Study Nuclear Winter with a radiative-convective climate model

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Abstract

1 Introduction

Planet Earth's climate dynamics can be studied with models of varying complexity depending on the subject of the study. Radiative-convective models (RCMs) provide an intermediate complexity approach to the simulation of climate, evaluating the atmospheric temperature profile in a vertical column where temperature is averaged over all latitudes and longitudes.

The present work intends to study the atmospheric temeprature profile averaged on the Northern Hemisphere in presence of conditions

2 Methods

The RCM developed for the present work approximates the atmosphere as a vertical column divided in a given number of layers.

2.1 Hypotheses and conventions

To simplify some calculations for radiative fluxes, the definition of wavenumber $\nu = \frac{1}{\lambda}$ is used in this work, where λ is the wavelenght.

Specific gas constant is used in thermodynamical relations, which is defined as the gas constant R divided by the gas molar mass. Henceforth, the symbol R_M is used for the specific gas constant.

Some assumptions are made on state and composition of Earth's atmosphere. Gravitational acceleration g is constant. Air parcels are supposed to be in hydrostatic equilibrium,

$$\mathrm{d}P = -\rho g \,\mathrm{d}z \tag{1}$$

with each component obeying the ideal gas law

$$P = \rho R_M T \tag{2}$$

where $\rho = \frac{m}{V}$ is the volumetric mass density of the air parcel.

Solar radiation is considered orthogonal to the plane which represents Earth's surface in the RCM. Constant value S_0 for total solar irradiance is used in calculations.

Solar spectral irradiance is peaked in the ultraviolet (UV) and visible bandwidths (henceforth the reference wavenumber about 2×10^4/cm), while spectral irradiance of Earth's surface is peaked at about 1×10^3/cm, in the infrared (IR) bandwidth. Due to this difference, the spectrum can be splitted at the wavenumber where the the distributions intersect and the two resulting spectra can be treated indipendently. The overlap between the two distributions is neglected and an upper limit for the accuracy of this approximation can be quantified by the ratio between radiant intensities of Sun and Earth's surface in each spectra, when the sources are considered as blackbodies (i.e. when the emission is maximum).

Data on constants used in this work are listed in table **??**.

2.2 Shortwave radiation

At wavelengths $\lambda < 4\,\mu\mathrm{m}$, solar radiation has much greater intensity than radiation emitted from Earth's surface and atmosphere. Both scattering and absorption by gases, aerosols and clouds of atmosphere dissipates solar radiation.[2, p. 469]

Specific intensity of solar radiation can be expressed by a differential equation whose resolution is complex even applying approximations and numerical methods.[2, p. 469]

Figure 1: MC complete.

Table 1: Data on constants used in the present work.

Symbol	Value	Unit	Notes
R	8.31446261815324	$\mathrm{m^3\,Pa/(K\,mol)}$	
g	9.80665	$\mathrm{m/s^2}$	
S_0	1361.0	$\mathrm{W/m^2}$	Nominal total solar irradiance, from [1]
R_{sun}	6.957×10^8	m	Nominal solar radius, from [1]

A lower complexity parametrisation is adopted instead, where the atmosphere is divided in a given number of layers and radiation is absorbed, scattered and reflected between each layes. Multiple reflections can occur from each layer but only one is considered in this model because successive reflections from amospheric layers have negligible intensities compared to the first one.[2, p. 470]

2.3 Longwave radiation

At wavelengths $\lambda \geq 4\,\mu\mathrm{m}$, solar radiation has lower intensities than radiation emitted by Earth's surface and atmosphere at the same wavelengths. Moreover, it presents negligible scattering in atmosphere with respect to absorption. For these reasons longwave radiation is considered to be emitted only by Earth's surface and atmosphere.[2, p. 468]

2.4 Numerical treatment

Euler's method is used to solve the Ordinary Differential Equation (ODE) for the time-dependant temperature function.[2, p. 472] A solution for each atmospheric layer is evaluated, hence the resulting values are triplets of temperature, altitude (i.e. proxy for the atmospheric layer) and time (i.e. simulation time). Further information on storage and plotting of data are presented in Section **??**.

Spectral bands are identified by two arrays: one listing the lower bound of each band, the other containing the width of each band. This choice simplify the use of functions for numerical integration. Values related to spectral bands are stored as integers.

3 Results

3.1 Stability analysis

4 Discussion

A Source code

In this section the C++ code used to obtain the results presented in this work is shown and commented.

First, the parametrization of the vertical coordinate is chosen among three alternatives: altitude z in m, atmospheric pressure P in Pa, coordinate $\sigma = \frac{P-P_{\mathrm{TOA}}}{P_S-P_{\mathrm{TOA}}}$ adimensional, with P_{TOA} pressure at the top of the modelised atmosphere and P_S pressure at the surface. One parametrisation can be written in term of another through a monotonic function (e.g. pressure decreases with altitude, cfr. Appendix ??). For the initial development the altitude z is chosen as vertical coordinate because it is more intuitive, moreover plots in TTAPS-I are expressed in terms of both z and P.

Second, the atmospheric layers are configured. In TTAPS-I 20 layers are used (cfr. [3, p. 396]) and they are numbered from the top of the atmosphere down as it is common in RCMs. The vertical coordinate refers to the center of each layer, with the excpetion of the last layer which is in direct contact with the surface and needs to be treated separately. Therefore two arrays are needed: one for the point and one with the corresponding layer thicknesses. Values are then assigned as double precision numbers. The value corresponding to the top of the atmosphere is set in a proper variable and a uniform distribution of layer thicknesses is assumed for ease.

To reduce the computation load of radiative fluxes, only some spectral intervals are considered and they are specific to each atmospheric layer, since the available absorbing gases and aerosols differ among layers. First absorbers for each layer are stored in an array, then the spectral bandwitdhs needed for each layer are evaluated. Spectral bandwidths are expressed in terms of wavenumbers with unit 1/cm to manage integer values or double precision values close to unity. For each chemical species the absorption intervals are identified by their width and central wavenumber, the former are obtained from the latter and using the exponential wide band model (cfr. [4, p. 360]). These two values are stored in separate arrays.

A.1 Classes

B Supplementary information

B.1 Plotting

Software Gnuplot is used to generate plots shown in this work. Output values from the simulation are stored in a DAT file with the following structure, line spacing between data blocks is important:

```
t[0]         P[0]         T[0][0]
...          ...          ...
t[0]         P[N_P - 1]   T[0][N_P - 1]

t[1]         P[0]         T[1][0]
...          ...          ...
t[1]         P[N_P - 1]   T[1][N_P - 1]

...

t[N_t - 1]   P[0]         T[N_t - 1][0]
...          ...          ...
t[N_t - 1]   P[N_P - 1]   T[N_t - 1][N_P - 1]
```

Value `N_t` is not known a priori since it is the number of temporal steps needed to reach convergence, instead `N_P` is the chosen number of atmospheric layers.

C Mathematical derivations

In this appendix mathematical derivations of some ancillary results and formulae used in the main text are explicitly shown.

C.1 Relation between pressure and altitude

A general result regarding planetary atmosphere is that atmospheric pressure decreases with increasing altitude. Theoretical relations which approximate this behaviour can be obtained. Hypotheses considered in Section ?? are valid.

If density is assumed constant, equation (??) can be solved easily resulting in a linear dependence of pressure P on altitude z,

$$P(z) = P_0 - \rho g(z - z_0) \quad , \tag{3}$$

where (z_0, P_0) is a reference point inside the atmosphere.

If density is not constant its expression is given by the ideal gas law (cfr. equation (**??**)) and, assuming constant temperature T, equation (**??**) results in:

$$\begin{aligned} \mathrm{d}P &= -\frac{Pg}{RT}\,\mathrm{d}z \iff \\ \iff \frac{\mathrm{d}P}{P} &= -\frac{g}{RT}\,\mathrm{d}z \iff \\ \iff \ln(P')\Big|_{P_0}^{P(z)} &= -\frac{g}{RT}z'\Big|_{z_0}^{z} \iff \\ \iff P(z) &= P_0 \exp\bigg(-\frac{g}{RT}(z-z_0)\bigg) \quad . \end{aligned} \tag{4}$$

This relation is not meaningful, since the aim of the work is to derive the non-constant temperature profile of the atmosphere. However, it can be used inside atmospheric layers where the temperature is considered constant (e.g. stratosphere).

A better approximation assumes non-constant density and constant lapse rate Γ, hence temperature depends linearly on altitude,

$$\Gamma = -\frac{\mathrm{d}T}{\mathrm{d}z} \iff T(z) = T_0 - \Gamma(z-z_0) \quad , \tag{5}$$

with T_0 temperature corresponding to reference altitude z_0. Using these assumptions and the density rewritten through the ideal gas law (**??**), equation (**??**) becomes

$$\begin{aligned} \mathrm{d}P &= -\frac{Pg}{RT}\bigg(-\frac{\mathrm{d}T}{\Gamma}\bigg) \iff \\ \iff \frac{\mathrm{d}P}{P} &= \frac{g}{R\Gamma}\frac{\mathrm{d}T}{T} \iff \\ \iff \ln(P')\Big|_{P_0}^{P(z)} &= \frac{g}{R\Gamma}\ln(T')\Big|_{T_0}^{T(z)} \iff \\ \iff P(z) &= P_0\bigg(\frac{T_0-\Gamma(z-z_0)}{T_0}\bigg)^{\frac{g}{R\Gamma}} \end{aligned} \tag{6}$$

Equation (**??**) can be used also with a piecewise constant lapse rate in altitude intervals where it is not null. Otherwise, in altitude intervals where lapse rate is null, equation (**??**) is valid with appropriate boundary conditions to ensure continuity between layers.

C.2 Radiometric quantities

Refer to [5] for more details on quantities reviewed in this section.

Consider electromagnetic radiation emitted by a point source. The total emitted power is called *radiant flux*, with unit W. The density of radiant flux with respect to a solid angle in the direction of emission is called *radiant intensity*, expressed in W/sr. When radiation interacts with a surface, i.e. it gets absorbed, transmitted or reflected, its radiant intensity distributed over the surface is measured through *radiance* in $\mathrm{W/(m^2\,sr)}$. If the area on which the radiation is incident is expressed through the solid angle it subtends, the integral of radiance over this solid angle is called *irradiance*, expressed in $\mathrm{W/m^2}$. Note that the coordinate system where the solid angles of radiant intensity and irradiance are defined may not be the same.

All previous quantities can be expressed as densities with respect to the wavelength or the wavenumber and the adjective *spectral* is prefixed to their names. Their units are divided by the respective spectral quantity (e.g. spectral radiance with wavenumber in 1/cm has units $\mathrm{W\,cm/(m^2\,sr)}$).

Spectral radiance of a blackbody is given by Planck's law

$$B_\nu(\nu, T) = 2hc^2\nu^5\frac{1}{e^{\frac{hc\nu}{k_B T}}-1} \quad , \tag{7}$$

where ν is the wavenumber in unit 1/cm (cfr. notation in section **??**), T in unit K is the temperature of the emitting body and the other quantities are constants (cfr. table **??**).

If the radiance is isotropic, i.e. it has not dependence on the direction of the radiation, the corresponding irradiance is proportional. For instance, if the radiation is absorbed by a hemispheric surface approximated by a blackbody, the spectral irradiance of the surface is

$$\begin{aligned} &\int B_\nu(\nu,T)\,\mathrm{d}\phi\sin(\theta)\,\mathrm{d}\theta\cos(\theta) = \\ &= B_\nu(\nu,T)\int_0^{2\pi}\mathrm{d}\phi\int_0^{\frac{\pi}{2}}\sin(\theta)\cos(\theta)\,\mathrm{d}\theta = \\ &= 2\pi B_\nu(\nu,T)\int_0^1\sin(\theta)\,\mathrm{d}(\sin(\theta)) = \\ &= 2\pi B_\nu(\nu,T)\frac{1}{2} = \pi B_\nu(\nu,T) \end{aligned} \quad , \tag{8}$$

where spherical coordinates are used to describe the surface and the term $\cos(\theta)$ considers the component of radiation along the normal of the infinitesimal solid angle.

References

[1] A. Prša, P. Harmanec, G. Torres, *et al.*, "Nominal values for selected solar and planetary quantities: Iau 2015 resolution b3," *The Astronomical Journal*, vol. 152, no. 2, p. 41, Aug. 2016. DOI: 10.3847/0004-6256/152/2/41. [Online]. Available: https://dx.doi.org/10.3847/0004-6256/152/2/41.

[2] V. Ramanathan and J. A. Coakley Jr., "Climate modeling through radiative-convective models," *Reviews of Geophysics*, vol. 16, no. 4, pp. 465–489, 1978. DOI: https://doi.org/10.1029/RG016i004p00465. eprint: https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/RG016i004p00465. [Online]. Available: https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/RG016i004p00465.

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[5] International Commission on Illumination, "CIE S 017:2020 ILV: International Lighting Vocabulary, 2nd edition," International Commission on Illumination, Standard, 2020.