

yacoub: a Python package for Simulating Generalized Fading Channels

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Abstract—We present a well tested Python-based library for simulating and computing generalized fading channels, named **yacoub**. We describe the applicability of **yacoub** using examples in recent communications systems challenges, namely: spectrum sensing, bit error rate computation, and parameter estimation in generalized fading channels. The development of **yacoub** open source and its code is available at <http://github.com/mirca/yacoub>.

I. INTRODUCTION

A. Note on notation

Scalars and random variables are denoted as *italic* small-case letters *e.g.* x ; vectors and random vectors are denoted as *italic*, boldface, small-case letters *e.g.* \mathbf{x} . The n -th component of a vector \mathbf{x} is denoted as x_n . A complex vector of length n is defined as $\mathbf{x} \in \mathbb{C}^{n \times 1}$. All vectors are column vectors. Matrices are denoted as *italic*, boldface, capital letters as in \mathbf{X} ; the identity matrix of order n is denoted as \mathbf{I}_n . We define a discrete-time circularly symmetric Gaussian process \mathbf{z} as any collection of random variables $\mathbf{z} = \mathbf{x} + j\mathbf{y}$, $j \triangleq \sqrt{-1}$, such that \mathbf{x} and \mathbf{y} are iid jointly Gaussian with zero mean vector and covariance matrix given by $\mathbb{E}[\mathbf{z}\mathbf{z}^\dagger]$, in which \mathbf{z}^\dagger means the conjugate transpose of \mathbf{z} . The expectation value wrt a random variable x is denoted as \mathbb{E}_x . The probability of an event A is denoted as $\mathbb{P}(A)$. The indicator function is denoted as $\mathbb{I}(\cdot)$ and it evaluates to 1 if its argument is true and 0 otherwise.

II. THE ACCEPTANCE-REJECTION SAMPLER IN LOG-SPACE

III. MAJORIZATION-MINIMIZATION ALGORITHMS

IV. EXAMPLES

A. Spectrum Sensing in Complex Generalized Fading Channels

The spectrum sensing problem consists in deciding whether or not a given channel frequency band is being occupied by a licensed (primary) user and, in case that such frequency band is available, how to opportunistically allocate secondary users such that the interference on the primary user is negligible.

From a probabilistic point of view, the spectrum sensing problem may be framed as a decision theory problem, as

follows

$$H_0 : \mathbf{y} = \mathbf{w}, \quad (1)$$

$$H_1 : \mathbf{y} = h\mathbf{s} + \mathbf{w}, \quad (2)$$

in which $\mathbf{y} \in \mathbb{C}^{n \times 1}$ is the decoded received vector signal, $\mathbf{w} \in \mathbb{C}^{n \times 1}$ is complex Gaussian noise process with zero mean vector and covariance matrix given as $\sigma^2 \mathbf{I}_n$, and h is the channel gain.

In [?], the authors have shown that the probability distribution of the energy statistic $\tilde{y} \triangleq \mathbf{y}^\dagger \mathbf{y}$ conditioned on the knowledge of h , in case that \mathbf{s} is an M -PSK signal such that every symbol has the same probability of occurrence, $\mathbb{P}(s_n = s) = \frac{1}{M}$, is given as

$$p(\tilde{y}|h, H_1) = 1 - Q_n \left(\sqrt{\frac{2n|h|^2 E_s}{\sigma^2}}, \sqrt{\frac{2\tilde{y}}{\sigma^2}} \right), \quad (3)$$

in which Q_n is the Marcum- Q function and E_s is the energy per symbol.

The pdf of \tilde{y} can be written using the Law of Total Expectation

$$p(\tilde{y}|H_1) = \mathbb{E}_h [p(\tilde{y}|h, H_1)] = \int_{-\infty}^{+\infty} p(\tilde{y}|h, H_1) p(h) dh. \quad (4)$$

Recall that the energy detection rule can be expressed as

$$d_\delta(\tilde{y}) = \mathbb{I}(\tilde{y} > \delta) \quad (5)$$

in which δ is a strictly positive real number known as energy threshold, and $d_\delta(\tilde{y}) = j$, $j \in \{0, 1\}$, means that the detector has decided in favor of the hypothesis H_j .

As a result, the probabilities of false alarm and miss detection can be written as

$$p_f \triangleq \mathbb{P}(d_\delta(\tilde{y}) = 1|H_0) = 1 - p(\delta|H_0), \quad (6)$$

$$p_d \triangleq \mathbb{P}(d_\delta(\tilde{y}) = 0|H_1) = \mathbb{E}_h [p(\delta, h|H_1)], \quad (7)$$

B. Parameter Estimation in Nakagami- m fading

$$p(\mathbf{h}) = \prod_{i=1}^n \frac{2m^m}{\Gamma(m)\Omega^m} h_i^{2m-1} \exp\left(-\frac{mh_i^2}{\Omega}\right) \quad (8)$$

$$= \left(\frac{2m^m}{\Gamma(m)\Omega^m}\right)^n \exp\left(-\frac{m \sum_{i=1}^n h_i^2}{\Omega}\right) \prod_{i=1}^n h_i^{2m-1} \quad (9)$$

$$\log p(\mathbf{h}) \cong n(m(\log m - \log \Omega) - \log \Gamma(m)) - m \left(\frac{\sum_{i=1}^n h_i^2}{\Omega} - 2 \sum_{i=1}^n \log h_i \right) \quad (10)$$

A direct maximum likelihood estimator for (10) has been investigated to be infeasible [?].

Therefore, we use a Majorization-Minimization algorithm to find smooth and easy to optimize upper bounds for $\log p(\mathbf{h})$.

Note that,

$$\begin{aligned} -\log \Gamma(m) &\geq -\log \Gamma(m_t) + (m - m_t) \\ &\quad + \left(m_t - \frac{1}{2}\right) \log(m_t - 1) \\ &\quad - \left(m - \frac{1}{2}\right) \log(m - 1), m \geq m_t > 1 \end{aligned} \quad (11)$$

and

$$m \log m \geq m \log m_t + m - m_t, \quad m > 0 \quad (12)$$

$$-m \log(m - 1) \geq -\log(m - 1) - (m - 1) \left(\log(m_t - 1) + \frac{m - 1}{m_t - 1} - 1 \right), \quad m > 1 \quad (13)$$

(in all inequalities, equality is achieved iff $m = m_t$).

Therefore,

C. BER in Complex $\alpha - \mu$ Fading

V. CONCLUSIONS

ACKNOWLEDGEMENT

The authors would like to thank the Federal University of Campina Grande (UFCG) and the Institute for Advanced Studies in Communications (Iecom) for supporting this research.

$$\begin{aligned}
-\log p(\mathbf{h}) \leq & -n \left(m \log m_t + 2(m - m_t) - m \log \Omega - \log \Gamma(m_t) + \left(m_t - \frac{1}{2} \right) \log(m_t - 1) - \frac{1}{2} \log(m - 1) \right. \\
& \left. - (m - 1) \left(\log(m_t - 1) + \frac{m - 1}{m_t - 1} - 1 \right) \right) + m \left(\frac{\sum_{i=1}^n h_i^2}{\Omega} - 2 \sum_{i=1}^n \log h_i \right), m \geq m_t > 1. \quad (14)
\end{aligned}$$

$$\frac{\partial g(m|m_t)}{\partial m} = -n \left(\log m_t + 2 - \log \Omega - \frac{1}{2(m - 1)} - (\log(m_t - 1) - 1) + \frac{-2m + 1}{m_t - 1} \right) + \frac{\sum_{i=1}^n h_i^2}{\Omega} - 2 \sum_{i=1}^n \log h_i = 0 \quad (15)$$

$$2(m - 1) \left(-n (\log m_t + 2 - \log \Omega - (\log(m_t - 1) - 1)) + \frac{\sum_{i=1}^n h_i^2}{\Omega} - 2 \sum_{i=1}^n \log h_i \right) + n - \frac{2n}{m_t - 1} (-2m^2 + 3m - 1) = 0 \quad (16)$$

$$2(m - 1)a + n - \frac{2n}{m_t - 1} (-2m^2 + 3m - 1) = 0 \quad (17)$$

$$m^2 \left(\frac{4n}{m_t - 1} \right) + m \left(\frac{6n}{m_t - 1} + 2a \right) - 2a + n \left(\frac{m_t + 1}{m_t - 1} \right) = 0 \quad (18)$$

$$(19)$$