Summary of *A tutorial on the free-energy framework for modelling perception and learning* by Rafal Bogacz

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University of Turin

Complex system in neuroscience, 12 December 2023

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2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.
4. Minimization of free energy can be seen as the base of many theories of perception.

Working hypotheses

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3. The state of a neuron is the result of the application of a monotonic function to the linear combination of states and synaptic weights of input neurons.

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2. In general inferred variable and sensory input are related by some smooth function.
3. Sensory input and stimulus are drafted from the same space.
4. Information gained and constantly updated from previous experience.

Exact solution of the inference problem

- Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{1}$$

1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus is the likelihood.

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- No implementation in simple biological systems.

1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus is the likelihood.
2. In general, marginal likelihood is difficult to evaluate.
3. Complex calculations and infinite nodes are needed to represent each value of the posterior.

Approximated solution of the inference problem

- Most likely value of the feature is a scalar variable $\phi \in \Omega_v$.

1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.

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1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.
2. The most likely feature value is the fixed point of the gradient descent method applied to the negative free energy.
3. Prediction errors can be introduced as new variables to extend the dynamical system.

Neural implementation

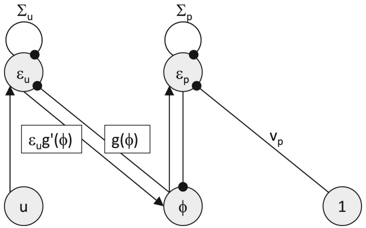
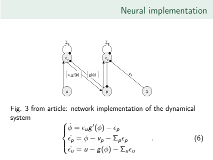


Fig. 3 from article: network implementation of the dynamical system

$$\begin{cases} \dot{\phi} = \epsilon_u g'(\phi) - \epsilon_p \\ \dot{\epsilon_p} = \phi - v_p - \Sigma_p \epsilon_p \\ \dot{\epsilon_u} = u - g(\phi) - \Sigma_u \epsilon_u \end{cases} . \quad (6)$$



- Hypotheses on local computation and Hebbian plasticity are satisfied.

Learning model parameters

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1. Introducing prediction errors as variables of the model allows to learn model parameters.

Learning relation between variable and stimulus

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Free energy framework

Multiple variables model

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Learning parameters

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Hierarchical structure implementation

Recover local plasticity

Recover local plasticity

Conclusion

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