

Introduction

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Radiative-convective equilibrium in a grey atmosphere

└ Introduction

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- Average vertical temperature profile $T(t, z)$ of atmosphere.

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1. The analysed quantity is the atmospheric temperature profile averaged over all latitudes and longitudes.

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- Average vertical temperature profile $T(t, z)$ of atmosphere.
- Radiative Transfer Equation (RTE).

1. The analysed quantity is the atmospheric temperature profile averaged over all latitudes and longitudes.
2. RTE describes radiative processes.

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- Radiative Transfer Equation (RTE).

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- Average vertical temperature profile $T(t, z)$ of atmosphere.
- Radiative Transfer Equation (RTE).
- Fluid dynamics equations.

1. The analysed quantity is the atmospheric temperature profile averaged over all latitudes and longitudes.
2. RTE describes radiative processes.
3. Fluid dynamics equations describe convective processes.

- Average vertical temperature profile $T(t, z)$ of atmosphere.
- Radiative Transfer Equation (RTE).
- Fluid dynamics equations.

Hypotheses

- Thermodynamic energy equation in Local Thermodynamic Equilibrium (LTE):

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_P} \frac{\partial q}{\partial z} \quad . \quad (1)$$

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└ Hypotheses

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- Radiative-convective equilibrium.

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└ Hypotheses

1. Thermodynamic energy equation describes average vertical temperature profile.
2. The study is conducted on an atmosphere in radiative-convective equilibrium.

- Thermodynamic energy equation in Local Thermodynamic Equilibrium (LTE):

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_P} \frac{\partial q}{\partial z} \quad . \quad (1)$$

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- Radiative-convective equilibrium.
- Grey atmosphere.

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Radiative-convective equilibrium in a grey atmosphere

└ Introduction

└ Hypotheses

1. Thermodynamic energy equation describes average vertical temperature profile.
2. The study is conducted on an atmosphere in radiative-convective equilibrium.
3. Quantities do not depend on the frequency of electromagnetic radiation.

- Thermodynamic energy equation in Local Thermodynamic Equilibrium (LTE):

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_P} \frac{\partial q}{\partial z} \quad (1)$$

- Radiative-convective equilibrium.
- Grey atmosphere.

Additional hypotheses

- Hypotheses on the planet.

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└ Additional hypotheses

1. Diurnal cycle, constant irradiance, constant Bond albedo, surface emits blackbody radiation, constant gravitational acceleration.

- Hypothesis on the planet.

Additional hypotheses

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└ Additional hypotheses

- Hypotheses on the planet.
- Hypotheses on the composition of atmosphere.

1. Diurnal cycle, constant irradiance, constant Bond albedo, surface emits blackbody radiation, constant gravitational acceleration.
2. Hydrostatic equilibrium, constant specific heat at constant pressure, scattering is neglected, absorption coefficient depends only on altitude, constant mass attenuation coefficient, ideal gas.

- Hypothesis on the planet.
- Hypothesis on the composition of atmosphere.

Additional hypotheses

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└ Additional hypotheses

- Hypotheses on the planet.
- Hypotheses on the composition of atmosphere.
- Hypotheses on total heat flux.
- Resulting thermodynamic energy equation:

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_p} \frac{\partial}{\partial z} (E_U - E_D) \quad . \quad (2)$$

1. Diurnal cycle, constant irradiance, constant Bond albedo, surface emits blackbody radiation, constant gravitational acceleration.
2. Hydrostatic equilibrium, constant specific heat at constant pressure, scattering is neglected, absorption coefficient depends only on altitude, constant mass attenuation coefficient, ideal gas.
3. Heat flux determined only by radiative and convective processes, two-stream approximation, numerical correction for convection.

- Hypothesis on the planet.
- Hypothesis on the composition of atmosphere.
- Hypothesis on total heat flux.
- Resulting thermodynamic energy equation:

$$\frac{\partial T}{\partial t} = -\frac{1}{\rho c_p} \frac{\partial}{\partial z} (E_U - E_D) \quad . \quad (2)$$

Vertical coordinates

- Relation between pressure and altitude:

$$P(z) = P_g \exp \left(- \frac{z - z_g}{z_0} \right) . \quad (3)$$

- Relation between optical depth and pressure:

$$\delta(P) = \frac{\mu_m}{g} (P - P_{\text{TOA}}) . \quad (4)$$

- Relation between optical depth and altitude:

$$\delta(z) = \frac{\mu_m}{g} \left(P_g \exp \left(- \frac{z - z_g}{z_0} \right) - P_{\text{TOA}} \right) . \quad (5)$$

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└ Vertical coordinates

1. Obtained from hydrostatic equilibrium and ideal gas law.
2. Obtained from definition of optical depth, hydrostatic equilibrium and hypotheses on attenuation coefficient.
3. Obtained by combining relations (3) and (4).

- Relation between pressure and altitude:

$$P(z) = P_g \exp \left(- \frac{z - z_g}{z_0} \right) . \quad (3)$$

- Relation between optical depth and pressure:

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Equations in radiative equilibrium

- RTE for non-scattering medium in LTE:

$$\frac{1}{\mu} \frac{\partial L}{\partial z} = B_\nu - L \quad . \quad (6)$$

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Radiative-convective equilibrium in a grey atmosphere

└ Radiative equilibrium

└ Equations in radiative equilibrium

- RTE describes radiance in a medium.

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Equations in radiative equilibrium

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- Integration over frequency and solid angle.
- Diffusion approximation: $\delta' = D\delta$.

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Radiative-convective equilibrium in a grey atmosphere

└ Radiative equilibrium

└ Equations in radiative equilibrium

- RTE describes radiance in a medium.
- To describe irradiances, RTE is integrated over the whole spectrum of radiation and over the solid angle of a hemisphere.

- RTE for non-scattering medium in LTE:

$$\frac{1}{\mu} \frac{\partial L}{\partial z} = B_\nu - L \quad . \quad (6)$$

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Equations in radiative equilibrium

- RTE for non-scattering medium in LTE:

$$\frac{1}{\mu} \frac{\partial L}{\partial z} = B_\nu - L \quad . \quad (6)$$

- Integration over frequency and solid angle.
- Diffusion approximation: $\delta' = D\delta$.
- Equations for irradiances:

$$-\frac{\partial}{\partial \delta'} E_U(t, \delta') = \sigma T(t, \delta')^4 - E_U(t, \delta') \quad , \quad (7)$$

$$\frac{\partial}{\partial \delta'} E_D(t, \delta') = \sigma T(t, \delta')^4 - E_D(t, \delta') \quad . \quad (8)$$

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Radiative-convective equilibrium in a grey atmosphere

└ Radiative equilibrium

└ Equations in radiative equilibrium

1. RTE describes radiance in a medium.
2. To describe irradiances, RTE is integrated over the whole spectrum of radiation and over the solid angle of a hemisphere.
3. Equations for irradiances and temperature are written in terms of δ' .

- RTE for non-scattering medium in LTE:

$$\frac{1}{\mu} \frac{\partial L}{\partial z} = B_\nu - L \quad . \quad (6)$$

- Integration over frequency and solid angle.

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- Steady state.

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Radiative-convective equilibrium in a grey atmosphere

- Radiative equilibrium

└ Initial Value Problem

1. Analytical solutions are found for temperature at the steady state.

- Steady state.
- Initial conditions on irradiances:

$$E_D(0) = 0 \quad , \quad (9)$$

$$E_U(0) = S_t \quad . \quad (10)$$

1. Analytical solutions are found for temperature at the steady state.
2. At TOA downward irradiance does not deposit energy and radiative equilibrium fixes upward irradiance, i.e. outgoing longwave radiation.

Initial Value Problem

- Steady state.
- Initial conditions on irradiances:

$$E_D(0) = 0 \quad , \quad (9)$$

$$E_U(0) = S_t \quad . \quad (10)$$

- Additional relations:

$$\frac{d}{d\delta'} (E_U(\delta') - E_D(\delta')) = 0 \quad , \quad (11)$$

$$E_U(\delta') - E_D(\delta') = S_t \quad . \quad (12)$$

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Radiative-convective equilibrium in a grey atmosphere

└ Radiative equilibrium

└ Initial Value Problem

- Analytical solutions are found for temperature at the steady state.
- At TOA downward irradiance does not deposit energy and radiative equilibrium fixes upward irradiance, i.e. outgoing longwave radiation.
- Hypotheses of atmosphere in radiative equilibrium at all altitudes and atmosphere transparent to radiation coming from outside the planet are equivalent to hypotheses of atmosphere in radiative equilibrium at TOA and steady state.

• Steady state.

• Initial conditions on irradiances:

$$E_D(0) = 0 \quad , \quad (9)$$

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Initial Value Problem

- Steady state.
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- Initial condition on temperature:

$$T(0) = \left(\frac{S_t}{2\sigma} \right)^{\frac{1}{4}} \quad . \quad (13)$$

Radiative-convective equilibrium in a grey atmosphere

└ Radiative equilibrium

└ Initial Value Problem

- Analytical solutions are found for temperature at the steady state.
- At TOA downward irradiance does not deposit energy and radiative equilibrium fixes upward irradiance, i.e. outgoing longwave radiation.
- Hypotheses of atmosphere in radiative equilibrium at all altitudes and atmosphere transparent to radiation coming from outside the planet are equivalent to hypotheses of atmosphere in radiative equilibrium at TOA and steady state.
- Initial condition on temperature is obtained by combining all conditions on irradiances.

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- Steady state.
- Initial conditions on irradiances:

$$E_D(0) = 0 \quad , \quad (9)$$

$$E_U(0) = S_t \quad . \quad (10)$$
- Additional relations:

$$\frac{d}{d\delta'} (E_U(\delta') - E_D(\delta')) = 0 \quad , \quad (11)$$

$$E_U(\delta') - E_D(\delta') = S_t \quad . \quad (12)$$
- Initial condition on temperature:

$$T(0) = \left(\frac{S_t}{2\sigma} \right)^{\frac{1}{4}} \quad . \quad (13)$$

Numerical solution

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└ Radiative equilibrium

└ Numerical solution

Radiative-convective equilibrium in a grey atmosphere

Marco Casari

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Radiative equilibrium

Radiative-convective equilibrium

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Radiative-convective equilibrium

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└ Radiative-convective equilibrium

└ Radiative-convective equilibrium

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