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University of Turin

Complex system in neuroscience, 12 December 2023

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Introduction

Predictive coding model of Rao and Ballard.

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Introduction

1. Prior predictions are compared to stimuli and the model parameters

1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.

. Partictive coding model of Rep and Balland

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Introduction

- Predictive coding model of Rao and Ballard.
- Free-energy model of Friston.

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Particitive coding model of Record Ballant.
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□Introduction

- 1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.
- 2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.



Predictive coding model of Rao and Ballard.

• Free-energy model of Friston.

Hebbian learning.

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-Introduction

. Particity coding model of Repart Ballant · Hebbie o leaded on.

- 1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.
- 2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
- 3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.

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Predictive coding model of Rao and Ballard.

- Free-energy model of Friston.
- Hebbian learning.
- Free energy minimization.

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. Particity coding model of Repart Ballant . Hebbie e lea criere.

· Face energy minimization

- 1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.
- 2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
- 3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.
- 4. Minimization of free energy can be seen as the base of many theories of perception.

-Working hypotheses

1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.

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Local computation.

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Working hypotheses

- Local computation.
- Local plasticity.

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Local computation.
 Local planticity

Working hypotheses

—Working hypotheses

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- 1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.
- 2. Synaptic plasticity depends only on the activities of pre-synaptic and post-synaptic neurons.

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Local computation.

Basic neuronal computation.

Local plasticity.

Working hypotheses

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-Working hypotheses

the state of its input neurons.

· Local compatation.

Working hypotheses

- 1. The state of a neuron is determined only by the synaptic weight and
- 2. Synaptic plasticity depends only on the activities of pre-synaptic and post-synaptic neurons.
- 3. The state of a neuron is the result of the application of a monotonic function to the linear combination of states and synaptic weights of input neurons.



Single variable model

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• Feature is a scalar variable $v \in \Omega_v$.

• Stimulus is a scalar variable $u \in \Omega_u$.

Single variable model

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Futus in a subs w in the z ∈ Ω.

Single variable model

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–Single variable model

-Single variable model

1. The model describes the inference of a single variable from a single sensory input.

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Conclusion

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_V \to \Omega_H$.

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Single variable model

Fortracing we have width σ ∈ Ω_σ.
Stimulus is a scale region by σ ∈ Ω_σ.
Relation between fortrace of attinuous is a differentiable function g : Ω_σ → Ω_σ.

Single variable model

- –Single variable model
- 1. The model describes the inference of a single variable from a single sensory input.
- 2. In general inferred variable and sensory input are related by some smooth function.



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Conclusion

Single variable model

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- Feature is a scalar variable $v \in \Omega_v$
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_v \to \Omega_u$.
- Sensory input p(u|v) is affected by gaussian noise and it has mean g(v) and variance Σ_u .

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—Single variable model

-Single variable model

Frates in a water we faile r ∈ Ω_r.
 Stimals is a scale region to ∈ Ω_r.
 Relation between first record attinuous is a differentiable function g: Ω_r → Ω_r.
 Section g: Ω_r → Ω_r.
 Section g: Ω_r → Ω_r.

g(r) 1 was 2,

- 1. The model describes the inference of a single variable from a single sensory input.
- 2. In general inferred variable and sensory input are related by some smooth function.
- 3. Sensory input and stimulus are drafted from the same space.

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Single variable model

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_V \to \Omega_u$.
- Sensory input p(u|v) is affected by gaussian noise and it has mean g(v) and variance Σ_u .
- Prior knowledge of the feature p(v) follows a gaussian distribution with mean v_p and variance Σ_p .

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz Single variable model

Single variable model

1. The model describes the inference of a single variable from a single sensory input.

Factors in a scalar variable r ∈ Ω_s.
 Stimulus is a scalar variable r ∈ Ω_s.
 Relation between factors and attimular is a differentiable.

function $g: \Omega_r \to \Omega_s$. • Surroughput p(x|r) is effected by gammin relational it but man g(r) and with rec Σ_s . • Prior troublegs of the feature p(r) follows a gammin distribution with man r_s and writers Σ_s .

- In general inferred variable and sensory input are related by some smooth function.
- 3. Sensory input and stimulus are drafted from the same space.
- 4. Information gained and constantly updated from previous experience.



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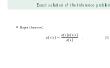
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Exact solution of the inference problem

• Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{2}$$

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Single variable model
Exact solution of the inference problem



1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus is the likelihood.

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Conclusion

Exact solution of the inference problem

• Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{1}$$

• Marginal likelihood of stimuli:

$$p(u) = \int_{\Omega_v} p(v)p(u|v) \, \mathrm{d}v \quad . \tag{2}$$

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz —Single variable model



Exact solution of the inference problem

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- 1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus is the likelihood.
- 2. In general marginal likelihood is difficult to evaluate.



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Exact solution of the inference problem

• Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{1}$$

• Marginal likelihood of stimuli:

$$p(u) = \int_{\Omega_v} p(v)p(u|v) \, \mathrm{d}v \quad . \tag{2}$$

• No implementation in simple biological systems.

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz Single variable model



- Exact solution of the inference problem
- 1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus is the likelihood.
- 2. In general marginal likelihood is difficult to evaluate.
- 3. Complex calculations and infinite nodes are needed to represent each value of the posterior.



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Approximated solution of the inference problem

Approximated solution of the inference problem

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-Neural implementation

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Single variable

Learning model parameters

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Learning model parameters

Learning model parameters

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Learning relation between variable and stimulus

Learning relation between variable and stimulus

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Learning parameters

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Hierarchical structure implementation

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Recover local plasticity

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