Sensitivity analysis of climate change risk assessment

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Contents

Chapter 1

The necessary background

1.1 Introduction

Climate risk assessment is becoming central in contemporary activities related in any way to the environment and whose assets could be affected by climate change. In particular, climate change risk assessment is a topic more and more organizations are considering in their decisions.

A climate change risk assessment for a given system is the analysis of the impacts of and the responses to climate change regarding that system. Various guidelines are available for these kind of risk assessments (cf. [**?**, **?**, **?**, **?**]) and slight variations of them are adopted by authors, but they do not specify precisely the practical details of the assessment. In particular, there is no objective method to choose the climate indicators used in the assessment, but they are selected according to their effectiveness in scientific literature and in previous assessments, combined with the personal experience of the authors. The choice of the indicators is far from objective.

1.1.1 Climate risk

But what is risk? Before introducing the methodology adopted to evaluate risk in the present work, first it is convenient to introduce a proper terminology. In this work definitions by International Organization for Standardization (ISO) are used for their concision. When some terms are not available, they are taken from Intergovernmental Panel on Climate Change (IPCC). Both sources have similar definitions for the same terms.

Risk is a general term which can be tailored to different contexts and applications as a measure of uncertain consequences on a system of interest.[1] A system is very broadly any concrete or abstract entity which can be affected by risk.

[1] Without delving into Philosophy, a source of change is needed to have consequences and it is specified by the definitions in use.

Example

Some possible systems which can be exposed to risks are any physical system, communities of people, an idea.

A paradigmatic example is the financial sector, where the concept of risk is widely known and is connected directly to economic value and the concept of portfolio, to the point that financial risk management can be considered a research field itself.[**?**] Examples on how other fields implement the concept are elaborated in [**?**, 14] and in [**?**].

In this work the risk related to climate change is: "effect of uncertainty", from [**?**].[2] IPCC proposes a similar definition, expanding on the entities involved (e.g. the possible systems) and the contexts in which the term is used, but focusing only on negative effects: "effect of uncertainty", from [**?**]. An important aspect of climate risk is that it originates both from impact of climate change, i.e. "effect on natural and human systems (3.3)", from [**?**], and any response to it, i.e. action enact to mitigate the effects of climate change or adapt to it. It is not common to see responses integrated into risk assessment, as exposed by [**?**, 492], and for the purposes of the present study they are neglected. Henceforth, terms climate risk and risk are used interchangeably.

To make the assessment easily extensible and modular, risk is defined as the result of the interaction of three elements, i.e. its determinants, namely hazard, exposure and vulnerability. Response is considered the fourth determinant of risk, when the adopted methodology includes it in the assessment. Definitions of determinants were introduced in [**?**, 69-70] and offer a change of direction from previous methodologies centered on the concept of vulnerability of the system instead of the overall risk (cf. [**?**] and [**?**]).

The hazard is defined by ISO as "potential source of harm", from [**?**] and is elaborated further by IPCC. In the following the term hazard is used to address to climate-related hazards without further specification. In [**?**, 2224] IPCC provides the term climatic impact-driver (CID) to address to climate-related physical phenomena with neutral effects (cf. [**?**, 10] or [**?**, 1871]). In other words, among the CIDs which can affect the system, only some may be regarded as hazards, depending on the risk assessed. Hazards used in the present work are selected among the taxonomy provided by European Union for climate change risk assessment (CCRA), to have a well-known and authoritative reference in the field.[**?**, 177]

The exposure of a system is determined by "presence of people, livelihoods, species or ecosystems, environmental functions, services, resources, infrastructure, or economic, social or cultural assets in places and settings

[2]Note that this definition is not specific to climate risk since no reference to climate is made.

that could be affected", from [?].

The vulnerability of a system is "propensity or predisposition to be adversely affected", from [?]. Properties of the system which determine its vulnerability may be classified further in sensitivity, i.e. "degree to which a system (3.3) or species is affected, either adversely or beneficially, by climate (3.4) variability or change", from [?], and adaptive capacity, i.e. "ability of systems (3.3), institutions, humans, and other organisms to adjust to potential damage, to take advantage of opportunities, or to respond to consequences", from [?]. This classification in general helps the analysis of the system and the identification of responses, e.g. adaptation measures may increase the adaptive capacity of some elements of the system.

Each determinant may be viewed as a collection of elements, which are of different nature depending on the determinant they belong to, but are addressed generically as drivers.[3] Physical elements of the system may be effectively considered as Drivers of this determinant.

Example

A tropical storm is a driver within the hazard determinant,[?, 15] income is a driver within the vulnerability determinant,[?, 493] airport structures (e.g. runways, aprons, terminals) in an airport (i.e. the system) are drivers within the exposure determinant.[?, 551] More examples are available in the references.

The concept of driver of risk is borrowed from [?] to allow a smooth extension to methodologies where risk is the result of complex interactions within and across determinants. In section 1.1.3 this topic is described further.

For a quantitative CCRA, numerical values must be associated to drivers. These values are called indicators and defined by ISO as: "quantitative, qualitative or binary variable that can be measured or described, in response to a defined criterion", from [?].[4] There can be more than one way to describe numerically the same driver, hence the choice is not unique and the resulting risk may be affected by it. In the following, the term indicator written alone refers to an indicator of a driver within the hazard determinant, to relax the lenghty wording. For the other determinants the full qualification is used.

Having introduced the definitions above, the various components of risk can be arranged as in figure 1.1. It sums up the relation between the various components, highlightning the fact that risk depends on drivers from three independent categories and are quantified possibly in multiple ways.

[3] When this terminology is not applied, it is common to refer to drivers with the name of the determinants they belong to, e.g. drivers within the vulnerability determinant are simply called vulnerabilities.

[4] The definition by IPCC is not as general because focuses only on the climate system and there is no specific term for the same concept applied to the other determinants.

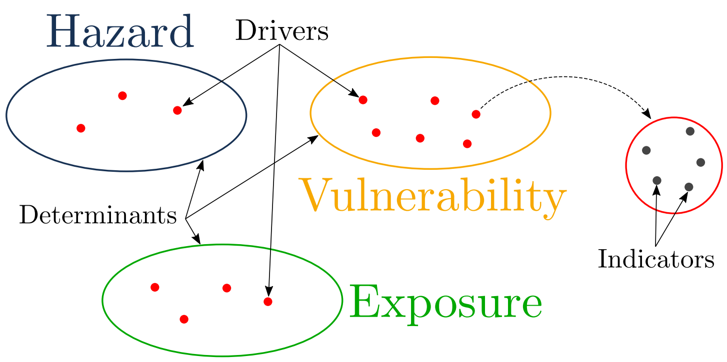


Figure 1.1: A possibile representation of the components of climate risk. Climate hazards can affect exposed and vulnerable elements of the system and determine a risk for it. Collectively these factors are called drivers and can grouped in the three independent determinants of risk: hazard, exposure and vulnerability. To provide quantitative results, each driver is described by numerical values, i.e. the indicators, each providing a different possible description or measure of the same driver.

1.1.2 Methodology

Restricting the treatise to climate-related applications is not sufficient to fix details on risk, e.g. how to evaluate it from its determinants. These implementation details depend on which methodology is chosen to perform the risk assessment, which is presented in this section and is defined operatively in section 2.1.

The methodology of CCRA applied in the present work follows [**?**] and its upgrade [**?**] to the concepts presented in section 1.1.1. The latter should be read in parallel with the former and superseds the outdated concepts.

This methodology is split in eight modules, each dependent on the previous ones. The following is an overview of them:

1. understand the context in which the assessment is framed and identify objectives, scope and resources involved;[**?**, 39-53]

2. identify risks and impacts affecting the system under study and determine drivers of hazard, exposure and vulnerability;[**?**, 26-41]

3. choose indicators for each driver of hazard, exposure and vulnerability;[**?**, 73-84]

4. collect data and quantify indicators;[**?**, 87-103]

5. normalise indicators to allow their comparison;[**?**, 105-119]

6. for each determinant, weight normalised indicators and aggregate them into a single value;[**?**, 121-131]

7. aggregate values for individual determinants into a single value for risk;[**?**, 133-141]

8. present the results of the CCRA.[**?**, 143-154]

When the vulnerability of the system is recalled, it is split into sensitivity and adaptive capacity if possible.

Even if a complete application of this methodology does not fall into the purposes of the present study, each module is briefly addressed in section **??** when case studies are treated.

1.1.3 Complex risk

Unlike other types of risk assessment, e.g. probabilistic risk assessment, CCRA focuses on interactions between drivers instead of estimating their likelihood.[**?**, 20-21] However, it is common to study each determinant in isolation. Instead, an integrated risk assessment which is able to relate drivers within and across determinants would be able to describe the overall risk more accurately.[**?**, 145-147]

Interacting elements are not considered mainly because it is more difficult to formalise the interactions. In [**?**] three categories of complex risk are proposed to build a framework which helps to address to complex interactions more easily, thus helping their adoption in CCRAs. Depending on what is the origin of risk, the categories are:[**?**, 493]

1. interacting drivers within the same determinant;

2. interacting drivers across different determinants;

3. interacting risks.

Response is taken into consideration, e.g. to introduce negative effects on vulnerability due to maladaptation. The list of complex risk adopted by IPCC in Sixth Assessment Report (AR6) is extended, to allow more granularity in the assessment.

Example

Flood risk in a geographical area is assessed. First, only artificial constructions are considered as system. The change in time of precipitation and temperature, i.e. drivers of risk within the hazard determinant, are aggregated to give a measure of the hazard. With this value and the analogous values of the other determinants, a category 1 risk can be evaluated, which measures the interaction of its determinants only defined by the methodology and lacking a particular meaning.
A collateral change in soil properties due to precipitation and temperature is added to the study. This results in a decrease of soil adaptive capacity, hence an increase of the vulnerability of buildings in that area. This interaction belongs to category 2.

A second iteration of the CCRA moves the focus to human activities in the area under study. When the corrispondent risk is evaluated, it can be merged with the risk value found previously to summarise the overall flood risk, which becomes a category 3 complex risk.

The methodology adopted in this work falls into category 1, as there is no particular relation between drivers within different determinants, except for the usual aggregation defined in module 7 needed to obtain a value for risk. Nevertheless, extending the current study by introducing complexity throught the other categories may be interesting to test the robustness of the results.

From a mathematical point of view, complex interactions between drivers translate to mathematical functions which relate indicators. They may be treated as additional indicators to consider in the aggregation of determinants.[**?**, 39-40] In the methodology used in this work, no particular interaction is considered between drivers across determinants. Indicators are related only by the functions which are chosen in the aggregation procedures of modules 6 and 7.

1.1.4 Structure of the document

The landview of terms and definitions used in climate change risk assessment is varied and this may cause confusion. For the sake of clarity, definitions are provided, along with the sources they are taken from. If no specification of the source is present, the definition is assumed to be taken from [**?**] or [**?**]. Terms which are present in both sources have equivalent definitions.

Definitions of terms used in the document are collected in the Glossary and are reachable by hyperlinks directly from the text in the digital version of the document.

1.2 Data

Climate data show great complexity in structure and availability (e.g. essential climate variables (ECVs) can be represented as multidimensional objects, some climate datasets are collections of ECVs). For these and other properties discussed in [**?**], climate data can be regarded as big data.

In this work a generic ECV T can be represented mathematically as a scalar function

$$T : S_{\text{lat}} \times S_{\text{lon}} \times S_{\text{time}} \to \mathbb{R} \tag{1.1}$$

where S_{lat}, S_{lon} and S_{time} are domains of latitude, longitude and time dimensions, respectively.[5] Every numerical value is equipped with proper units of

[5] In contexts related to Machine Learning these objects are called tensors. Since they

measurement, to represent physical quantities correctly. As a consequence, the codomain in equation (1.1) is partially wrong: with an abuse of notation, it represents only the magnitude of the ECV and does not consider the unit of measurement. This is a small exception to simplify the notation and in the remainder of this document units of measurement are always addressed explicitly.

A more practical representation of T is a multidimensional array, where values in the domain are coordinates associated to each dimension and each entry of the array is the result of T evaluated on those coordinates. Figure 1.2 shows this representation visually. In the following, this representation is

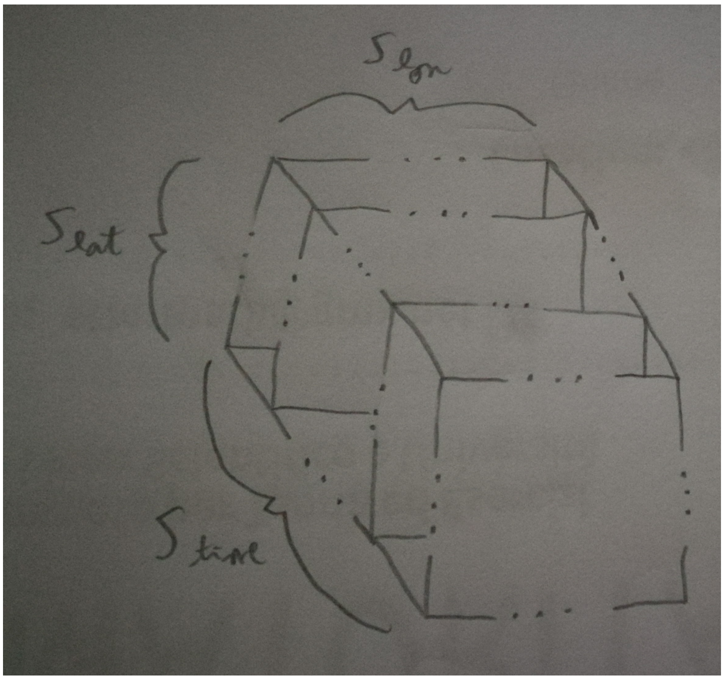


Figure 1.2: Representation of a generic ECV as multidimensional array.

used to simplify the discussion and same ECV name is used both for the function and the multidimensional array.

Example

Near-Surface Air Temperature, symbol tas, is available for some coordinates and timestamps. It can be seen as the scalar function in equation (1.1), which associates each value in set $S_{\text{lat}} \times S_{\text{lon}} \times S_{\text{time}}$ to a value with unit K, or it can be represented as the multidimensional

may not satisfy the mathematical definition of a tensor, in particular the map may not be multilinear and the numerical sets may not be vector spaces, no reference to such objects is made in this work.

array in figure 1.2, where each entry is function (1.1) evaluated at the corresponding coordinates.

As a visual aid when generic symbols are used, capital letters represent both functions and multidimensional arrays, the former being followed by the arguments in parentheses when there is an explicit reference to their values. Instead, lowercase letters are used for functions and values which are one dimensional.

Normals used as reference depends only on spatial coordinates, hence they are functions

$$\bar{T} : S_{\mathrm{lat}} \times S_{\mathrm{lon}} \to \mathbb{R} \tag{1.2}$$

or equivalently, multidimensional arrays with spatial coordinates only. Normals are evaluated as explained in [**?**, 6] using an averaging period S_{clim} specific to each case study (cf. section **??**).

Indicators of drivers within the hazard determinant are functions of ECVs and additional parameters. In general they are aggregated over the temporal dimension and are non-linear functions of their arguments. An indicator I can be defined mathematically as

$$I : S_{\mathrm{lat}} \times S_{\mathrm{lon}} \times S_{\mathrm{y}} \times \prod_{p \in P_I} S_p \to \mathbb{R} \tag{1.3}$$

where S_{y} is a set of the years considered during the analysis, P_I is the set of parameters for that indicator and S_p is the set of values available for each parameter $p \in P_I$.[6] In this work their evaluation is performed for each year, i.e. the indicator has yearly resolution or is evaluated with yearly frequency. As a consequence, the elements of set S_{y} are references to the years taken into account during calculations.

An indicator can be represented as a multidimensional array, similarly to ECVs. The dependence of an indicator on ECVs is not clear in the definition given by equation (1.3), but in the following this is made explicit by the context or by the definition of the indicator.

Example

The indicator $\mathrm{TX_x}$ is evaluated for the period 1991-2020 with yearly frequency. This indicator is the monthly maximum value of daily maximum temperature,[**?**] hence:

- it depends on ECV tasmax defined at daily frequency over the considered period,

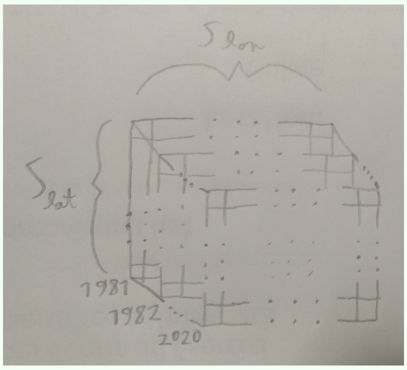
$$S_{\mathrm{time}} = \left\{ t : \begin{array}{l} t \text{ day from 1st January 1991} \\ \text{to 31st December 2020} \end{array} \right\} \quad ;$$

[6] As a symbolic shortcut, if $P_I = \varnothing$ then the indicator is defined only over $S_{\mathrm{lat}} \times S_{\mathrm{lon}} \times S_{\mathrm{y}}$.

- spatial dimensions are not specified, hence the evaluation is performed for each point of an arbitrary set $S_{\mathrm{lat}} \times S_{\mathrm{lon}}$;
- no additional parameters are required, $P_{\mathrm{TX_x}} = \varnothing$;
- the outcome is a scalar value for each year in the period,

$$S_{\mathrm{y}} = \{1991, 1992, \ldots, 2020\} \quad ;$$

- the multidimensional array representation of the indicator is in the following figure, where each entry is a real value with unit K:



Indicators of drivers within exposure and vulnerability determinants may be defined similarly as the hazard determinant as scalar functions depending on specific variables characterising the system.

Chapter 2

The math behind risk

2.1 Methods

2.1.1 Evaluation of indicators

This section explains the process to obtain a scalar value for each determinant, following the methodology presented in section 1.1.2. Modules 3, 4, 5 and 6 are recalled. Suppose a set of drivers is chosen. The selection of drivers for the actual case studies, i.e. module 2, is performed and argued in section **??**.

Module 3 may be stated symbolically as follow. For each driver within the hazard determinant, a set $\mathcal{H}_j$ of indicators is obtained. A similar procedure is carried out for drivers of exposure and vulnerability, resulting in sets $\mathcal{E}_i$ and $\mathcal{V}_k$, respectively. Indices i, j and k are present to clarify that each set is related to a different driver and the way these sets are indexed is not important (e.g. each of them can be a sequence of integers where each one refers to a different driver, they can be the names of the drivers they refer to). If the distinction between drivers of sensitivity and adaptive capacity is made, then it is not explicit in the indices. The reason is that these drivers are treated mathematically the same way, as explained below.

Note that in module 3 no evaluation is performed yet, hence the elements of each set are just scalar functions. In other words, they are just descriptions of how they quantify the driver.

The methodology suggests to avoid double counting of drivers by allocating each of them in one determinant only.[**?**, 29] This implies to have different indicators, even if they are defined in a similar way.

Example

The risk of water scarcity affecting a location is assessed. This example is inspired by [**?**, 46]. The system is the location under study.

One driver of exposure and two of vulnerability are chosen, they are respectively:

e1 presence of farmers in the region;

v1 insufficient know-how about irrigation systems;

v2 weak institutional setting for water management.

Both drivers of vulnerability describe the adaptive capacity of the system. All drivers are conveniently identified by lables.
An indicator for each driver is chosen, the resulting sets are:

$$\begin{aligned}
\mathcal{E}_{\text{e1}} &= \left\{\text{“number of farmers in the region”}\right\} ,\\
\mathcal{V}_{\text{v1}} &= \left\{\text{“number of farmers trained in improved irrigation techniques”}\right\} ,\\
\mathcal{V}_{\text{v2}} &= \left\{\text{“number of local water co-operations”}\right\} .
\end{aligned}$$

Note that all indicators consist in counting some elements of the system. Nevertheless, they are different from each other since they refer to different elements.

In module 4 the evaluation of indicators on climate and system data occurs. Concerning the system, the data collection step is explained in section **??** and scalar values are readily obtained for the elements in sets $\mathcal{E}_i$ and $\mathcal{V}_k$ for any driver i or k. See paragraphs about data in section **??** for the definition of the indicators of exposure and vulnerability used to describe each system.

A more convoluted path is needed to evaluate indicators of hazard. They are functions defined by equation (1.3), hence they depend on climate data and additional parameters. The former are presented in section 1.2, the latter are chosen as explained in the following paragraphs.

For each indicator, a set of values of its parameters is defined. Ideally these sets would be continuous, to explore the whole space of possible configurations of parameters. However, for limitations intrinsic to the analysis tools, only a finite and small number of values can be considered (in the following these discrete sets are called intervals to preserve generality). How values in these intervals are selected depends on the nature of the parameters. The selection is ultimately arbitrary, to allow greater control, e.g. remove values which are not interesting for the analysis, and to apply a form of non-parametric sampling.

To simply the explanation, consider driver j and an indicator $I \in \mathcal{H}_j$. This indicator depends on some parameters, which are collected in set P_I, and on some ECVs. The chosen interval for a parameter $p \in P_I$ is a set of scalar values, possibly with units, denoted by S_p.

If parameter p is related to a ECV (e.g. threshold on tas), its values are sampled from the distribution of the ECV. First all data available for the ECV of interest are collected in a single sample, with the following conditions:

- temporal coordinates belong to the averaging period S_{clim} chosen for the normals;
- spatial coordinates are ignored.

This procedure has the side effects of removing the dependence of parameter values from spatial and temporal coordinates and to increase the sample size. The sampling is not affected by existing spatial correlation between data, because it is non parametric and regards only the possible values of the ECV and not their spatial distribution. In fact, the probability of having any value v for the ECV T in the sample is

$$\mathcal{P}(v) = \frac{1}{|S_{\text{lat}}||S_{\text{lon}}||S_{\text{clim}}|} \sum_{y \in S_{\text{lat}}} \sum_{x \in S_{\text{lon}}} \sum_{t \in S_{\text{clim}}} \mathbb{1}[T(y,x,t) = v] \tag{2.1}$$

This is a frequentist probability and gets more accurate the larger the sample size is, by definition. Second, the empirical quantile function (QF) of data is built from the sample, to acknowledge the shape of their true probability distribution. Minimum and maximum values are treated as the first and last quantiles, respectively. Since the number of data is large but limited, this curve is an approximation of the inverse function of the cumulative distribution function (CDF) and missing data are interpolated linearly. Third, values are chosen with the aim to sample the true probability distribution uniformly. The density of points may be increased where needed to have a better description of the shape of the distribution.[1]

If parameter p is not related to a ECV (e.g. window size for moving averages), some heuristic is applied and explained case by case where values are presented in section **??**.

After S_p is defined for every $p \in P_I$, the indicator I is finally evaluated. This results in elements of set $\mathcal{H}_j$ being multidimensional arrays given by equation (1.3), for every driver j of the hazard. Different indicators may have different parameters, but they depend on ECVs which same spatial and temporal coordinates $S_{\text{lat}} \times S_{\text{lon}} \times S_{\text{clim}}$, hence they have same temporal frequency, i.e. same S_{y}.

[1] Why do not use derivative-based methods to set the density of points? The reason is again greater control on the selected values: there is no need to evaluate the derivative of the QF in every point, just within the subintervals which are interesting for the analysis. Note that the QF is obtained by linear interpolation between existent data points, hence the slope is already evaluated internally and the derivative is a piecewise constant function.

These results need further elaboration. In fact, for every driver i and k, $\mathcal{E}_i$ and $\mathcal{V}_k$ contain scalar values which encapsulate information about the system for the chosen temporal and spatial coordinates. To obtain an analogous result for drivers of hazard, indicators are aggregated over spatial and temporal dimensions. The spatial aggregation is performed by using empirical orthogonal function (EOF) analysis as a dimensionality reduction technique. [2] Consider any indicator I in its multidimensional array representation, then the following procedure is applied for each element in $\prod_{p\in P_I} S_p$:

1. assign $S_{\mathrm{lat}} \times S_{\mathrm{lon}}$ as the feature dimension of the design matrix;
2. assign S_{y} as the sample dimension of the design matrix;
3. apply the EOF analysis to the design matrix;
4. keep the first PC only, i.e. the coefficients corresponding to the EOF which maximises the variance in the sample dimension.

This procedure returns a time series for each combination of parameter values in $\prod_{p\in P_I} S_p$, effectively mapping I to a multidimensional array with coordinates in $S_{\mathrm{y}} \times \prod_{p\in P_I} S_p$. Next, the temporal aggregation is performed. It consists in averaging the multidimensional array over the temporal dimension with a sample average. After these aggregations, for every j each element $I \in \mathcal{H}_j$ depends only on parameter values in $\prod_{p\in P_I} S_p$.

To execute module 5, first the scale of each indicator is defined. In this work all indicators are numeric values and may have both metric or categorical scales (i.e. values are distributed uniformly or not, respectively, cf. [**?**, 109]). When metric scales are involved, the methodology suggests to apply the min-max normalisation. Instead, in the present work indicators which are scalar values are not transformed,[3] while indicators which depend on parameters are standardised, i.e. substituted by their z-score. More in detail, for any indicators I undergoing normalisation, the new value is

$$\frac{I-\mu_I}{\sigma_I} \tag{2.2}$$

where the sample mean

$$\mu_I = \frac{1}{\prod_{p\in P_I}|S_p|} \sum_{\underline{x}\in\prod_{p\in P_I} S_p} I(\underline{x}) \tag{2.3}$$

[2]Terminology is varied. Here the eigenvectors, i.e. spatial patterns, are referred to as EOFs and their coefficients, i.e. temporal patterns, are the principal components (PCs). The object containig data is called design matrix with samples, i.e. observations, organised in rows and their features, i.e. variables which describe them, in columns. For further clarification on the terminology see [**?**, 626-627] and for a recap on EOF analysis see [**?**, 6502-6503] and [**?**, 1121-1122].

[3]Formally they are divided by a unit value of their quantity to obtain dimensionless values, which are trivially compatible and easily used as arguments in mathematical functions.

and the sample standard deviation

$$\sigma_I = \sqrt{\frac{1}{\prod_{p\in P_I}|S_p| - 1} \sum_{\underline{x}\in\prod_{p\in P_I} S_p} \big(I(\underline{x}) - \mu_I\big)^2} \tag{2.4}$$

are calculated using values obtained for the reference period S_{clim}.[?, 84] To simplify the notation, $\underline{x}$ represents the set of arguments passed to the indicator, in other words the coordinates of I as multidimensional array. The advantage of equation (2.2) with respect to min-max normalisation is to be flexible when new values are introduced, in fact they are measured in terms of statistics of the reference period without breaking the normalisation.[4] To normalise categorical scales, the methodology suggests first to group the values in five classes, then replace them with specific values in the range $[0, 1]$, see [?, 115-116]. Higher normalised values are associated to more negative impacts. However, this case does not occur in the present study since every indicator with categorical scale is a constant value. Not normalising some indicators may seem wrong since normalisation is a requisite to compare them, but the reason is explained in the next paragraphs.

For module 6, the weight w_I of each indicator I is set to 1, because no particular influence on the final risk is known a priori. Since indicators are unique, there is not ambiguity to identify their weights with their names as labels. This choice of weighting has not effects on the final risk value, as explained in section 2.1.2. Then, for each determinant the weighted mean of its indicators is computed. Since all indicators are weighted equally, the result equals the arithmetic mean. In [?, 51] this process is represented graphically with a single indicator for each driver. The results are a scalar value for exposure

$$E = \frac{1}{\sum_i \sum_{I\in\mathcal{E}_i} w_I} \sum_i \sum_{I\in\mathcal{E}_i} w_I I \quad , \tag{2.5}$$

a scalar value for vulnerability

$$V = \frac{1}{\sum_k \sum_{I\in\mathcal{V}_k} w_I} \sum_k \sum_{I\in\mathcal{V}_k} w_I I \tag{2.6}$$

and a scalar function for hazard $H : \prod_j \prod_{I\in\mathcal{H}_j} \prod_{p\in P_I} S_p \to \mathbb{R}$, which depends on all parameters, defined as

$$H(\underline{x}) = \frac{1}{\sum_j \sum_{I\in\mathcal{H}_j} w_I} \sum_j \sum_{I\in\mathcal{H}_j} w_I I(\underline{x}_I) \tag{2.7}$$

where $\underline{x}_I$ is the sequence of values for parameters which are arguments of I. Note that the there is no need to treat drivers of adaptive capacity and sensitivity separately because the aggregation procedure is applied equally to them.

[4] See section A.1 for further insight on why min-max normalisation is not used.

2.1.2 Evaluation of risk

Scalar values representing each determinant of risk are aggregated into a single value. The aggregation procedure is a weighted mean, see module 7, and the weights assigned to each determinant are $w_E = 1$, $w_H = 1$ and $w_V = 1$. Then, analogously to the aggregation of hazard indicators in equation 2.7, the value for risk is a scalar function $R : \prod_j \prod_{I \in \mathcal{H}_j} \prod_{p \in P_I} S_p \to \mathbb{R}$ which depends on all parameters:

$$R(\underline{x}) = \frac{w_E E + w_H H(\underline{x}) + w_V V}{w_E + w_H + w_V} \quad . \tag{2.8}$$

The risk value is a linear function of normalised hazard indicators. This can be seen easily by manipulating equation (2.8),

$$R(\underline{x}) = c_0 + \frac{w_H H(\underline{x})}{w_E + w_H + w_V} = c_0 + c_1 \sum_j \sum_{I \in \mathcal{H}_j} w_I I(\underline{x}_I) \tag{2.9}$$

with $c_0 = \frac{w_E E + w_V V}{w_E + w_H + w_V}$ and $c_1 = \frac{w_H}{(w_E + w_H + w_V) \sum_j \sum_{I \in \mathcal{H}_j} w_I}$, and it holds true as long as the aggregation procedures for risk and hazard are linear.

One of the problems which justifies this work can be stated using equation (2.8): given different choices of indicators or parameters, the resulting risk value can be the same, as long as the differences in values balance out.

Example

Risk is assessed for a system, E and V are known and only one driver of hazard is considered. Two sets of different indicators are prepared, $\mathcal{H}'$ and $\mathcal{H}''$, functions may differ only in the values of their parameters.These alternatives would be equivalent to describe the driver mathematically from different points of view.
Even if all indicators have different values after the calculation steps, the aggregated values of hazard H' and H'' for sets $\mathcal{H}'$ and $\mathcal{H}''$, respectively, are ideally equal. The same same risk value R results from the aggregation, since E and V do not depend on the choices regarding the hazard. This is the expected outcome, because changing the mathematical description of the physical phenomenon should not change the final risk.

A final non-linear transformation is performed on risk value R, to simplify the presentation and comparison of risk values. Values obtained from all the combinations of parameters are classified accordingly to five categories, in increasing order of severity:[**?**, 53]

1. very low;

2. low;

3. intermediate;

4. high;

5. very high.

This transformation can be formalised as a piecewise function $r : \prod_j \prod_{I \in \mathcal{H}_j} \prod_{p \in P_I} S_p \rightarrow \{\text{very low}, \text{low}, \text{intermediate}, \text{high}, \text{very high}\}$, where the thresholds for each piece are the quantiles of the image of $\prod_j \prod_{I \in \mathcal{H}_j} \prod_{p \in P_I} S_p$ through function R for the reference period:

$$r(\underline{x}) = \begin{cases} \text{very low} & R(\underline{x}) < q_1 \\ \text{low} & q_1 \leq R(\underline{x}) < q_2 \\ \text{intermediate} & q_2 \leq R(\underline{x}) < q_3 \\ \text{high} & q_3 \leq R(\underline{x}) < q_4 \\ \text{very high} & R(\underline{x}) \geq q_4 \end{cases} . \tag{2.10}$$

Thresholds q_1, q_2, q_3 and q_4 are calculated from risk values of the reference period because hazard drivers are supposed to change risk and impacts with time. Moreover, the pieces of the function reflect the fact that the image of R is not bounded, due to the aggregated values of determinants being not bounded by the chosen normalisation. This allows to account for extreme values of risk in periods different from the reference.

Module 7 address the possible aggregation of multiple sub-risks to an overall risk value.[**?**, 54] This task is not pursued because a single risk is studied in this work, i.e. the climate risk, and because it should be treated as a complex risk belonging to category 3, as discussed in section 1.1.3.

Appendix A

The supplementary information

A.1 Min-max normalisation

When min-max normalisation is applied to a given indicator, it is rescaled to the $[0, 1]$ interval, with higher values associated to more negative impacts. If multiple temporal periods are involved, one is chosen as reference for the extreme values.[?, 85]

There is the implicit assumption is that the image of the indicator is bounded and its extremes are known. This is not always true, hence the extremes must be set in alternative ways (e.g. by discussing with experts, by consulting the literature, by analysing the system), see [?, 113-115].

Once the extremes are found, the min-max normalisation of an indicator can be evaluated easily. Denote its image as $X \subset \mathbb{R}$ and the extremes as $x_{\max} = \max X$ and $x_{\min} = \min X$. Then the min-max normalisation applied to $x \in X$ is

$$\frac{x - x_{\min}}{x_{\max} - x_{\min}} \quad . \tag{A.1}$$

This procedure is not chosen because of two downsides:

1. In the present work a single system at a time is analysed, not multiple ones. This means that a single value representing the system exists for any given indicator and trivially it corresponds to its minimum and maximum values, making the outcome of the normalisation undefined.

2. In periods different from the reference there may be values of the indicator which exceed the extremes. This results in normalised values which fall outside the interval $[0, 1]$, invalidating the purpose of the normalisation.

Examples where min-max normalisation is applied as suggested by the methodology are: [?, 6], [?, 6] and [?, 74].

A.2 Alternative aggregation for risk

Some methodologies suggest to evaluate risk as product of the aggregated values of exposure, hazard and vulnerability:

$$R = E\,H\,V \quad . \tag{A.2}$$

This procedure is used in contexts preceding the concept of risk introduced in [**?**] and it is still used in later articles (see [**?**, 7] and [**?**, 6]).

This procedure is used because it admits a null risk value, i.e. negligible impacts, when either of the values for its determinant is null. This reasoning is intuitive, but hides the request that values for determinants have exactly value zero when their impacts are negligible or balance out, which depends on the aggregation procedure used to evaluate them from their indicators. Moreover, it neglects the possibility to weight each determinant differently.

With respect to how risk is evaluated in the present work (see section 2.1.2 and equation (A.2)), two considerations can be made on this procedure. First, it is possible to rewrite equation (2.8) as equation (A.2) through a continuous transformation,

$$\begin{aligned} R'(\underline{x}) &= \exp\big(R(\underline{x})\big) = \\ &= \exp\left(\frac{w_E E}{w_E + w_H + w_V}\right) \exp\left(\frac{w_H H(\underline{x})}{w_E + w_H + w_V}\right) \exp\left(\frac{w_V V}{w_E + w_H + w_V}\right) = \\ &= E'\,H'(\underline{x})\,V' \quad . \end{aligned} \tag{A.3}$$

This allows to apply the considerations of the present study to CCRAs which evaluate risk as in equation (A.2). Note that an exact value of $R'(\underline{x}) = 0$ can not be obtained if real data and functions are used in the methodology. Second, in equation (A.2) R is linear with respect to H. As a consequence, if equation (A.2) is used instead of equation (2.8) and every other detail of the methodology follows section 2.1.2, then the risk value R is still a linear function of normalised hazard indicators:

$$R(\underline{x}) = E\,V \sum_j \sum_{I \in \mathcal{H}_j} w_I I(\underline{x}_I) \quad . \tag{A.4}$$

Definitions

adaptation "process of adjustment to actual or expected climate (3.4) and its effects", from [?]. 3

adaptive capacity "ability of systems (3.3), institutions, humans, and other organisms to adjust to potential damage, to take advantage of opportunities, or to respond to consequences", from [?]. 3

climatological standard normal "averages of climatological data computed for the following consecutive periods of 30 years: 1 January 1981-31 December 2010, 1 January 1991-31 December 2020, and so forth", from [?, 2].. *see* average

determinant Any component of risk, i.e. hazard, exposure, vulnerability, response, from [?, 493].. 2

driver individual components of determinants, from [?, 493]. *see* determinant, hazard, vulnerability, exposure & response, 3

exposure "presence of people, livelihoods, species or ecosystems, environmental functions, services, resources, infrastructure, or economic, social or cultural assets in places and settings that could be affected", from [?]. 2

hazard "potential source of harm", from [?]. 2

impact "effect on natural and human systems (3.3)", from [?]. 2

indicator "quantitative, qualitative or binary variable that can be measured or described, in response to a defined criterion", from [?]. 3

normal "period averages computed for a uniform and relatively long period comprising at least three consecutive ten-year periods", from [?, 2].. *see* period average, 8

period average “averages of climatological data computed for any period of at least ten years starting on 1 January of a year ending with the digit 1”, from [**?**, 2].. *see* average

response action enact to mitigate the effects of climate change or adapt to it. 2

risk “effect of uncertainty”, from [**?**]. 1

sensitivity “degree to which a system (3.3) or species is affected, either adversely or beneficially, by climate (3.4) variability or change”, from [**?**]. 3

vulnerability “propensity or predisposition to be adversely affected”, from [**?**]. *see* sensitivity & adaptive capacity, 2

Acronyms

AR6 Sixth Assessment Report.

CCRA climate change risk assessment.

CDF cumulative distribution function.

CID climatic impact-driver.

ECV essential climate variable.

EOF empirical orthogonal function.

IPCC Intergovernmental Panel on Climate Change.

ISO International Organization for Standardization.

PC principal component.

QF quantile function.

Symbols

$\diamondsuit$ arbitrary argument or mathematical object.

$|\diamondsuit|$ absolute value or, if the argument is a set, cardinality.

$\mathbb{1}[\diamondsuit]$ symbolic representation of a conditional test: returns 1 if the condition in square brackets is satisfied, else 0 (i.e. generalisation of a characteristic function of the argument in square brackets).