Summary of A tutorial or the free-energy framework for modelling perception and learning

Marco Casari

Introduction

Single variable model

Multiple variable model

Canalusia

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Marco Casari

University of Turin

Complex system in neuroscience, 12 December 2023

for modelling perception and learning by Rafal Bogacz

Summary of A tutorial on the free-energy framework

Summary of A tutorial on the free-ener framework for modelling perception ar learning by Rafal Bogacz

> Marco Casari University of Turin

Complex system in neuroscience, 12 December 2023

•

2023-12-09

Introduction

-Introduction

Marco Casari

Introduction

• Predictive coding model of Rao and Ballard.

1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.

Summary of A tutorial on the free-energy framework

Introduction

Introduction

Summary of A tutorial on the free-energy framework 2023-12-09 for modelling perception and learning by Rafal Bogacz Introduction

-Introduction

1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.

. Predictive coding model of Rao and Ballard

2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.

Predictive coding model of Rao and Ballard.

• Free-energy model of Friston.

Introduction

Marco Casari

Introduction

- Predictive coding model of Rao and Ballard.
- Free-energy model of Friston.
- Hebbian plasticity.

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz 2023-12-Introduction

-Introduction

1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.

. Predictive coding model of Rao and Ballard

· Hebbian plasticity

- 2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
- 3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.



Introduction

Marco Casari

Introduction

- Predictive coding model of Rao and Ballard.
- Free-energy model of Friston.
- Hebbian plasticity.
- Free energy minimization.

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Introduction

2023-1

-Introduction

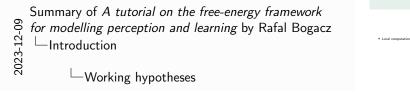
. Predictive coding model of Rao and Ballard

- 1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.
- 2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
- 3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.
- 4. Minimization of free energy can be seen as the base of many theories of perception.

Introduction

Working hypotheses

Local computation.



1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.

Working hypotheses



Introduction

Local computation.

Local plasticity.

Working hypotheses

2023-12-09

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz -Introduction

· Local plasticity.

Working hypotheses

-Working hypotheses

- 1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.
- 2. Synaptic plasticity depends only on the activities of pre-synaptic and post-synaptic neurons.

Introduction

Working hypotheses

Local computation.

- Local plasticity.
- Basic neuronal computation.

2023-12-

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz -Introduction

Local elasticity

Working hypotheses

-Working hypotheses

- 1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.
- 2. Synaptic plasticity depends only on the activities of pre-synaptic and post-synaptic neurons.
- 3. The state of a neuron is the result of the application of a monotonic function to the linear combination of states and synaptic weights of input neurons.

Single variable model

free-energy framework for modelling perception and learning by Rafal

Marco Casari

Introduction

Single variable model

> Multiple variables model

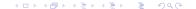
~ . .

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz
Single variable model
Single variable model

1. The model describes the inference of a single variable from a single sensory input.

Single variable model



ummary of

the
free-energy
framework
for modellin
perception
and learning

Marco Casari

Introductio

Single variable model

Multiple variables model

C l

Single variable model

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_V \to \Omega_U$.

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz —Single variable model

-Single variable model

sensory input.

Feature is a scalar variable v ∈ Ω_v.
 Stimulus is a scalar variable u ∈ Ω_u.
 Relation between feature and stimulus is a differentiable function g : Ω_v → Ω_u.

Single variable model

- 1. The model describes the inference of a single variable from a single
- 2. In general inferred variable and sensory input are related by some smooth function.



ummary of

Single variable model

free-energy framework for modellin perception and learnin by Rafal

Marco Casari

Introductio

Single variable model

Multiple variables model

Conclusion

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_V \to \Omega_U$.
- Sensory input p(u|v) is affected by gaussian noise and it has mean g(v) and variance Σ_u .

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Single variable model

- Stimulus is a scalar variable u ∈ Ω_u.
 Relation between feature and stimulus is a differentiable function g : Ω_v → Ω_u.
- Sensory input p(u|v) is affected by gaussian noise and it has mean x(v) and variance Σ

 igspace Single variable model

- 1. The model describes the inference of a single variable from a single sensory input.
- 2. In general inferred variable and sensory input are related by some smooth function.
- 3. Sensory input and stimulus are drafted from the same space.



immary of

free-energy framework for modelling perceptions and learning by Rafal

Marco Casari

Introductio

Single variable model

Multiple variables model

Conclusio

Single variable model

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_V \to \Omega_u$.
- Sensory input p(u|v) is affected by gaussian noise and it has mean g(v) and variance Σ_u .
- Prior knowledge of the feature p(v) follows a gaussian distribution with mean v_p and variance Σ_p .

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Single variable model

Stimulus is a scalar variable ir = 0.

- 1. The model describes the inference of a single variable from a single sensory input.
- 2. In general inferred variable and sensory input are related by some smooth function.
- 3. Sensory input and stimulus are drafted from the same space.
- 4. Information gained and constantly updated from previous experience.



ummary of

free-energy framework for modelling perception and learning by Rafal

Marco Casari

Introductio

Single variable model

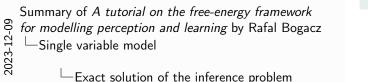
Multiple variables

C l

Exact solution of the inference problem

• Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{2}$$



1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus is the likelihood.

Exact solution of the inference problem



ummary of

free-energy framework for modelling perception and learning by Rafal Bogacz

Marco Casari

Introduction

Single variable model

Multiple variables model

Conclusion

Exact solution of the inference problem

• Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{1}$$

Marginal likelihood of stimuli:

$$p(u) = \int_{\Omega_{V}} p(v)p(u|v) \, \mathrm{d}v \quad . \tag{2}$$

Summary of *A tutorial on the free-energy framework*for modelling perception and learning by Rafal Bogacz
Single variable model

Exact solution of the inference problem



- 1. Knowledge of feature depending on a given stimulus is the posterior.
- is the likelihood.

 2. In general, marginal likelihood is difficult to evaluate.

Prior knowledge on the feature is the prior, distribution of stimulus

◆ロト ◆御 ト ◆ 恵 ト ◆ 恵 ・ 夕 Q (*)

Single variable model

Exact solution of the inference problem

Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{1}$$

Marginal likelihood of stimuli:

$$p(u) = \int_{\Omega_v} p(v)p(u|v) \, \mathrm{d}v \quad . \tag{2}$$

• No implementation in simple biological systems.

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz -Single variable model

Exact solution of the inference problem

2023-12-09



- 1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus
- 2. In general, marginal likelihood is difficult to evaluate.

is the likelihood.

3. Complex calculations and infinite nodes are needed to represent each value of the posterior.



mmary of

free-energy framework for modelling perception and learning by Rafal

Marco Casari

Introducti

Single

variable model

Multiple variables model

Canalusia

Approximated solution of the inference problem

• Most likely value of the feature is a scalar variable $\phi \in \Omega_{\nu}$.

(□) (□) (□) (□) (□) (□) (□)

Summary of *A tutorial on the free-energy framework*for modelling perception and learning by Rafal Bogacz
Single variable model

Approximated solution of the inference problem

1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.

Approximated solution of the inference problem

mmary of

free-energy framework for modellin perception and learning by Rafal

Marco Casari

Introduction

Single

variable model

variables model

Canalusia

Approximated solution of the inference problem

- Most likely value of the feature is a scalar variable $\phi \in \Omega_{\nu}$.
- Equivalent to minimize negative free energy:

$$F(v,u) = \ln(p(v)) + \ln(p(u|v)) \quad . \tag{3}$$

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz —Single variable model

2023-12-

Approximated solution of the inference problem

1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.

Approximated solution of the inference problem

2. The most likely feature value is the fixed point of the gradient descent method applied to the negative free energy.



ummary of tutorial on

free-energy framework for modelling perception and learning by Rafal

Marco Casari

Introductio

Single variable model

Multiple variables model

Conclusion

Approximated solution of the inference problem

- Most likely value of the feature is a scalar variable $\phi \in \Omega_{\nu}$.
- Equivalent to minimize negative free energy:

$$F(v,u) = \ln(p(v)) + \ln(p(u|v)) \quad . \tag{3}$$

• Prediction errors:

$$\epsilon_{p} = \frac{v - v_{p}}{\Sigma_{p}} \quad , \tag{4}$$

$$\epsilon_u = \frac{u - g(v)}{\Sigma_u} \quad . \tag{5}$$

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz —Single variable model

2023-12-09

errors:		
	$\epsilon_p = \frac{\nu - \nu_p}{\Sigma_p}$	(4)
	$\epsilon_u = \frac{u - g(v)}{\Sigma_u}$	(5)

Approximated solution of the inference problem

Approximated solution of the inference problem

- 1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.
- 2. The most likely feature value is the fixed point of the gradient descent method applied to the negative free energy.
- 3. Prediction errors can be introduced as new variables to extend the dynamical system.



ımmary of tutorial on

free-energy framework for modelling perception and learning by Rafal

Marco Casari

Introduction

Single variable model

Multiple variables model

Conclusio

Neural implementation

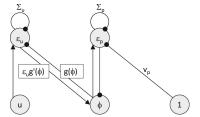
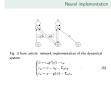


Fig. 3 from article: network implementation of the dynamical system

$$\begin{cases} \dot{\phi} = \epsilon_{u} g'(\phi) - \epsilon_{p} \\ \dot{\epsilon_{p}} = \phi - \nu_{p} - \Sigma_{p} \epsilon_{p} \\ \dot{\epsilon_{u}} = u - g(\phi) - \Sigma_{u} \epsilon_{u} \end{cases}$$
 (6)

Summary of *A tutorial on the free-energy framework* for modelling perception and learning by Rafal Bogacz —Single variable model

—Neural implementation



 Hypotheses on local computation and Hebbian plasticity are satisfied.



Learning model parameters

Summary of A tutorial on the free-energy framework 2023-12-09 for modelling perception and learning by Rafal Bogacz -Single variable model

Learning model parameters

1. Introducing prediction errors as variables of the model allows to learn model parameters.

Learning model parameters

Marco Casari

Single variable model

Learning relation between variable and stimulus

Summary of *A tutorial on the free-energy framework*for modelling perception and learning by Rafal Bogacz

Single variable model

Learning relation between variable and stimulus

Learning relation between variable and stimulus

Marco Casari

Introductio

Single variable model

Multiple variables

Free energy framework

2023-12-09

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz —Single variable model

Free energy framework

Free energy framework

Marco Casari

Introductio

Single variable model

Multiple variables model

Multiple variables model

2023-12-09

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Multiple variables model

Multiple variables model

ı

└─Multiple variables model

Marco Casari

Introduction

Single variable model

Multiple variables model

Learning parameters

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Learning parameters

-Multiple variables model

2023-12-09

Learning parameters

Marco Casari

Introduction

Single variable model

Multiple variables model

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Hierarchical structure implementation

-Multiple variables model

2023-12-09

 \sqsubseteq Hierarchical structure implementation

Marco Casari

Introduction

oingie variable model

Multiple variables model

2023-12-09

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz -Multiple variables model

Recover local plasticity

Marco Casari

Multiple variables model

Conclusion

Conclusion

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz Conclusion

-Conclusion

2023-12-09

└─ Conclusion