarily. Pressure decreases with altitude from standard value P_{g} at ground level to value P_{TOA}, which is calculated using equation 8.

Each layer is identified by a value of z corresponding to the upper boundary of the layer and by its thickness. The corresponding values of pressure are evaluated at each time instant with equation 8.

2.1 Hypotheses and conventions

To simplify some calculations for radiative fluxes, the definition of wavenumber $\nu = \frac{1}{\lambda}$ is used in this work, where λ is the wavelenght of the radiation.

Specific gas constant is used in thermodynamical relations, which is defined as the gas constant R divided by the gas molar mass. Henceforth, specific gas constant is represented with symbol R_m.

Some assumptions are made on state and composition of Earth's atmosphere. Gravitational acceleration g is constant. Atmospheric layers are supposed to be in hydrostatic equilibrium described by

$$\mathrm{d}P = -\rho g \,\mathrm{d}z \tag{1}$$

and gaseous components obey the ideal gas law

$$P = \rho R_m T \quad , \tag{2}$$

where ρ is the volumetric mass density of the gas. Each layer is homogeneous in the sense that quantities affecting radiative calculations are not dependent on spatial position inside the layer (e.g. gas densities, extintion coefficients and specific heats at constant pressure c_P are constant inside the layer). The average Earth's surface considered in the model is approximated as an emitting blackbody with temperature T_{earth}. Average Bond albedo for Earth is identified with constant α.

Solar radiation is made by parallel rays forming an angle of incidence ε with the normal of the plane-parallel atmospheric layers. Sun's surface is approximated as an emitting blackbody with temperature T_{sun}. Constant value S_0 for total solar irradiance is used in calculations.

Solar spectral irradiance extends in the infrared (IR), visible (VIS) and ultraviolet (UV) bands, while spectral irradiance of Earth's surface is peaked in the IR band. The difference of their shapes allows the division of the spectrum at the intersection of the curves. The wavenumber of the intersection point is ν_{div}, radiation with lesser wavenumbers is called longwave radiation, otherwise it is addressed as shortwave radiation. The two resulting spectra are treated independently and the overlap between curves is neglected. Accuracy of these approximations and value of ν_{div} are discussed in section 4.1.

Data on constants used in this work are listed in table 1.

2.2 Shortwave radiation

At wavelengths $\lambda < 4\,\mu\text{m}$, solar radiation has much greater intensity than radiation emitted from Earth's surface and atmosphere. Both scattering and absorption by gases, aerosols and clouds of atmosphere dissipates solar radiation.[5, p. 469]

Specific intensity of solar radiation can be expressed by a differential equation whose resolution is complex even applying approximations and numerical methods.[5, p. 469]

A lower complexity parametrisation is adopted instead, where the atmosphere is divided in a given number of layers and radiation is absorbed, scattered and reflected between each layes. Multiple reflections can occur from each layer but only one is considered in this model because successive reflections from amospheric layers have negligible intensities compared to the first one.[5, p. 470]

2.3 Longwave radiation

At wavelengths $\lambda \geq 4\,\mu\text{m}$, solar radiation has lower intensities than radiation emitted by Earth's surface and atmosphere at the same wavelengths. Moreover, it presents negligible scattering in atmosphere with respect to absorption. For these reasons longwave radiation is considered to be emitted only by Earth's surface and atmosphere.[5, p. 468]

2.4 Temperature equation

Intermediate layers of the atmosphere are treated as open systems where in general air parcels can move vertically, therefore temperature is determined by horizontally averaging the thermodynamic energy equation. Details on the derivation are in [5, p. 466]. The resulting differential equation describing temperature variation with time across layer z is

$$\frac{\partial}{\partial t}T(t,z) = -\frac{1}{\rho c_P}\frac{\mathrm{d}}{\mathrm{d}z}q(z) \tag{3}$$

where $q(z)$ is the total power exchanged due to heat transfer, which depends on altitude z also through the temperature. In particular, $q(z) = \Phi_L(z) + \Phi_S(z) + q_C(z)$, where $\Phi_L(z)$ and $\Phi_S(z)$ are the total radiant fluxes generated by longwave and shortwave radiations, respectively, and $q_C(z)$ is the heat exchanged by convection.

Equation (3) can be rewritten using equation (1) to drop the dependency on layer density. Moreover, $q_C(z)$ is neglected because it would require the resolution of equations from fluid dynamics. A numerical correction is adopted instead, which is described in section 2.6. With these considerations, the actual equation for temperature is:

$$\frac{\partial}{\partial t}T(t,z) = -\frac{g}{c_P}\frac{\mathrm{d}}{\mathrm{d}P}\big(\Phi_L(z) + \Phi_S(z)\big) \quad . \tag{4}$$

2.5 Numerical formulation of the model

Euler's method is used to solve the Ordinary Differential Equation (ODE) for the time-dependant temperature function.[5, p. 472] A solution for each atmospheric layer is calculated, hence the resulting values are triplets of temperature, altitude (i.e. proxy for the atmospheric layer) and time (i.e. simulation time). Further information on storage and plotting of data are presented in Section C.1.

Spectral bands are identified by two arrays: one listing the lower bound of each band, the other containing the width of each band. This choice simplify the use of functions for numerical integration. Values related to spectral bands are stored as integers.

Layers are numbered downward starting from 0 at the TOA layer.

Table 1: Data on constants used in the present work.

Symbol	Value	Unit	Notes
R	8.31446261815324	J/(K mol)	
g	9.80665	$\mathrm{m/s^2}$	
S_0	1361.0	$\mathrm{W/m^2}$	Nominal total solar irradiance, from [1]
R_{sun}	6.957×10^8	m	Nominal solar radius, from [1]
T_{sun}	5772	K	Solar surface temperature, from [1]
T_{earth}	288.15	K	Earth's surface temperature based on [2]
α	0.3		Value compatible with various measures, cfr. [3, p. 1281]
P_{g}	1.013250×10^5	Pa	Standard pressure at ground level, from [2, p. 2]
$c_{P,\mathrm{air}}$	1.004×10^3	J/(K kg)	Specific heat at constant pressure of air, from [4, p. 16]
z_{g}	0	m	Nominal ground level

2.6 Convective adjustment

3 Results

3.1 Stability analysis

4 Discussion

4.1 Division of the electromagnetic spectrum

Spectral irradiances of Sun and Earth's surfaces are obtained from blackbody spectral radiance (cfr. appendix B.2). They are compared inside Earth's atmosphere, whose thickness is neglected with respect to the other spatial quantities involved in these calculations, hence the comparison is performed directly on Earth's surface. Given a generic coordinate system, attenuation of solar radiation is considered: spectral radiant intensity of Sun is obtained by multiplying the spectral radiance by the area of a spherical surface with radius R_{sun}, then the result is divided by the area of a spherical surface with radius 1 au to measure solar radiance at Earth's surface. The resulting solar spectral irradiance is

$$E_{\mathrm{sun}}(\nu) = (1-\alpha)\left(\frac{R_{\mathrm{sun}}}{1\,\mathrm{au}}\right)^2 \pi B_\nu(\nu, T_{\mathrm{sun}}) \quad , \tag{5}$$

while Earth's surface spectral irradiance is

$$E_{\mathrm{earth}}(\nu) = \pi B_\nu(\nu, T_{\mathrm{earth}}) \quad . \tag{6}$$

Figure 1 shows the plots of equations (5) and (6) and the division of the spectrum at the wavenumber ν_{div} where the curves intersect. Value $\nu_{\mathrm{div}} \approx 2154/\mathrm{cm}$ is obtained numerically (cfr. appendix A.1).

Considering only interval $[100/\mathrm{cm}, \nu_{\mathrm{div}}]$, the largest contribution to the total irradiance is brought by Earth's surface and solar irradiance is neglected, being about 2.2 % of the total irradiance. The removed irradiance is 0.65 % of the total Sun's surface irradiance. Instead, in $[\nu_{\mathrm{div}}, 100\,000/\mathrm{cm}]$ Earth's surface irradiance is 0.16 % of the total irradiance in this interval and is neglected. The removed quantity is 0.55 % of the total Earth's surface irradiance.

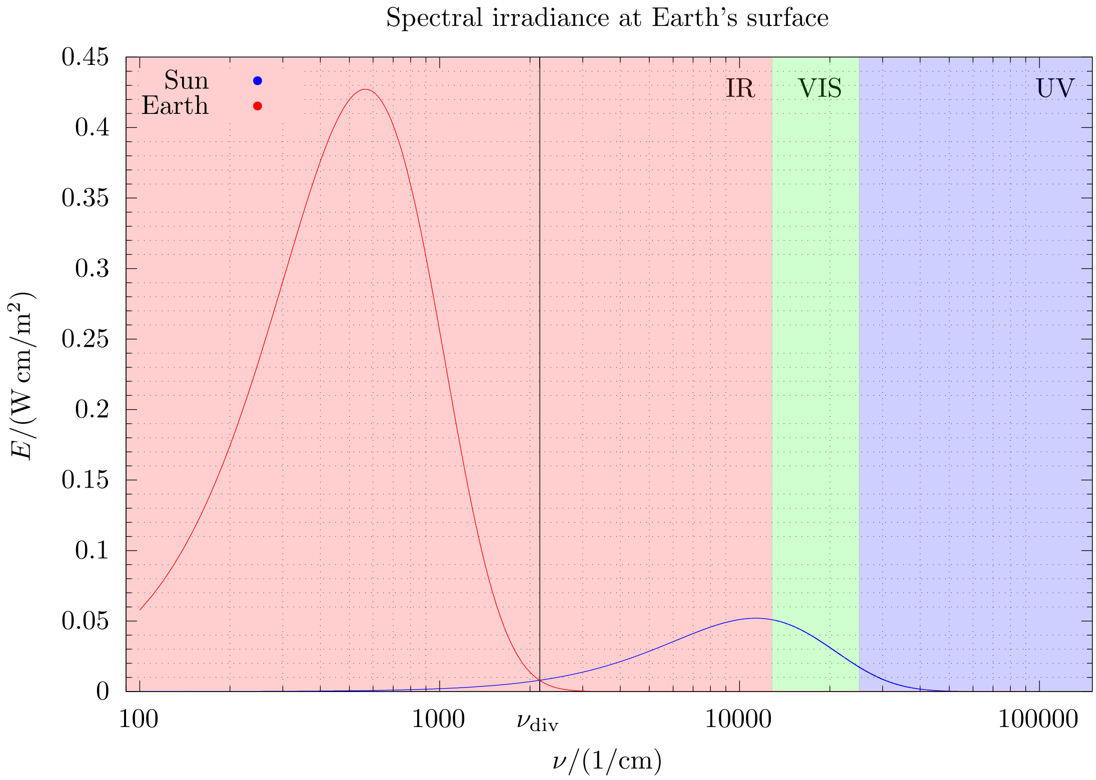

Figure 1: The vertical black line marks the division of the spectrum at ν_{div}. In the longwave region $[100/\mathrm{cm}, \nu_{\mathrm{div}}]$ Earth's surface brings the largest contribution to the total irradiance, in the shortwave region $[\nu_{\mathrm{div}}, 100\,000/\mathrm{cm}]$ solar radiation dominates instead. Infrared (IR), visible (VIS) and ultraviolet (UV) spectral bands as specified by [6] are shown in background.

A Source code

In this section the C++ code used to obtain the results presented in this work is shown and commented.

First, the parametrization of the vertical coordinate is chosen among three alternatives: altitude z in m, atmospheric pressure P in Pa, coordinate $\sigma = \frac{P - P_{\mathrm{TOA}}}{P_S - P_{\mathrm{TOA}}}$ adimensional, with P_{TOA} pressure at the top of the modelised atmosphere and P_S pressure at the surface. One parametrisation can be written in term of another through a monotonic function (e.g. pressure decreases with altitude, cfr. Appendix B.1). For the initial development the altitude z is chosen as vertical coordinate because it is more intuitive, moreover plots in TTAPS-I are expressed in terms of both z and P.

Second, the atmospheric layers are configured. In TTAPS-I 20 layers are used (cfr. [7, p. 396]) and they are numbered from the top of the atmosphere down as it is common in RCMs. The vertical coordinate refers to the center of each layer, with the excpetion of the last layer which is in direct contact with the surface and needs to be treated separately. Therefore two arrays are needed: one for the point and one with the corresponding layer thicknesses. Values are then assigned as double precision numbers. The value corresponding to the top of the atmosphere is set in a proper variable and a uniform distribution of layer thicknesses is assumed for ease.

To reduce the computation load of radiative fluxes, only some spectral intervals are considered and they are specific to each atmospheric layer, since the available absorbing gases and aerosols differ among layers. First absorbers for each layer are stored in an array, then the spectral bandwitdhs needed for each layer are evaluated. Spectral bandwidths are expressed in terms of wavenumbers with unit 1/cm to manage integer values or double precision values close to unity. For each chemical species the absorption intervals are identified by their width and central wavenumber, the former are obtained from the latter and using the exponential wide band model (cfr. [8, p. 360]). These two values are stored in separate arrays.

A.1 Hypotheses

A.2 Classes

B Mathematical derivations

In this appendix mathematical derivations of some ancillary results and formulae used in the main text are explicitly shown.

B.1 Relation between pressure and altitude

A general result regarding planetary atmosphere is that atmospheric pressure decreases with increasing altitude. Theoretical relations which approximate this behaviour can be obtained. Hypotheses considered in Section 2.1 are valid.

If density is assumed constant, equation (1) can be solved easily resulting in a linear dependence of pressure P on altitude z,

$$P(z) = P_0 - \rho g (z - z_0) \quad , \tag{7}$$

where (z_0, P_0) is a reference point inside the atmosphere.

If density is not constant its expression is given by the ideal gas law (cfr. equation (2)) and, assuming constant temperature T, equation (1) results in:

$$\begin{aligned}
&\mathrm{d}P = -\frac{Pg}{R_m T}\,\mathrm{d}z \iff \\
\iff &\frac{\mathrm{d}P}{P} = -\frac{g}{R_m T}\,\mathrm{d}z \impliedby \\
\impliedby &\ln(P')\Big|_{P_0}^{P(z)} = -\frac{g}{R_m T} z'\Big|_{z_0}^{z} \iff \\
\iff &P(z) = P_0 \exp\left(-\frac{g}{R_m T}(z - z_0)\right) \quad .
\end{aligned} \tag{8}$$

This relation is not meaningful, since the aim of the work is to derive the non-constant temperature profile of the atmosphere. However, it can be used inside atmospheric layers where the temperature is considered constant (e.g. stratosphere).

A better approximation assumes non-constant density and constant lapse rate Γ, hence temperature depends linearly on altitude,

$$\Gamma = -\frac{\mathrm{d}T}{\mathrm{d}z} \impliedby T(z) = T_0 - \Gamma(z - z_0) \quad , \tag{9}$$

with T_0 temperature corresponding to reference altitude z_0. Using these assumptions and the density rewritten through the ideal gas law (2), equation (1) becomes

$$\begin{aligned}
&\mathrm{d}P = -\frac{Pg}{R_m T}\left(-\frac{\mathrm{d}T}{\Gamma}\right) \iff \\
\iff &\frac{\mathrm{d}P}{P} = \frac{g}{R_m \Gamma}\frac{\mathrm{d}T}{T} \impliedby \\
\impliedby &\ln(P')\Big|_{P_0}^{P(z)} = \frac{g}{R_m \Gamma}\ln(T')\Big|_{T_0}^{T(z)} \iff \\
\iff &P(z) = P_0 \left(\frac{T_0 - \Gamma(z - z_0)}{T_0}\right)^{\frac{g}{R_m \Gamma}}
\end{aligned} \tag{10}$$

Equation (10) can be used also with a piecewise constant lapse rate in altitude intervals where it is not null. Otherwise, in altitude intervals where lapse rate is null, equation (8) is valid with appropriate boundary conditions to ensure continuity between layers.

B.2 Radiometric quantities

Refer to [6] for more details on quantities reviewed in this section.

Consider electromagnetic radiation emitted by a point source. The total emitted power is called *radiant flux*, with unit W. The density of radiant flux with respect to a solid angle in the direction of emission is called *radiant intensity*, expressed in W/sr. When radiation interacts with a surface, i.e. it gets absorbed, transmitted or reflected, its radiant intensity distributed over the surface is measured through *radiance* in W/(m^2 sr). If the area on which the radiation is incident is expressed through the solid angle it subtends, the integral of radiance over this solid angle is called *irradiance*, expressed in W/m^2. Note that the coordinate system where the solid angles of radiant intensity and irradiance are defined may not be the same.

All previous quantities can be expressed as densities with respect to the wavelength or the wavenumber and the adjective *spectral* is prefixed to their names. Their units are divided by the respective spectral quantity (e.g. spectral radiance with wavenumber in 1/cm has units $\mathrm{W\,cm/(m^2\,sr)}$).

Spectral radiance of a blackbody is given by Planck's law

$$B_\nu(\nu, T) = 2hc^2\nu^3 \frac{1}{e^{\frac{hc\nu}{k_B T}} - 1} \quad , \tag{11}$$

where ν is the wavenumber in unit 1/cm (cfr. notation in section 2.1), T in unit K is the temperature of the emitting body and the other quantities are constants (cfr. table 1). Note that Planck's law has different form when it is expressed in terms of wavelength, due to its definition as density and the resulting change of variables:

$$B_\lambda(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_B T}} - 1} \quad . \tag{12}$$

If radiance is isotropic, i.e. it has not dependence on the direction of the radiation, the corresponding irradiance is proportional. For instance, if the radiation is absorbed by a hemispheric surface approximated by a blackbody, the spectral irradiance of the surface is

$$\begin{aligned} &\int B_\nu(\nu, T)\,\mathrm{d}\phi \sin(\theta)\,\mathrm{d}\theta \cos(\theta) = \\ &= B_\nu(\nu, T) \int_0^{2\pi} \mathrm{d}\phi \int_0^{\frac{\pi}{2}} \sin(\theta)\cos(\theta)\,\mathrm{d}\theta = \\ &= 2\pi B_\nu(\nu, T) \int_0^1 \sin(\theta)\,\mathrm{d}(\sin(\theta)) = \\ &= 2\pi B_\nu(\nu, T)\frac{1}{2} = \pi B_\nu(\nu, T) \end{aligned} \quad , \tag{13}$$

where spherical coordinates are used to describe the surface and the term $\cos(\theta)$ considers the component of radiation along the normal of the infinitesimal solid angle.

B.3 Common quantities used in atmospheric sciences

Earth's surface horizontal profile is not uniform, hence altitude and pressure near ground level could present sudden variations. In models where this is taken into consideration, the sigma coordinate system is commonly used instead, defined by

$$\sigma = \frac{P - P_{\mathrm{TOA}}}{P_{\mathrm{g}} - P_{\mathrm{TOA}}} \quad . \tag{14}$$

An alternative quantity evaluated in place of $T(t, z)$ for a given parcel of fluid is the potential temperature

$$\theta(t, z) = T(t, z)(\frac{P_0}{P(z)})^{\frac{R_m}{c_P}} \quad , \tag{15}$$

where P_0 is a reference pressure and quantities $P(z)$, R_m and c_P refer to the fluid.

C Supplementary information

C.1 Plotting

Software gnuplot is used to generate plots shown in this work. Output values from the simulation are stored in a DAT file with the structure reproduced in listing 1.

Listing 1: Structure of DAT file containing output values. The line spacing between data blocks is mandatory and ellipsis `...` indicates a logical continuation of the sequences. Number `N_t` of temporal steps needed to reach convergence is not known a priori, instead `N` is the chosen number of atmospheric layers. Quantities `P`, `sigma` and `theta` are added in case of need for additional plots.

```
t[0]         z[0]     T[0][0]            P[0]     sigma[0]     theta[0][0]
...          ...      ...                ...      ...          ...
t[0]         z[N - 1] T[0][N - 1]        P[N - 1] sigma[N - 1] theta[0][N - 1]

t[1]         z[0]     T[1][0]            P[0]     sigma[0]     theta[1][0]
...          ...      ...                ...      ...          ...
t[1]         z[N - 1] T[1][N - 1]        P[N - 1] sigma[N - 1] theta[1][N - 1]

...

t[N_t - 1] z[0]     T[N_t - 1][0]      P[0]     sigma[0]     theta[N_t - 1][0]
...          ...      ...                ...      ...          ...
t[N_t - 1] z[N - 1] T[N_t - 1][N - 1] P[N - 1] sigma[N - 1] theta[N_t - 1][N - 1]
```

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