Airport Security Queues

Introduction

The scenario we are simulating is airport security queues with a single server per line. The service rate follows a truncated normal distribution with parameters $\mu = 0.5\ min$, $\sigma = 1/6\ min$, and the arrival time distribution follows an exponential distribution with $\lambda = 10\ ppl/min$. The model is then complicated by an addition of extra security screening with probability $p = 0.05$ and the extra service distribution that follows a truncated normal distribution with parameters $\mu = 2\ min$, $\sigma = 2\ min$.

The scenario is modeled using the M/G/1 queueing model, since it corresponds most closely to the situation described above due to the arrival rate distribution, freedom to choose any service rate distribution and a single server per line. Other queuing theory models such as M/D/1 or M/G/c would not capture the key features of the scenario, since the former assumes a deterministic service rate and the latter assumes multiple servers per line.

In the M/G/1 queueing model, *M* stands for the Memoryless arrival process (arrivals are modeled by Poisson distribution, the time between arrivals is exponentially distributed, and the arrival times are assumed to be independent). *G* stands for general service time distribution, meaning that service time can be described by any probability distribution. In this project, we are using a truncated normal distribution with parameters $\mu = 0.5\ min$, $\sigma = 1/6\ min$ like described in the scenario above. The model also assumes 1 server attending to customers in every queue. Other assumptions the model is making is that service and arrival times are independent, FIFO queue discipline is preserved, queues have infinite capacity, and arrivals are constant throughout the day. Despite the inherent assumptions of the model, it provides a reasonable approximation of airport queues because 1) in most cases (except for families, etc) arrivals are independent, especially under longer periods of time, corresponding to the Memoryless property; 2) most security lines have only 1 main server; 3) FIFO queue discipline is preserved with an exception of special classes of passengers. While there are cases where the model assumptions outlined above do not hold (eg. priority passes, families, line cutters, etc), the M/G/1 model allows one to capture the key features of the scenario.

In order to examine the model's behavior under different conditions such as the addition of a security officer check, it might be more efficient to run simulation experiments with M/G/1 to capture the new conditions. The variables that can be used in a simulated model include the state of a particular queue/traveler, the status of a senior officer, the number of queues, the queue length, ρ (utilization), and waiting time. The model's parameters are: the arrival rate (λ), service/extra screening time (μ), service/extra screening time variance (σ), the probability of extra screening (*p*) and number of queues (*n*). The update rules are that the traveler joins the shortest queue, the fact that the rest of the queue should wait while a person is undergoing additional screening.[1]

Simulation Analysis

One queue, no senior officer, various arrival rates

While the scenario being modeled implies $\lambda = 10$ travelers per min (arrival rate), and $\mu = 0.5$ min per traveler (service rate), one cannot compare a simulated model with these parameters to a theoretical model, assuming there is a single queue. This is because with such parameters $\rho = \lambda\mu$ (utilization) is greater than 1, implying that the arrival rate is greater or equal to the service rate. In such cases, the system is unstable and both the queue length and waiting times grow infinitely, never reaching equilibrium. Hence, in such a case one would not be able to compare the simulated results with the theoretical ones. In order to avoid this problem, one can either implement more queues and calculate the length and waiting times per queue, or change the service/arrival rates.

In this test, I am assessing whether my simulated model works as expected on the most basic level (1 service station, no senior security officer) by comparing it with the theoretical model. That is why in this test case I am keeping the number of queues at 1, holding the service rate at 0.5 min per traveler, and not yet implementing the senior officer. Hence, in order to avoid

[1] **#modeling:** In the introduction section of the report I have explained that the chosen model provides a reasonable approximation of the key features of the scenario. More specifically, I outlined the assumptions the model is making and explained that these assumptions hold in most cases in the real world and correspond to the given scenario. Moreover, I compared the chosen theoretical model to other models, M/D/1 and M/G/c, and explained why M/G/1 was chosen over them. I also clearly outlined the rules (passengers join the shortest queue), parameters (arrival rate, service rate, etc) and variables (the queue length, waiting times, etc) of the model.

over utilization, I am setting the arrival rate parameter to be between 0.1 and 1.9.

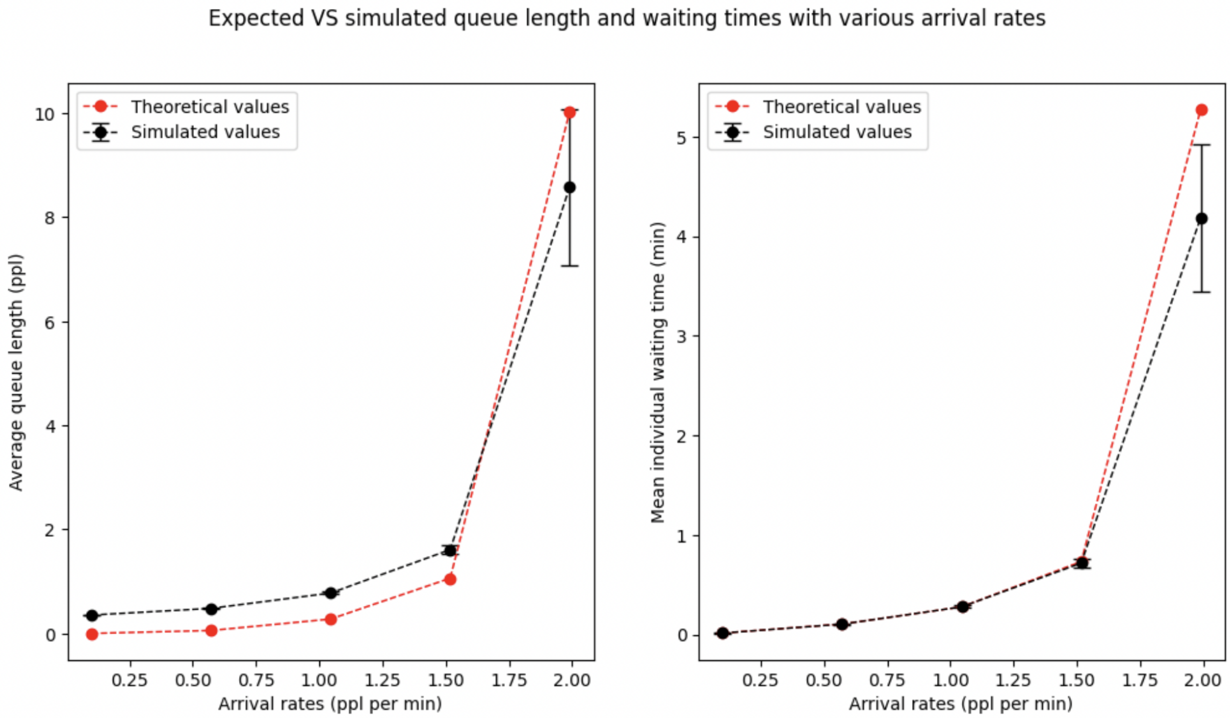

Figure 1. This figure demonstrates simulated and theoretical average queue lengths (left) and individual waiting times (right) corresponding to various arrival rates (λ = 0.1, 0.55, 1,1.45, 1.9). The service rate is held constant at $\mu = 0.5$ *min/traveler. The simulation is run and calculations are performed with an assumption of 1 service station in the airport. The probability of additional check by a senior security officer is not implemented. The simulation is run for 200 trials and for 1000 min. The formula used for average waiting time is:* $W = \frac{\rho\tau}{2(1-\rho)}(1 + \frac{\sigma^2}{\tau^2})$ *. The formula used for average queue length is:* $L = \lambda W$.

While there is some discrepancy between the simulated and theoretical values on the plots above, the values follow the same general trend and overlap. We can see that at arrival rates up to λ = 1.45 ppl/min, the simulated average queue length is greater than the expected value by ~0.5 units. At λ = 1.9 ppl/min, the theoretical value is in the 95% confidence interval of the simulated result, which implies that observed values are in line with the expected value. On the right subplot, we see that the waiting times align perfectly until λ = 1.9 ppl/min. This difference suggests that the system might have not yet reached equilibrium and should be run for a longer period of time to see a closer match. Generally, we see greater confidence intervals, higher waiting times, and longer queue lengths as we approach higher arrival rates due to the fact that ρ is approaching 1 and the system is starting to destabilize. Overall, the plots showcase that the observed values are aligned with the expected values and, hence, the simulation is implemented correctly on a basic level.

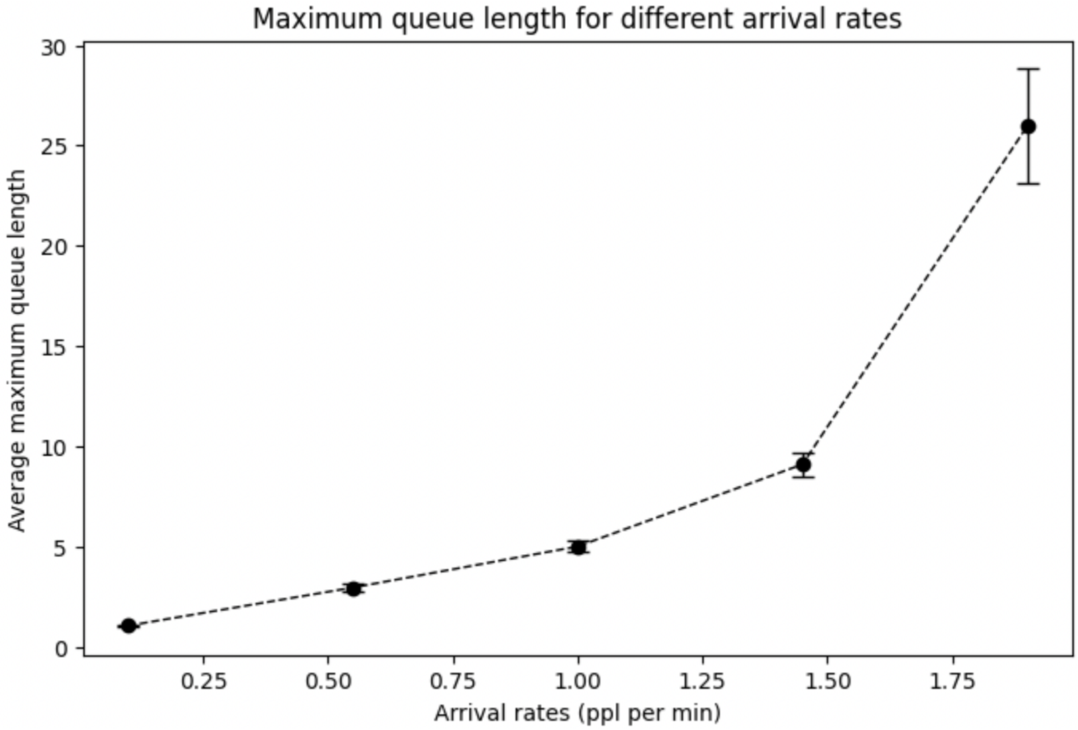

Figure 2. This figure shows the average maximum queue length over 200 trials for various arrival rates (λ = 0.1, 0.55, 1,1.45, 1.9). The service rate is held constant at $\mu = 0.5$ *min/traveler. The simulation is run and calculations are performed with an assumption of 1 service station in the airport. The probability of additional checks by a senior security officer is not implemented. The simulation is run for 1000 min. The formula used for average waiting time is:* $W_n = \frac{\rho\tau/n}{2(1-\rho)}(1 + \frac{\sigma^2}{\tau/n^2})$ *. The formula used for average queue length is:* $L = \lambda W_n$ *.*

Figure 2 shows the maximum queue length without a comparison to the expected values. While I did not derive the formula to make the comparison, we can see that the maximum queue lengths align with the average values. Specifically, we see that the longest queue is ~ 26 people, while the longest average queue (Figure 1) is ~ 9 people. Moreover, the general trend follows the trend displayed in Figure 1, corroborating the idea that the model produces reasonable results. Hence, we can conclude that the metrics align, and the simulated model produces reasonable results for the 1 queue, no senior officer, and various arrival rates.

Multiple queues, no senior security officer

While the simulated model corresponds to the expected values for a scenario with 1 service station, this assumption simplifies the situation dramatically. To be able to gain deeper insights into the real-life problem we are modeling, it is necessary to implement all the key features of the system such as multiple service stations. In this test case, I am adding multiple queues into the simulated model and adjusting theoretical calculations by dividing the arrival rate by the number of stations. This test allows me to assess if multiple service stations were implemented correctly into the model.

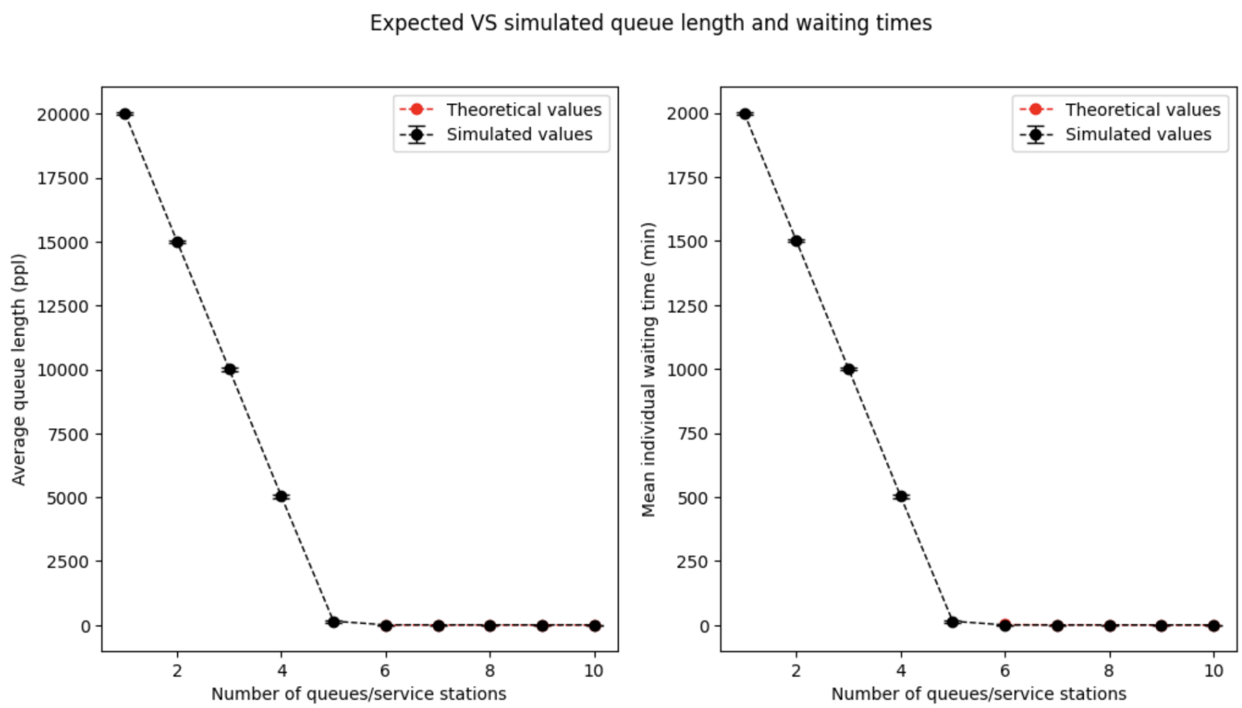

Figure 3. This figure showcases the average queue length (left) and individual waiting times (right) corresponding to various numbers of service stations (up to 10 service stations). This average was computed over 100 trials, and the simulation was run for 500 minutes. The service and arrival rates are held constant at the rates described in the scenario. This simulated model did not implement a senior security officer. The theoretical values are only computed for scenarios where the number of queues is > 6.

This simulation allows us to simulate the real-life scenario closer, while still allowing for theoretical comparison. It should be noted that the theoretical values were calculated for scenarios where the number of queues exceeds 6. This is because under 6 queues, the queue arrival rate is the same or exceeds the service rate leading to queues and waiting times increasing infinitely. This is partly reflected in the plots where we see how the queue length and waiting time decrease drastically as the number of queues reaches 5. The expected values that are present on the plot coincide with the simulated values very closely, indicating that the simulated model works as expected and simulates the scenario correctly.

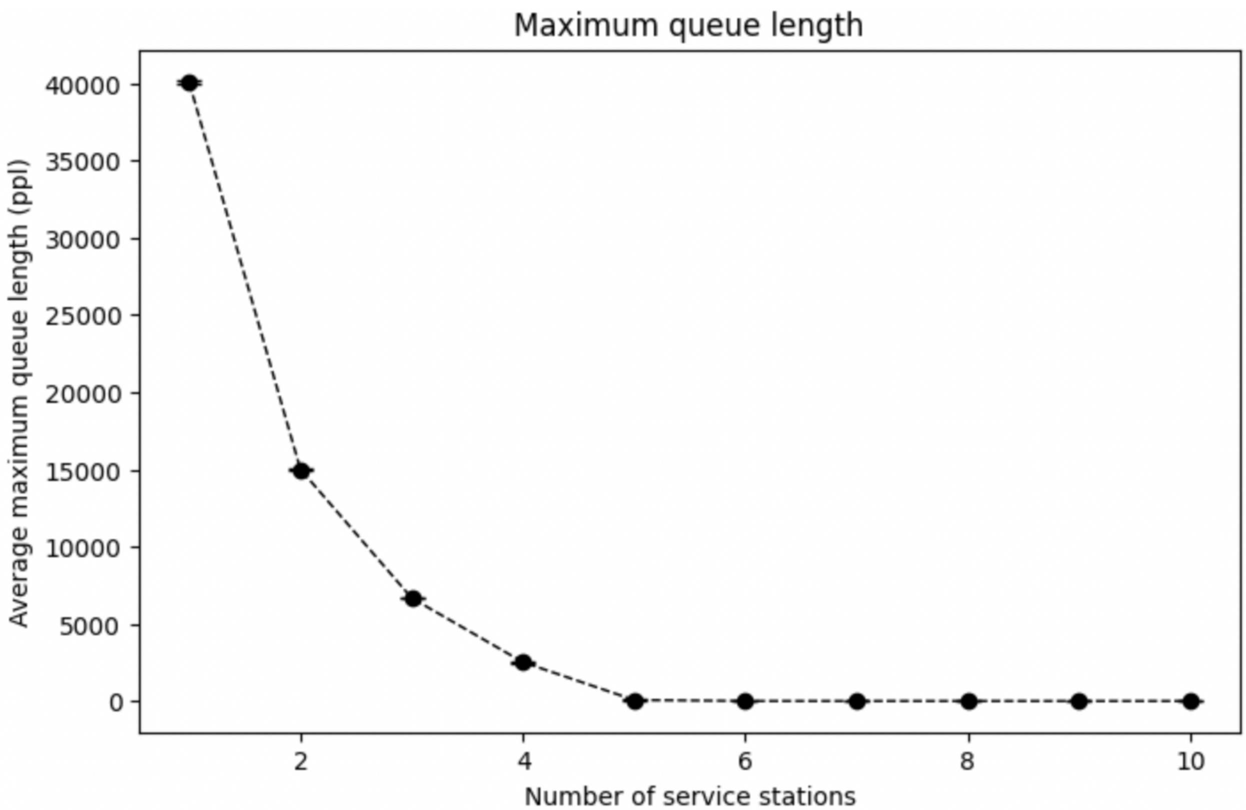

Figure 4. This figure shows the average maximum queue length over a different number of service stations (1 to 10). The simulation was run for 500 minutes for 100 trials. The general trend corresponds to those in Figure 3.

While there are no theoretical values on the plot, we see that the general trend in Figure 4 corresponds to the one in Figure 3, and the maximum values are approximately twice greater than the average queue length values. All the statistics align, indicating that a simulated model with multiple queues produces reasonable results.

Multiple queues with a senior security officer

To model the given scenario fully, I am implementing a chance of an extra security screening, which was given as p = 0.05 (5%) in the problem description. To do so, I am generating a random number between 0 and 1 and comparing it to the given threshold. If the number is lower than the threshold, then the traveler will undergo additional screening that follows a truncated normal distribution with $\mu = 2$ min and $\sigma = 2$ min. It should be noted that the simulation assumes only one senior officer in the airport, meaning that if several people need the officer at approximately the same time, they will have to wait until the officer finishes processing the travelers before them. Running the simulated model with the officer gave me the following results:

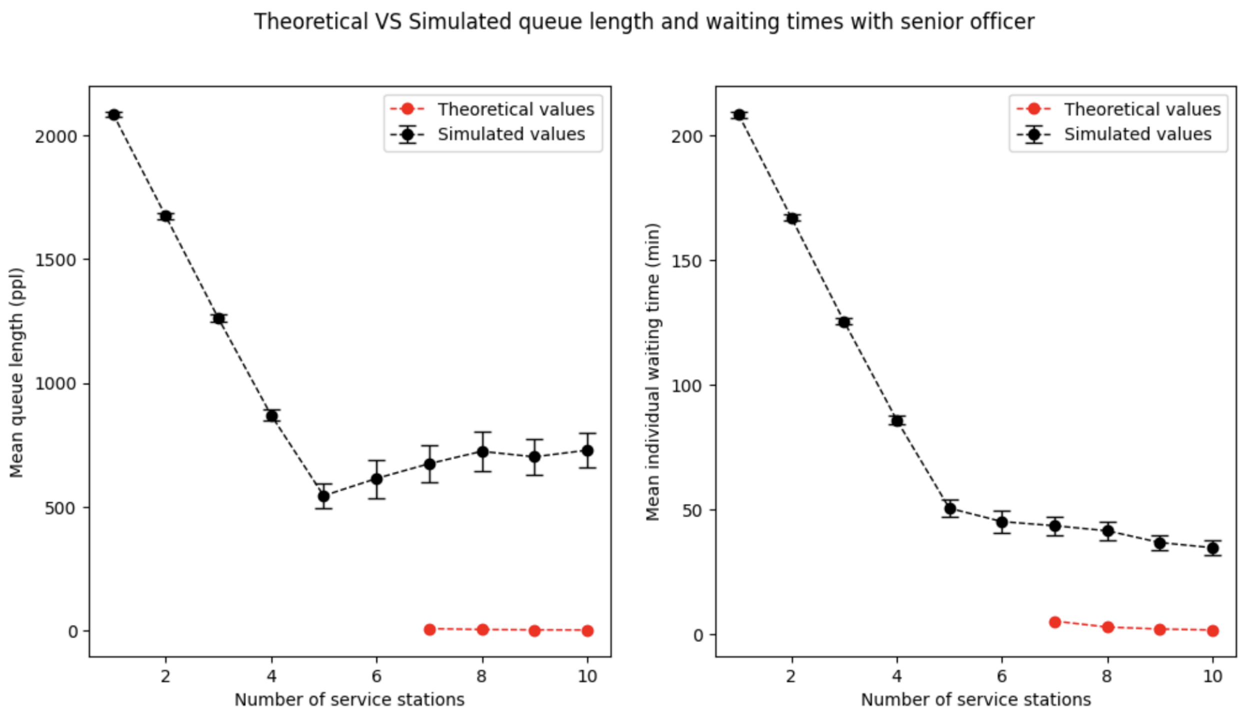

Figure 5. The figure showcases the average queue length and individual waiting time over a different number of service stations. The models implement the probability (p = 0.05) of having to undergo an additional check by a senior security officer. The simulation was run for 500 simulation minutes for 300 trials. The theoretical values are only calculated for the scenarios with the number of stations exceeding 6. The formula used for average waiting time is $W_n = \frac{\rho\tau/n}{2(1-\rho)}(1 + \frac{\sigma^2}{\tau/n^2})$, *where* $\sigma = \sigma_1 + p\sigma_2$ *and* $\tau = \mu_1 + p\mu_2$. *The formula used for average length is* $L = \frac{\lambda}{n}W$.

Figure 5 shows a big discrepancy between the expected and observed results. While we cannot compare the results for all the number of service stations due to the reasons outlined earlier in the report (ρ value must be < 1), we can still observe that the average length differs from the expected length by >500 people, while the waiting time difference is approximately ~50 min. The reason behind the discrepancy can be the fact that the simulation only has one senior security officer for all the queues. For instance, if there are 100 people at the airport (given the arrival rate of 10 travelers per minute, 100 travelers would be at the airport 10 minutes after the opening), roughly 5 of them will have to call for a senior officer. Since the additional service takes on average 2 more minutes for every traveler, people in need of additional screening would accumulate and the waiting time for the queues would increase because people would have to wait for the senior officer. Moreover, if there is a person already waiting for the senior officer in a queue and it happens to be the shortest queue, newly arrived travelers will join that queue not knowing that they will have to wait for the security officer to come to the person before them and also serve that person. This leads to increased average queue length and waiting time.

While I adjusted the theoretical model calculation to account for the probability of additional security checks, the model assumes that a senior security officer is readily available as soon as they are needed. The theoretical calculations do not account for the waiting time in between being served by the senior officer, which results in a big difference in theoretical versus observed results. Hence, the discrepancy is expected.

Another interesting finding that Figure 5 showcases is that the average queue length reaches its minimum when the number of service stations equals 5 and then the length gradually picks back up. One reason for this might be that as the number of available queues increases initially, the number arrival rate per queue decreases leading to a lesser load. However, as the number of queues increases beyond 5, the fact that there is only a single senior officer might influence the queue length. While each individual queue has fewer people, the total number of travelers in the system does not decrease. This means that as more and more people come to the airport, an increasing number of them require additional screening, causing several other queues to wait and accumulate passengers. Hence, the “dip” when the number of service stations equals 5 can be the combination of a decreased arrival rate per queue and a “bottleneck” effect created by a single available senior officer.

However, on the subplot to the right, we can see that the average waiting time continues to decrease gradually. This might suggest that the reduction in the arrival rate can be outweighing the longer queue length. [2] [3] [4]

Conclusion

Having compared a less complicated simulated version of the model to the theoretical expectations, the report showed that the simulated model corresponds to the expected values closely. The results obtained after the implementation of an additional security check probability did not coincide with the expected values. However, this can be attributed to the specific conditions of the real-life scenario we are presented with. Specifically, the fact that the airport only has one senior security officer.

Having analyzed the model above, one can see that the simulation shows a noticeable dip in the average queue length when the number of service stations reaches 5, while the average waiting time keeps decreasing at a lower rate. This can be caused by the decreased load per queue and the "bottleneck" created by a singular senior officer. Based on the findings presented in this report, the airport is recommended to implement 5 service stations in order to optimize the average queue length.

However, it should be noted that the simulation is based on several assumptions outlined in the introduction section of this report, and that further analysis is necessary to determine the optimal number of service stations. Potential further tests may be conducted with respect to different service and arrival rates for the simulation (with the senior officer). These tests would

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[2] **#cs166-TheoreticalAnalysis:** The section "Simulation Analysis" includes theoretical modeling computations for a model 1) with varying arrival rates (1 service station, no senior officer), 2) with varying number of service stations (no senior officer), and 3) with varying number of service stations and a senior security officer. More specifically, I successfully computed the expected queue length and waiting time for all three scenarios and compared them to the observed results. I also offered explanations and analysis as to why the results for model 3) do not coincide with the theoretical values.

[3] **#cs166-EmpiricalAnalysis:** The results obtained from models 1) with varying arrival rates (1 service station, no senior officer), 2) with varying number of service stations (no senior officer), and 3) with varying number of service stations and a senior security officer are analyzed using the following statistics: average queue length, and average waiting times. The graphs include both theoretical and simulated results which allows for a meaningful comparison. Moreover, the plots contain a 95% confidence interval obtained by running the simulations multiple times. My captions are insightful and clear.

[4] **#dataviz:** I have provided data visualizations for every alteration of the model I ran as well as the corresponding theoretical results. My data visualizations allow for a meaningful comparison between the theoretical and simulated values. The visualizations are easy to navigate. They include legends 95% confidence intervals computed by running experiments multiple times >100, and insightful captions.

allow one to see how sensitive the system is to changes in these parameters. Moreover, incorporating more than 1 senior security officer can offer insights into easing the bottleneck effect created by a single officer and, potentially, a different optimal number of service stations.[5] [6] [7] [8]

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AI use statement:

I used ChatGPT to generate my docstrings. I provided my code and explained what it is doing to ChatGPT, then it would format them for me.

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[5] **#cs166-PythonImplementation:** The code submitted via Jupyter notebook file implements a working simulating and provides several test cases to show that it produces expected results (comparison of the simulation with various arrival rates with theoretical values, and comparison of the simulated results for multiple queues with the theoretical values). Appropriate data structures and algorithms were used to create an OOP-based implementation of the simulation. Moreover, useful visualizations are provided to show that the simulation is working and producing expected results (plots with theoretical vs simulated values).

[6] **#cs110-CodeReadability:** The code submitted via Jupyter notebook contains useful docstrings explaining the purpose of functions and class methods as well as helpful in-line comments elaborating on the code. Moreover, separate code blocks are followed or preceded by a text block or a comment explaining what the code is doing.

[7] **#composition:** The written part of the report is easy to follow, concise, unambiguous and communicates the relevant information effectively. This report only includes the information necessary for understanding the model and simulation.

[8] **#professionalism:** The report follows the established guidelines and presents the work professionally. The report uses correct mathematical notation, has insightful and clear figures with captions and legends, as well as a well-organized Python notebook that corroborates the findings presented in the report.