Elliptic Integrals

- The period of the pendulum for arbitrary initial amplitude θ_0 is given by

$$T = 4\sqrt{\frac{L}{g}}F\left(\frac{\pi}{2}, k\right)$$

where **L** is the length of the pendulum, **g** is the gravitational acceleration, **k = sin(θ_0/2)** is related to the initial angle, while **F(ϕ,k)** is the incomplete elliptic integral of the first kind defined by

$$\boxed{F(\phi, k) = \int_0^{\phi} \frac{du}{\sqrt{1 - k^2 \sin^2 u}}}$$

- Here ϕ = am (F,k) is also called the Jacobi amplitude[1].

- Assume **L/g = 1** and **k = 0.8** throughout the exercise.

[1]Note that ϕ is *not* the angular displacement but it is related to it by $\sin\phi = \frac{\sin(\theta/2)}{\sin(\theta_0/2)}$

Elliptic Integrals

- Write a code to compute

  1. The oscillation period **T**;
  2. The relative error when compared to the small-angle approximation;
  3. For the more general case, find $\boldsymbol{\phi}$**(t)** by inverting the equation **t = F(**$\boldsymbol{\phi}$**,k)** at regularly spaced interval **t = 0.0, 0.1, 0.2, … , 30.0**. A plot can be used for the purpose. Use both Bisection and Newton method in your implementation.

- Provide a single .pdf document including
  - the result of point 1) and 2);
  - the plot in point 3);
  - Any extra explanation you may consider useful;
  - the C++ code (max 100 lines, do not include quadrature rules or root solvers).

- Time can be passed through functions using a global variable (e.g. `static double g_time`), whose value is updated in the `main()` function.