Radiativeconvective equilibrium in a grey atmospher

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# Radiative-convective equilibrium in a grey atmosphere

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Complex systems in climate physics, 3 October 2023



#### Radiative-convective equilibrium in a grey atmosphere

Radiative-convective equilibrium in a grey atmosphere

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Complex systems in climate physics, 3 October 2023

- A radiative-convective model is used to study a grey atmosphere.
- Comparison between numerical and analytical solutions is possible in radiative equilibrium.

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Introduction

• Average vertical temperature profile T(t, z) of atmosphere.

1. The analysed quantity is the atmospheric temperature profile averaged over all latitudes and longitudes.

# Introduction

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Average vertical temperature profile T(t, z) of atmosphere.

- Average vertical temperature profile T(t, z) of atmosphere.
- Radiative Transfer Equation (RTE).

- 1. The analysed quantity is the atmospheric temperature profile averaged over all latitudes and longitudes.
- 2. RTE describes radiative processes.

## Introduction



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Average vertical temperature profile T(t, z) of atmosphere.

- 1. The analysed quantity is the atmospheric temperature profile averaged over all latitudes and longitudes.
- 2. RTE describes radiative processes.
- 3. Fluid dynamics equations describe convective processes.

• Average vertical temperature profile T(t,z) of atmosphere.

• Radiative Transfer Equation (RTE).

Fluid dynamics equations.

• Thermodynamic energy equation in Local Thermodynamic Equilibrium (LTE):

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_P} \frac{\partial q}{\partial z} \quad . \tag{1}$$

1. Thermodynamic energy equation describes average vertical temperature profile.

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└─Introduction

└─Hypotheses



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#### Introduction

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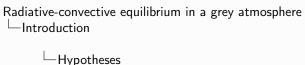
Conclusion

# Hypotheses

• Thermodynamic energy equation in Local Thermodynamic Equilibrium (LTE):

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho_{CP}} \frac{\partial q}{\partial z} \quad . \tag{1}$$

• Radiative-convective equilibrium.





Radiative-convective equilibrium.

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- 1. Thermodynamic energy equation describes average vertical temperature profile.
- 2. The study is conducted on an atmosphere in radiative-convective equilibrium.



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#### Introduction

Radiative equilibrium

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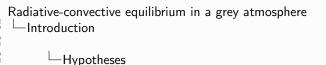
Conclusion

## Hypotheses

• Thermodynamic energy equation in Local Thermodynamic Equilibrium (LTE):

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_P} \frac{\partial q}{\partial z} \quad . \tag{1}$$

- Radiative-convective equilibrium.
- Grey atmosphere.





- 1. Thermodynamic energy equation describes average vertical temperature profile.
- 2. The study is conducted on an atmosphere in radiative-convective equilibrium.
- 3. Quantities do not depend on the frequency of electromagnetic radiation.

#### Additional hypotheses

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-Additional hypotheses

Hypotheses on the planet.

Additional hypotheses

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Hypotheses on the planet.

1. Diurnal cycle, constant irradiance, constant Bond albedo, surface emits blackbody radiation, constant gravitational acceleration.

## Additional hypotheses

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Hypotheses on the planet.
 Hypotheses on the composition of atmosphere

Additional hypotheses

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\_\_Additional hypotheses

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- Hypotheses on the planet.
- Hypotheses on the composition of atmosphere.

- 1. Diurnal cycle, constant irradiance, constant Bond albedo, surface emits blackbody radiation, constant gravitational acceleration.
- 2. Hydrostatic equilibrium, constant specific heat at constant pressure, scattering is neglected, absorption coefficient depends only on altitude, constant mass attenutation coefficient, ideal gas.

#### Additional hypotheses

-Additional hypotheses

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Additional hypotheses

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- Hypotheses on the planet.
- Hypotheses on the composition of atmosphere.
- Hypotheses on total heat flux.
- Resulting thermodynamic energy equation:

$$\frac{\partial T}{\partial t} = -\frac{1}{\varrho c_{P}} \frac{\partial}{\partial z} (E_{U} - E_{D}) \quad . \tag{2}$$

- 1. Diurnal cycle, constant irradiance, constant Bond albedo, surface emits blackbody radiation, constant gravitational acceleration.
- 2. Hydrostatic equilibrium, constant specific heat at constant pressure, scattering is neglected, absorption coefficient depends only on altitude, constant mass attenutation coefficient, ideal gas.
- 3. Heat flux determined only by radiative and convective processes, two-stream approximation, numerical correction for convection.

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• Relation between pressure and altitude:

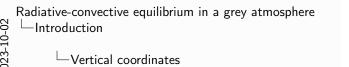
$$P(z) = P_{\rm g} \exp\left(-\frac{z - z_{\rm g}}{z_0}\right) \quad . \tag{3}$$

• Relation between optical depth and pressure:

$$\delta(P) = \frac{\mu_{\rm m}}{\sigma} (P - P_{\rm TOA}) \quad . \tag{4}$$

• Relation between optical depth and altitude:

$$\delta(z) = \frac{\mu_{\rm m}}{g} \left( P_{\rm g} \exp\left(-\frac{z - z_{\rm g}}{z_{\rm 0}}\right) - P_{\rm TOA} \right) \quad . \tag{5}$$





- 1. Obtained from hydrostatic equilibrium and ideal gas law.
- 2. Obtained from definition of optical depth, hydrostatic equilibrium and hypotheses on attenutation coefficient.
- 3. Obtained by combining relations (3) and (4).

Equations in radiative equilibrium

• RTE for non-scattering medium in LTE:

$$\frac{1}{\mu} \frac{\partial L}{\partial z} = B_{\nu} - L \quad . \tag{6}$$

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Equations in radiative equilibrium

. RTE for non-scattering medium in LTE:

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1. RTE describes radiance in a medium.

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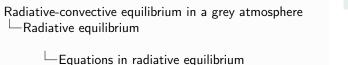
Conclusion

#### Equations in radiative equilibrium

• RTE for non-scattering medium in LTE:

$$\frac{1}{\mu} \frac{\partial L}{\partial z} = B_{\nu} - L \quad . \tag{6}$$

- Integration over frequency and solid angle.
- Diffusion approximation:  $\delta' = D\delta$ .



\* RTE for non-scattering medium in LTE:  $\frac{1}{\mu}\frac{\partial L}{\partial z}=B_{\nu}-L\quad.$  \* Integration over frequency and solid angle.

Equations in radiative equilibrium

• Diffusion approximation:  $\delta' = D\delta$ .

1. RTE describes radiance in a medium.

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2. To describe irradiances, RTE is integrated over the whole spectrum of radiation and over the solid angle of a hemisphere.



#### Equations in radiative equilibrium

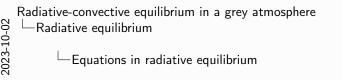
• RTE for non-scattering medium in LTE:

$$\frac{1}{\mu} \frac{\partial L}{\partial z} = B_{\nu} - L \quad . \tag{6}$$

- Integration over frequency and solid angle.
- Diffusion approximation:  $\delta' = D\delta$ .
- Equations for irradiances:

$$-\frac{\partial}{\partial \delta'} E_{\mathsf{U}}(t, \delta') = \sigma T(t, \delta')^{4} - E_{\mathsf{U}}(t, \delta') \quad , \tag{7}$$

$$-\frac{\partial}{\partial \delta'} E_{\mathsf{U}}(t, \delta') = \sigma T(t, \delta')^{4} - E_{\mathsf{U}}(t, \delta') \quad , \tag{7}$$
$$\frac{\partial}{\partial \delta'} E_{\mathsf{D}}(t, \delta') = \sigma T(t, \delta')^{4} - E_{\mathsf{D}}(t, \delta') \quad . \tag{8}$$



 $\frac{\partial}{\partial t}$ En(t,  $\delta'$ ) =  $\sigma T$ (t,  $\delta'$ )<sup>4</sup> - En(t,  $\delta'$ ) . (8)

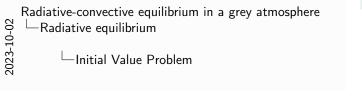
Equations in radiative equilibrium

- 1. RTF describes radiance in a medium.
- 2. To describe irradiances, RTE is integrated over the whole spectrum of radiation and over the solid angle of a hemisphere.
- 3. Equations for irradiances and temperature are written in terms of  $\delta'$ .



#### Initial Value Problem

• Steady state.



1. Analytical solutions are found for temperature at the steady state.

Initial Value Problem

Steady state.

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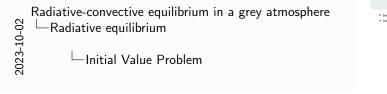
Radiative equilibrium

#### Initial Value Problem

- Steady state.
- Initial conditions on irradiances:

$$E_{\mathsf{D}}(0) = 0 \quad , \tag{9}$$

$$E_{D}(0) = 0$$
 , (9)  
 $E_{U}(0) = S_{t}$  . (10)



1. Analytical solutions are found for temperature at the steady state.

Initial Value Problem

2. At TOA downward irradiance does not deposit energy and radiative equilibrium fixes upward irradiance, i.e. outgoing longwave radiation.



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Radiative equilibrium

#### Initial Value Problem

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- Steady state.
- Initial conditions on irradiances:

$$E_{\mathsf{D}}(0) = 0 \quad , \tag{9}$$

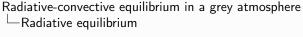
$$E_{\rm U}(0) = S_{\rm t}$$
 . (10)

Additional relations:

$$\frac{\mathrm{d}}{\mathrm{d}\delta'} \left( E_{\mathsf{U}}(\delta') - E_{\mathsf{D}}(\delta') \right) = 0 \quad , \tag{11}$$

$$E_{\mathsf{U}}(\delta') - E_{\mathsf{D}}(\delta') = S_{\mathsf{t}} \quad . \tag{12}$$

$$E_{\mathsf{U}}(\delta') - E_{\mathsf{D}}(\delta') = S_{\mathsf{t}} \quad . \tag{12}$$





Initial Value Problem

└ Initial Value Problem

- 1. Analytical solutions are found for temperature at the steady state.
- 2. At TOA downward irradiance does not deposit energy and radiative equilibrium fixes upward irradiance, i.e. outgoing longwave radiation.
- 3. Hypotheses of atmosphere in radiative equilibrium at all altitudes and atmopshere transparent to radiation coming from outside the planet are equivalent to hypotheses of atmopshere in radiative equilibrium at TOA and steady state.

#### Initial Value Problem

- Steady state.
- Initial conditions on irradiances:

$$E_{\rm D}(0)=0 \quad , \tag{9}$$

$$E_{\rm U}(0) = S_{\rm t}$$
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Additional relations:

$$\frac{\mathrm{d}}{\mathrm{d}\delta'} \left( E_{\mathsf{U}}(\delta') - E_{\mathsf{D}}(\delta') \right) = 0 \quad , \tag{11}$$

$$E_{\mathsf{U}}(\delta') - E_{\mathsf{D}}(\delta') = S_{\mathsf{t}} \quad . \tag{12}$$

$$F_{\square}(\delta') - F_{\square}(\delta') = S_{+} \qquad (12)$$

Initial condition on temperature:

$$T(0) = \left(\frac{S_{\rm t}}{2\sigma}\right)^{\frac{1}{4}} \quad . \tag{13}$$



Radiative-convective equilibrium in a grey atmosphere —Radiative equilibrium



Initial Value Problem

- 1. Analytical solutions are found for temperature at the steady state.
- 2. At TOA downward irradiance does not deposit energy and radiative equilibrium fixes upward irradiance, i.e. outgoing longwave radiation.
- 3. Hypotheses of atmosphere in radiative equilibrium at all altitudes and atmopshere transparent to radiation coming from outside the planet are equivalent to hypotheses of atmopshere in radiative equilibrium at TOA and steady state.
- 4. Initial condition on temperature is obtained by combining all conditions on irradiances.

☐ Analytical solution

Analytical solution

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Temperature:

$$T(\delta) = \left(\frac{S_{\rm t}}{2\sigma}(1+D\delta)\right)^{\frac{1}{4}}$$
 (14)

Irradiances:

$$E_{\mathsf{U}}(\delta) = \frac{S_{\mathsf{t}}}{2}(2 + D\delta) \quad , \tag{15}$$

$$E_{\mathsf{D}}(\delta) = \frac{S_{\mathsf{t}}}{2}D\delta \quad . \tag{16}$$

$$E_{\rm D}(\delta) = \frac{S_{\rm t}}{2} D\delta \quad . \tag{16}$$

#### Numerical solution

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☐ Numerical solution

Numerical solution

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## Radiative-convective equilibrium

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