

- Predictive coding model of Rao and Ballard.

2023-12-11

Summary of *A tutorial on the free-energy framework for modelling perception and learning* by Rafal Bogacz

└ Introduction

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1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.

- Predictive coding model of Rao and Ballard.
- Free-energy model of Friston.

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2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.

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- Predictive coding model of Rao and Ballard.
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- Hebbian plasticity.

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2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.

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- Predictive coding model of Rao and Ballard.
- Free-energy model of Friston.
- Hebbian plasticity.
- Free energy minimization.

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1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.
2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.
4. Minimization of free energy can be seen as the base of many theories of perception.

- Local computation.

1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.

Working hypotheses

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- Working hypotheses

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Working hypotheses

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- Working hypotheses

- Local computation.
- Local plasticity.
- Basic neuronal computation.

1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.
2. Synaptic plasticity depends only on the activities of pre-synaptic and post-synaptic neurons.
3. The state of a neuron is the result of the application of a monotonic function to the linear combination of states and synaptic weights of input neurons.

- Local computation.
- Local plasticity.
- Basic neuronal computation.

Single variable model

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.

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- Single variable model

- └ Single variable model

1. The model describes the inference of a single variable from a single sensory input.

- Feature is a scalar variable $v \in \Omega_v$.
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Single variable model

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g : \Omega_v \rightarrow \Omega_u$.

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- Single variable model

- └ Single variable model

1. The model describes the inference of a single variable from a single sensory input.
2. In general inferred variable and sensory input are related by some smooth function.

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_v \rightarrow \Omega_u$.

Neural implementation

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└ Single variable model

└ Neural implementation

- Note that hypotheses and Hebbian plasticity are satisfied.

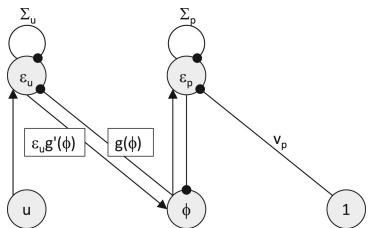


Fig. 3 from article: network implementation of the dynamical system

$$\begin{cases} \dot{\phi} = \varepsilon_u g'(\phi) - \varepsilon_p \\ \dot{\varepsilon}_p = \phi - v_p - \Sigma_p \varepsilon_p \\ \dot{\varepsilon}_u = u - g(\phi) - \Sigma_u \varepsilon_u \end{cases} \quad (6)$$

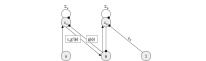


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Learning model parameters

- Choose model parameters to maximize $p(u)$.
- Equivalent to maximize negative free energy with respect to parameters:

$$\frac{\partial F}{\partial v_p} = \frac{\phi - v_p}{\Sigma_p} \quad , \quad (7)$$

$$\frac{\partial F}{\partial \Sigma_p} = \frac{1}{2} \left(\frac{(\phi - v_p)^2}{\Sigma_p^2} - \frac{1}{\Sigma_p} \right) \quad , \quad (8)$$

$$\frac{\partial F}{\partial \Sigma_u} = \frac{1}{2} \left(\frac{(u - g(\phi))^2}{\Sigma_u^2} - \frac{1}{\Sigma_u} \right) \quad . \quad (9)$$

- Hebbian plasticity is satisfied using prediction errors.

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- Single variable model
 - Learning model parameters

1. Model parameters are mean and variance of variables.
2. The fixed point of this dynamical system exists only as sample mean over the occurred events of perception.
3. Without prediction errors, the computation is still local.

- Choose model parameters to maximize $p(u)$.
- Equivalent to maximize negative free energy with respect to parameters:

$$\frac{\partial F}{\partial v_0} = \frac{\phi - v_0}{\Sigma_0}, \quad (7)$$

$$\frac{\partial F}{\partial \Sigma} = \frac{1}{2} \left(\frac{(\phi - v_\phi)^2}{\Sigma^2} - \frac{1}{\Sigma} \right), \quad (8)$$

$$\frac{\partial F}{\partial u} = \frac{1}{2} \left(\frac{(u - g(\phi))^2}{2} - \frac{1}{2} \right). \quad (9)$$

- Hebbian plasticity is satisfied using prediction errors

Recover local plasticity

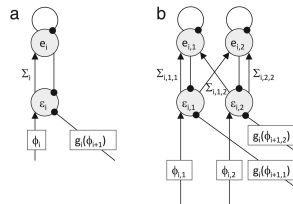


Fig. 7 from article: networks satisfying local plasticity for (a) single variable model and (b) multiple variables model. They implement the generalized dynamical system

$$\begin{cases} \dot{\vec{e}}_i = \vec{\phi}_i - g_i(\vec{\phi}_{i+1}) - \vec{e}_i \\ \dot{\vec{e}}_i = \mathbf{\Sigma}_i \vec{e}_i - \vec{e}_i \end{cases} \quad (19)$$

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└ Multiple variables model

└ Recover local plasticity

- The update rule for the model parameters is Hebbian and contains the learning rate as hyperparameter of the model.
- Convergence of prediction errors to the sample variances is guaranteed if the most likely feature values change at slower time scales.

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