<u>**Part VI: Calculus II Final Project**</u>

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We are working with rates of changes along with definite integrals for this project. These topics can help us understand how traffic builds up over a busy road. Rates of change tell us the number of cars that enter or leave a road per minute and integrals help us add up the number of cars that passed in that certain time period and how much traffic has been built up. Traffic congestion is an issue where cars build up over time and integrals are used in Calculus II for measuring things that accumulate overtime. These ideas help us understand how traffic builds up and when the road becomes crowded.

Our project talks about traffic congestion in a busy area and we try to understand how cars build up throughout the day. Everyday travel becomes difficult, traffic slows down, and people become trapped for extended periods of time when there are more vehicles on a road than it can accommodate. Workers, students, families, delivery trucks, and even emergency vehicles that must get to locations fast are all impacted by this. Heavy traffic brings in problems like air pollution, noise, and causes health hazards who live close by. Many of the neighborhoods have a lack of resources which causes the people to commute for long periods of time and they do not have access to clean air. By using integration to prove how traffic builds up, our project can show one of the world's biggest issues and suggest solutions to transportation and road construction so that the system is fair for everyone.

Cars enter a busy urban road at a variable rate over the course of a day. How many total vehicles pass through this area during a four-hour rush period, and how does this information help us assess congestion and mobility within that community?

$R(t) = 18 + 6cos(\frac{\pi t}{4})$

- **R(t)**: Rate at which cars enter the road at a particular time (Cars per minute).
- **t**: Time in hours, $0 \leq t \leq 8$.

- **18**: Base number of cars travelling on the road, even during non-peak hours.
- **Cosine**: Predicts the increase and decrease of traffic based on rush hour patterns.

$$C = \int_0^4 R(t)\,dt \qquad C = \int_0^4 [18 + 6cos(\frac{\pi t}{4})]\,dt$$

- **C:** Total number of cars that passed during the 4 hours.

This definite integral is a representation of how traffic builds up over time, adding small changes in vehicle flow. Since the number of cars entering the road rises and falls due to commuting patterns, integrating R(t) provides a realistic way of measuring congestion during busy hours of the day. This directly relates to our project by showing how concepts in Calculus II, including rates of change and integration, are useful in explaining how mobility is shaped in urban areas. Knowledge of cumulative traffic can help determine when roads become overcrowded, the duration of delays involved, and how infrastructure limitations typically affect marginalized communities that rely on these routes daily.

R(t) is given in minutes while t is standardized to hours. Therefore, dt would need to be converted to account for the change from minutes to hours. This is shown below:

If R(t) is in minutes, it would need to be multiplied by 60 to convert to an hour to be in level as the unit of time (R(t) x 60).

$$C = \int_0^4 R(t)\,.\,60\,dt = 60\int_0^4 [18 + 6cos(\frac{\pi t}{4})]\,dt$$

Split the integral for simplicity:

$$C = 60\,.\,[\int_0^4 18 + \int_0^4 6cos(\frac{\pi t}{4})]$$

1. $\int_0^4 18\,dt = (18\,.\,4) - (18\,.\,0) = 72$

- 72 cars per hour

2 $\int_0^4 6cos(\frac{\pi t}{4}) = 6\int_0^4 cos(\frac{\pi t}{4})$

$u = \frac{\pi t}{4} \qquad du = \frac{\pi}{4}dt \qquad dt = \frac{4}{\pi}du$

$6\int_0^4 cos(u) . \frac{4}{\pi}du = \frac{24}{\pi}\int_0^4 cos(u) = \frac{24}{\pi}sin(u) = \frac{24}{\pi}sin(\frac{\pi t}{4})$

$[\frac{24}{\pi}sin(\frac{\pi(4)}{4})] - [\frac{24}{\pi}sin(\frac{\pi(0)}{4})] = 0 - 0 = 0$

- 0 cars per hour

Combine the separate integrals:

$C = 60(72 - 0) = 60 . 72 = 4320$

- R(cars/minute) x 60(mins/hour) x 4 hour time period = 4320 cars

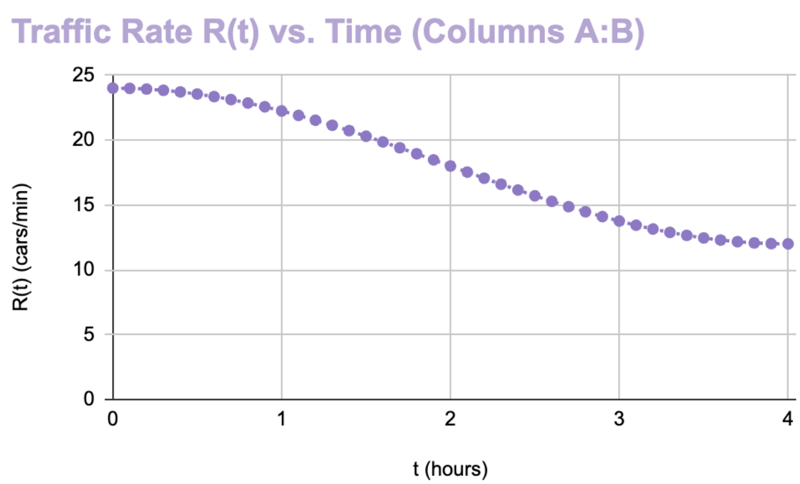
To support the analysis of phantom traffic and its mathematical context, a supplementary website has been developed to provide readers with interactive and visual materials (a). All supplemental visual components for this project including the interactive graphs, data files, YouTube demonstration, and 3D AR model are available on the project website: https://mirasha296.github.io/VisualDemo/.

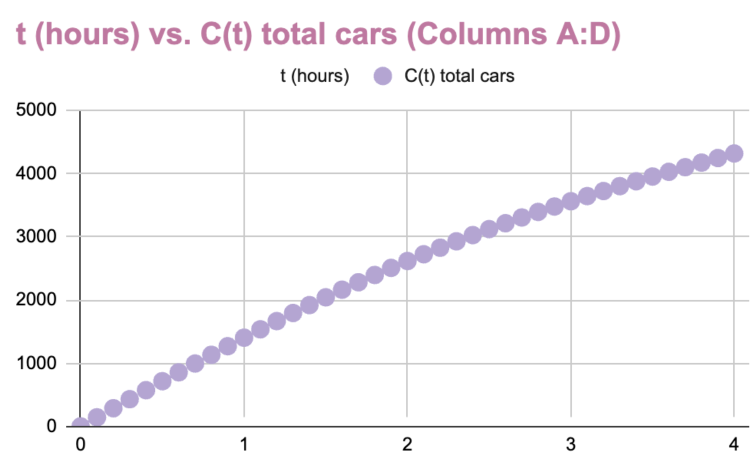
The site includes an embedded YouTube video that presents a real-time example of the formation of phantom traffic waves, illustrating how small fluctuations in driver behavior can escalate into large-scale congestion even without physical obstacles. In addition to the video, the website hosts links to all components of the project (Parts I–VI), allowing readers to explore the full progression of the mathematical modeling and problem-solving process.

Two interactive graphs are provided to demonstrate the rate of change within the corresponding Calculus II problem examined in the project. These graphs are accompanied by downloadable CSV and PDF files to allow readers to review or analyze the underlying data independently. Part VI of the project also includes an external data visualization created in Google Sheets (b), offering a clear representation of the final analytical outcomes.

To contextualize the mathematical findings within real-world applications, the website also features a 3D walkable augmented reality (AR) demonstration. This AR model presents a conceptual traffic-management solution designed to mitigate congestion by improving road clarity and reorganizing vehicular flow (c). Together, these visual, interactive, and data-driven components serve to enhance the reader's understanding of phantom traffic behavior and the potential for applied mathematical strategies to address it. To view all repository and code files, visit the Github Repo link at https://github.com/mirasha296/VisualDemo.

The calculations and their corresponding visual representations show a decreasing rate of change and an increase in the definite integral values. To put it differently, during the four-hour rush period, traffic becomes heavier, even though fewer cars are entering. This pattern reveals how traffic has been built up over time in a theoretical model, helping explain why heavy traffic forms even when fewer cars appear to enter. Through mathematical modeling, we can conclude that entering





the busy road at the end of the peak hours can help drivers save a significant amount of time, especially for those who live far from their workplaces. From another perspective, it can help policymakers arrange limited government resources more effectively, helping groups of people in lower socio-economic classes who have limited commuting options. While it is true that the theoretical findings reflect the traffic congestion in everyday life and offer meaningful policy suggestions, it is also important to recognize the limitations of this project due to mathematical assumptions and simplifications. In real life situations, factors such as road accidents and construction are excluded from mathematical modeling due to their situational complexity.

In a society where the topic of human rights captures significant public attention, building mathematical models grounded in social justice empowers marginalized groups to fight for their rights by means of education. As demonstrated in this project, rate of change and definite integrals in Calculus II serve as tools to model traffic congestion. By discovering the time of least traffic in the mathematical model, this finding not only helps drivers determine when to leave workplaces, but also offers practical insights to policymakers. In essence, mathematics becomes a tool for just-based decision making, which extends far beyond mathematical calculations.

References:

Massachusetts Institute of Technology (MIT). (2009, June 9). Formation of a "phantom traffic jam" [Video]. YouTube. https://youtube.com/watch?v=Q78Kb4uLAdA