Study Nuclear Winter with a radiative-convective climate model

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Abstract

1 Introduction

2 Methods

2.1 Hypotheses and conventions

To simplify some calculations for radiative fluxes, the definition of wavenumber $\nu = \frac{1}{\lambda}$ is used in this work, where λ is the wavelenght.

Specific gas constant is used in thermodynamical relations, which is defined as the gas constant divided by the gas molar mass. In this work the symbol R is used for the specific gas constant.

Some assumptions are made on the dynamical state and the composition of Earth's atmosphere. Gravitational acceleration is constant (cfr. Table **??** for values). Air parcels are supposed to be in hydrostatic equilibrium,

$$\mathrm{d}P = -\rho g \,\mathrm{d}z \tag{1}$$

with each component obeying the ideal gas law

$$P = \rho R T \tag{2}$$

where $\rho = \frac{m}{V}$ is the volumetric mass density of the air parcel.

2.2 Longwave radiation

At wavelengths $\lambda \geq 4\,\mu\mathrm{m}$, solar radiation has lower intensities than radiation emitted by Earth's surface and atmosphere at the same wavelengths. Moreover, it presents negligible scattering in atmosphere with respect to absorption. For these reasons longwave radiation is considered to be emitted only by Earth's surface and atmosphere.[1, p. 468]

2.3 Shortwave radiation

At wavelengths $\lambda < 4\,\mu\mathrm{m}$, solar radiation has much greater intensity than radiation emitted from Earth's surface and atmosphere. Both scattering and absorption by gases, aerosols and clouds of atmosphere dissipates solar radiation.[1, p. 469]

Specific intensity of solar radiation can be expressed by a differential equation whose resolution is complex even applying approximations and numerical methods.[1, p. 469]

A lower complexity parametrisation is adopted instead, where the atmosphere is divided in a given number of layers and radiation is absorbed, scattered and reflected between each layes. Multiple reflections can occur from each layer but only one is considered in this model because successive reflections from amospheric layers have negligible intensities compared to the first one.[1, p. 470]

2.4 Numerical approach

Euler's method is used to solve the Ordinary Differential Equation (ODE) for the time-dependant temperature function.[1, p. 472] A solution for each atmospheric layer is evaluated, hence the resulting values are triplets of temperature, altitude (i.e. proxy for the atmospheric layer) and time (i.e. simulation time). Further information on storage and plotting of data are presented in Section B.1.

Spectral bands are identified by two arrays: one listing the lower bound of each band, the other containing the width of each band. This choice simplify the use of functions for numerical integration. Values related to spectral bands are stored as integers.

3 Results

3.1 Stability analysis

4 Discussion

A Source code

In this section the C++ code used to obtain the results presented in this work is shown and commented.

A.1 Classes

B Supplementary information

B.1 Plotting

Software Gnuplot is used to generate plots shown in this work. Output values from the simulation are stored in a DAT file with the following structure, line spacing between data blocks is important:

```
t[0]        P[0]        T[0][0]
...         ...         ...
t[0]        P[N_P - 1]  T[0][N_P - 1]

t[1]        P[0]        T[1][0]
...         ...         ...
t[1]        P[N_P - 1]  T[1][N_P - 1]

...

t[N_t - 1]  P[0]        T[N_t - 1][0]
...         ...         ...
t[N_t - 1]  P[N_P - 1]  T[N_t - 1][N_P - 1]
```

Value `N_t` is not known a priori since it is the number of temporal steps needed to reach convergence, instead `N_P` is the chosen number of atmospheric layers.

C Mathematical derivations

In this appendix mathematical derivations of some ancillary results and formulae used in the main text are explicitly shown.

C.1 Relation between pressure and altitude

A general result regarding planetary atmosphere is that atmospheric pressure decreases with increasing altitude. Theoretical relations which approximate this behaviour can be obtained. Hypotheses considered in Section 2.1 are valid.

If density is assumed constant, equation (1) can be solved easily resulting in a linear dependence of pressure P on altitude z,

$$P(z) = P_0 - \rho g(z - z_0) \quad , \tag{3}$$

where (z_0, P_0) is a reference point inside the atmosphere.

If density is not constant its expression is given by the ideal gas law (cfr. equation (2)) and, assuming constant temperature T, equation (1) results in:

$$\begin{aligned} &\mathrm{d}P = -\frac{Pg}{RT}\,\mathrm{d}z \iff \\ &\iff \frac{\mathrm{d}P}{P} = -\frac{g}{RT}\,\mathrm{d}z \impliedby \\ &\impliedby \ln(P')\Big|_{P_0}^{P(z)} = -\frac{g}{RT} z'\Big|_{z_0}^{z} \iff \\ &\iff P(z) = P_0 \exp\left(-\frac{g}{RT}(z - z_0) \right) \quad . \end{aligned} \tag{4}$$

This relation is not meaningful, since the aim of the work is to derive the non-constant temperature profile of the atmosphere. However, it can be used inside atmospheric layers where the temperature is considered constant (e.g. stratosphere).

A better approximation assumes non-constant density and constant lapse rate Γ, hence temperature depends linearly on altitude,

$$\Gamma = -\frac{\mathrm{d}T}{\mathrm{d}z} \impliedby T(z) = T_0 - \Gamma(z - z_0) \quad , \tag{5}$$

with T_0 temperature corresponding to reference altitude z_0. Using these assumptions and the density rewritten through the ideal gas law (2), equation (1) becomes

$$\begin{aligned} &\mathrm{d}P = -\frac{Pg}{RT}\left(-\frac{\mathrm{d}T}{\Gamma} \right) \iff \\ &\iff \frac{\mathrm{d}P}{P} = \frac{g}{R\Gamma}\frac{\mathrm{d}T}{T} \impliedby \\ &\impliedby \ln(P')\Big|_{P_0}^{P(z)} = \frac{g}{R\Gamma} \ln(T')\Big|_{T_0}^{T(z)} \iff \\ &\iff P(z) = P_0 \left(\frac{T_0 - \Gamma(z - z_0)}{T_0} \right)^{\frac{g}{R\Gamma}} \end{aligned} \tag{6}$$

Equation (6) can be used also with a piecewise constant lapse rate in altitude intervals where it is not null, otherwise equation (4) is valid with appropriate boundary conditions to ensure continuity.

References

[1] V. Ramanathan and J. A. Coakley Jr., "Climate modeling through radiative-convective models," *Reviews of Geophysics*, vol. 16, no. 4, pp. 465–489, 1978. DOI: https://doi.org/10.1029/RG016i004p00465. eprint: https://agupubs.onlinelibrary.wiley.com/doi/pdf/10.1029/RG016i004p00465. [Online]. Available: https://agupubs.onlinelibrary.wiley.com/doi/abs/10.1029/RG016i004p00465.