

Part VI: Calculus II Final Project

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We are working with rates of changes along with definite integrals for this project. These topics can help us understand how traffic builds up over a busy road. Rates of change tell us the number of cars that enter or leave a road per minute and integrals help us add up the number of cars that passed in that certain time period and how much traffic has been built up. Traffic congestion is an issue where cars build up over time and integrals are used in Calculus II for measuring things that accumulate overtime. These ideas help us understand how traffic builds up and when the road becomes crowded.

Our project talks about traffic congestion in a busy area and we try to understand how cars build up throughout the day. Everyday travel becomes difficult, traffic slows down, and people become trapped for extended periods of time when there are more vehicles on a road than it can accommodate. Workers, students, families, delivery trucks, and even emergency vehicles that must get to locations fast are all impacted by this. Heavy traffic brings in problems like air pollution, noise, and causes health hazards who live close by. Many of the neighborhoods have a lack of resources which causes the people to commute for long periods of time and they do not have access to clean air. By using integration to prove how traffic builds up, our project can show one of the world's biggest issues and suggest solutions to transportation and road construction so that the system is fair for everyone.

Cars enter a busy urban road at a variable rate over the course of a day. How many total vehicles pass through this area during a four-hour rush period, and how does this information help us assess congestion and mobility within that community?

$$R(t) = 18 + 6\cos\left(\frac{\pi t}{4}\right)$$

- **R(t)**: Rate at which cars enter the road at a particular time (Cars per minute).
- **t**: Time in hours, $0 \leq t \leq 8$.

- **18:** Base number of cars travelling on the road, even during non-peak hours.
- **Cosine:** Predicts the increase and decrease of traffic based on rush hour patterns.

$$C = \int_0^4 R(t) dt \quad C = \int_0^4 [18 + 6\cos(\frac{\pi t}{4})] dt$$

- **C:** Total number of cars that passed during the 4 hours.

This definite integral is a representation of how traffic builds up over time, adding small changes in vehicle flow. Since the number of cars entering the road rises and falls due to commuting patterns, integrating $R(t)$ provides a realistic way of measuring congestion during busy hours of the day. This directly relates to our project by showing how concepts in Calculus II, including rates of change and integration, are useful in explaining how mobility is shaped in urban areas. Knowledge of cumulative traffic can help determine when roads become overcrowded, the duration of delays involved, and how infrastructure limitations typically affect marginalized communities that rely on these routes daily.

$R(t)$ is given in minutes while t is standardized to hours. Therefore, dt would need to be converted to account for the change from minutes to hours. This is shown below:

If $R(t)$ is in minutes, it would need to be multiplied by 60 to convert to an hour to be in level as the unit of time ($R(t) \times 60$).

$$C = \int_0^4 R(t) \cdot 60 dt = 60 \int_0^4 [18 + 6\cos(\frac{\pi t}{4})] dt$$

Split the integral for simplicity:

$$C = 60 \cdot \left[\int_0^4 18 + \int_0^4 6\cos(\frac{\pi t}{4}) \right]$$

$$1. \quad \int_0^4 18 dt = (18 \cdot 4) - (18 \cdot 0) = 72$$

- 72 cars per hour

$$2 \quad \int_0^4 6 \cos\left(\frac{\pi t}{4}\right) dt = 6 \int_0^4 \cos\left(\frac{\pi t}{4}\right) dt$$

$$u = \frac{\pi t}{4} \quad du = \frac{\pi}{4} dt \quad dt = \frac{4}{\pi} du$$

$$6 \int_0^4 \cos(u) \cdot \frac{4}{\pi} du = \frac{24}{\pi} \int_0^4 \cos(u) du = \frac{24}{\pi} \sin(u) = \frac{24}{\pi} \sin\left(\frac{\pi t}{4}\right)$$

$$\left[\frac{24}{\pi} \sin\left(\frac{\pi(4)}{4}\right) \right] - \left[\frac{24}{\pi} \sin\left(\frac{\pi(0)}{4}\right) \right] = 0 - 0 = 0$$

- 0 cars per hour

Combine the separate integrals:

$$C = 60 (72 - 0) = 60 \cdot 72 = 4320$$

- $R(\text{cars/minute}) \times 60(\text{mins/hour}) \times 4 \text{ hour time period} = 4320 \text{ cars}$