Summary of *A tutorial on the free-energy framework for modelling perception and learning* by Rafal Bogacz

Marco Casari

University of Turin

Complex system in neuroscience, 12 December 2023

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- In this presentation the main topics of the paper are presented, in the same order they appear in the paper.

Introduction

- Predictive coding model of Rao and Ballard.

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2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.
4. Minimization of free energy can be seen as the base of many theories of perception.

Working hypotheses

- Local computation.

1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.

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3. The state of a neuron is the result of the application of a monotonic function to the linear combination of states and synaptic weights of input neurons.

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1. The model describes the inference of a single variable from a single sensory input.
2. In general inferred variable and sensory input are related by some smooth function.
3. Sensory input and stimulus are drafted from the same space.
4. Information gained and constantly updated from previous experience.

Exact solution of the inference problem

- Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{1}$$

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1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus is the likelihood.
2. In general, marginal likelihood is difficult to evaluate.
3. Complex calculations and infinite nodes are needed to represent each value of the posterior.

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- Prediction errors:

$$\varepsilon_p = \frac{v - v_p}{\Sigma_p} \quad , \tag{4}$$

$$\varepsilon_u = \frac{u - g(v)}{\Sigma_u} \quad . \tag{5}$$

1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.
2. The most likely feature value is the fixed point of the gradient descent method applied to the negative free energy.
3. Prediction errors can be introduced as new variables to extend the dynamical system and to allow learning.

Neural implementation

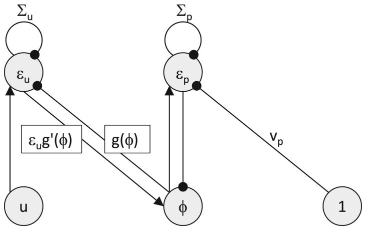
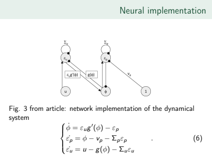


Fig. 3 from article: network implementation of the dynamical system

$$\begin{cases} \dot{\phi} = \varepsilon_u g'(\phi) - \varepsilon_p \\ \dot{\varepsilon_p} = \phi - v_p - \Sigma_p \varepsilon_p \\ \dot{\varepsilon_u} = u - g(\phi) - \Sigma_u \varepsilon_u \end{cases} . \tag{6}$$



- Hypotheses on local computation and Hebbian plasticity are satisfied.

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1. Model parameters are variables mean and variance.
2. Feature and stimulus joint probability, hence free energy, is maximized instead of the marginal likelihood.
3. Parameters are updated each time with different samples hence the convergence is guaranteed only as mean over all samples.
4. Introducing prediction errors as variables of the model allows to learn model parameters.

Learning relation between variable and stimulus

2023-12-09

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└Single variable model

└Learning relation between variable and stimulus

Free energy framework

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Multiple variables model

└Multiple variables model

Learning parameters

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Hierarchical structure implementation

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Recover local plasticity

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Recover local plasticity

Conclusion

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