Summary of A tutorial on the free-energy framework for modelling perception

Marco Casari

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Single variable model

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Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Marco Casari

University of Turin

Complex system in neuroscience, 12 December 2023



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Complex system in neuroscience, 12 December 2023

• In this presentation the main topics of the paper are presented, in the same order they appear in the paper.

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Introduction

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Introduction

• Predictive coding model of Rao and Ballard.

1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.

Summary of A tutorial on the free-energy framework

Introduction

Introduction

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1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.

. Predictive coding model of Rao and Ballard

2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.

Predictive coding model of Rao and Ballard.

• Free-energy model of Friston.

Introduction

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Introduction

- Predictive coding model of Rao and Ballard.
- Free-energy model of Friston.
- Hebbian plasticity.

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-Introduction

1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.

. Predictive coding model of Rao and Ballard

· Hebbian plasticity

- 2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
- 3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.



Introduction

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Introduction

- Predictive coding model of Rao and Ballard.
- Free-energy model of Friston.
- Hebbian plasticity.
- Free energy minimization.

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Introduction

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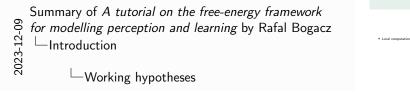
. Predictive coding model of Rao and Ballard

- 1. Prior predictions are compared to stimuli and the model parameters are updated considering prediction errors, features corresponding to receptive fields in the the primary sensory cortex are learned.
- 2. Weight stimuli by their noise, learn features using their covariance, implement attentional modulation changing the variance of attended features.
- 3. Synaptic strenght is changed proportionally to activities of pre-synaptic and post-synaptic neurons.
- 4. Minimization of free energy can be seen as the base of many theories of perception.

Introduction

Working hypotheses

Local computation.



1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.

Working hypotheses



Introduction

Local computation.

Local plasticity.

Working hypotheses

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· Local plasticity.

Working hypotheses

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- 1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.
- 2. Synaptic plasticity depends only on the activities of pre-synaptic and post-synaptic neurons.

Introduction

Working hypotheses

Local computation.

- Local plasticity.
- Basic neuronal computation.

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Local elasticity

Working hypotheses

-Working hypotheses

- 1. The state of a neuron is determined only by the synaptic weight and the state of its input neurons.
- 2. Synaptic plasticity depends only on the activities of pre-synaptic and post-synaptic neurons.
- 3. The state of a neuron is the result of the application of a monotonic function to the linear combination of states and synaptic weights of input neurons.

Single variable model

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Introduction

Single variable model

> Multiple variables model

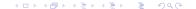
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- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.

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Single variable model
Single variable model

1. The model describes the inference of a single variable from a single sensory input.

Single variable model



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Single variable model

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_V \to \Omega_U$.

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz —Single variable model

-Single variable model

sensory input.

Feature is a scalar variable v ∈ Ω_v.
 Stimulus is a scalar variable u ∈ Ω_u.
 Relation between feature and stimulus is a differentiable function g : Ω_v → Ω_u.

Single variable model

- 1. The model describes the inference of a single variable from a single
- 2. In general inferred variable and sensory input are related by some smooth function.



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Single variable model

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- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_V \to \Omega_U$.
- Sensory input p(u|v) is affected by gaussian noise and it has mean g(v) and variance Σ_u .

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Single variable model

- Stimulus is a scalar variable u ∈ Ω_u.
 Relation between feature and stimulus is a differentiable function g : Ω_v → Ω_u.
- Sensory input p(u|v) is affected by gaussian noise and it has mean x(v) and variance Σ

 igspace Single variable model

- 1. The model describes the inference of a single variable from a single sensory input.
- 2. In general inferred variable and sensory input are related by some smooth function.
- 3. Sensory input and stimulus are drafted from the same space.



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Single variable model

- Feature is a scalar variable $v \in \Omega_v$.
- Stimulus is a scalar variable $u \in \Omega_u$.
- Relation between feature and stimulus is a differentiable function $g: \Omega_V \to \Omega_u$.
- Sensory input p(u|v) is affected by gaussian noise and it has mean g(v) and variance Σ_u .
- Prior knowledge of the feature p(v) follows a gaussian distribution with mean v_p and variance Σ_p .

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz

Single variable model

Stimulus is a scalar variable ir = 0.

- 1. The model describes the inference of a single variable from a single sensory input.
- 2. In general inferred variable and sensory input are related by some smooth function.
- 3. Sensory input and stimulus are drafted from the same space.
- 4. Information gained and constantly updated from previous experience.



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Single variable model

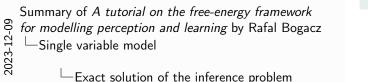
Multiple variables

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Exact solution of the inference problem

• Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{2}$$



1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus is the likelihood.

Exact solution of the inference problem



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Exact solution of the inference problem

• Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{1}$$

Marginal likelihood of stimuli:

$$p(u) = \int_{\Omega_{V}} p(v)p(u|v) \, \mathrm{d}v \quad . \tag{2}$$

Summary of *A tutorial on the free-energy framework*for modelling perception and learning by Rafal Bogacz
Single variable model

Exact solution of the inference problem



- 1. Knowledge of feature depending on a given stimulus is the posterior.
- is the likelihood.

 2. In general, marginal likelihood is difficult to evaluate.

Prior knowledge on the feature is the prior, distribution of stimulus

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Single variable model

Exact solution of the inference problem

Bayes theorem:

$$p(v|u) = \frac{p(v)p(u|v)}{p(u)} \quad . \tag{1}$$

Marginal likelihood of stimuli:

$$p(u) = \int_{\Omega_v} p(v)p(u|v) \, \mathrm{d}v \quad . \tag{2}$$

• No implementation in simple biological systems.

Summary of A tutorial on the free-energy framework for modelling perception and learning by Rafal Bogacz -Single variable model

Exact solution of the inference problem

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- 1. Knowledge of feature depending on a given stimulus is the posterior. Prior knowledge on the feature is the prior, distribution of stimulus
- 2. In general, marginal likelihood is difficult to evaluate.

is the likelihood.

3. Complex calculations and infinite nodes are needed to represent each value of the posterior.



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Approximated solution of the inference problem

• Most likely value of the feature is a scalar variable $\phi \in \Omega_{\nu}.$



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Approximated solution of the inference problem

1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.

Approximated solution of the inference problem

Most likely value of the feature is a scalar variable o ⊂ O...

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Approximated solution of the inference problem

- Most likely value of the feature is a scalar variable $\phi \in \Omega_{\nu}$.
- Equivalent to minimize negative free energy with respect to the feature:

$$F(v,u) = \ln(p(v)) + \ln(p(u|v)) \quad . \tag{3}$$



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Approximated solution of the inference problem

1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.

Approximated solution of the inference problem

2. The most likely feature value is the fixed point of the gradient descent method applied to the negative free energy.

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Approximated solution of the inference problem

- Most likely value of the feature is a scalar variable $\phi \in \Omega_{\nu}$.
- Equivalent to minimize negative free energy with respect to the feature:

$$F(v,u) = \ln(p(v)) + \ln(p(u|v)) \quad . \tag{3}$$

Prediction errors:

$$\varepsilon_p = \frac{v - v_p}{\Sigma_p} \quad , \tag{4}$$

$$\varepsilon_u = \frac{u - g(v)}{\Sigma_u} \quad . \tag{5}$$



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Approximated solution of the inference problem

Approximated solution of the inference problem Most Buly value of the feature is a scalar variable $\phi \in \Omega_r$. Equivalent to minimize negation free energy with respect to the feature: $F(\nu, v) = \ln (\rho(v)) + \ln (\rho(u|v)) \qquad \qquad (3)$ Prediction errors: $\varepsilon_{x,y} = \frac{\nu - \nu_y}{2}, \qquad (4)$

- 1. Evaluating the mode of the posterior instead of the whole function is more biologically plausible.
- 2. The most likely feature value is the fixed point of the gradient descent method applied to the negative free energy.
- 3. Prediction errors can be introduced as new variables to extend the dynamical system and to allow learning.

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Neural implementation

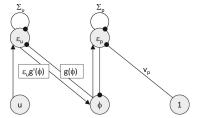
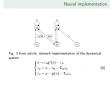


Fig. 3 from article: network implementation of the dynamical system

$$\begin{cases} \dot{\phi} = \varepsilon_{u} g'(\phi) - \varepsilon_{p} \\ \dot{\varepsilon_{p}} = \phi - v_{p} - \Sigma_{p} \varepsilon_{p} \\ \dot{\varepsilon_{u}} = u - g(\phi) - \Sigma_{u} \varepsilon_{u} \end{cases}$$
(6)

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—Neural implementation



 Hypotheses on local computation and Hebbian plasticity are satisfied.



Learning model parameters

• Choose model parameters to maximize p(u).

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Learning model parameters

Choose model parameters to maximize p(u)

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Learning model parameters

1. Model parameters are variables mean and variance.

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Learning model parameters

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- Choose model parameters to maximize p(u).
- Equivalent to maximize negative free energy with respect to parameters.

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ivalent to maximize negative free energy with respectarameters.

Learning model parameters

1. Model parameters are variables mean and variance.

-Learning model parameters

2. Feature and stimulus joint probability, hence free energy, is maximized instead of the marginal likelihood.

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Learning model parameters

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- Choose model parameters to maximize p(u).
- Equivalent to maximize negative free energy with respect to parameters.

Summary of *A tutorial on the free-energy framework* for modelling perception and learning by Rafal Bogacz —Single variable model

-Learning model parameters

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Learning model parameters

- 1. Model parameters are variables mean and variance.
- 2. Feature and stimulus joint probability, hence free energy, is maximized instead of the marginal likelihood.
- 3. Parameters are updated each time with different samples hence the convergence is guaranteed only as mean over all samples.
- 4. Introducing prediction errors as variables of the model allows to learn model parameters.



Learning relation between variable and stimulus

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Learning relation between variable and stimulus

Learning relation between variable and stimulus

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Learning parameters

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Hierarchical structure implementation

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Recover local plasticity

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