`yacoub`: a Python package for Simulating Generalized Fading Channels

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***Abstract*—We present a well tested Python-based library for simulating and computing generalized fading channels, named `yacoub`. We describe the applicability of `yacoub` using examples in recent communications systems challenges, namely: spectrum sensing, bit error rate computation, and parameter estimation in generalized fading channels. For the latter, we develop an iterative algorithm based on the Majorization-Minimization framework. The development of `yacoub` is open source and its code is avaliable at `http://github.com/mirca/yacoub`.**

I. Introduction

Note on notation

Scalars and random variables are denoted as *italic* small-case letters *e.g.* x; vectors and random vectors are denoted as *italic*, boldface, small-case letters *e.g.* $\boldsymbol{x}$. The $n-$th component of a vector $\boldsymbol{x}$ is denoted as x_n. A complex vector of length n is defined as $\boldsymbol{x} \in \mathbb{C}^{n\times 1}$. All vectors are column vectors. Matrices are denoted as *italic*, boldface, capital letters as in $\boldsymbol{X}$; the identity matrix of order n is denoted as $\boldsymbol{I}_n$. We define a discrete-time circularly symmetric Gaussian process $\boldsymbol{z}$ as any collection of random varibles $\boldsymbol{z} = \boldsymbol{x} + j\boldsymbol{y}$, $j \triangleq \sqrt{-1}$, such that $\boldsymbol{x}$ and $\boldsymbol{y}$ are iid jointly Gaussian with zero mean vector and covariance matrix given by $\mathbb{E}\left[\boldsymbol{z}\boldsymbol{z}^\dagger\right]$, in which $\boldsymbol{z}^\dagger$ means the conjugate transpose of $\boldsymbol{z}$. The expectation value wrt a random variable x is denoted as $\mathbb{E}_x$. The probability of an event A is denoted as $\mathbb{P}(A)$. The indicator function is denoted as $\mathbb{I}(\cdot)$ and it evaluates to one if its argument is true and zero otherwise.

II. The Acceptance-Rejection sampler in log-space

III. Majorization-Minimization Algorithms

IV. Examples

A. Spectrum Sensing in Complex Generalized Fading Channels

The spectrum sensing problem consists in deciding whether or not a given channel frequency band is being occupied by a licensed (primary) user and, in case that such frequency band is available, how to opportuniscally allocate secondary users such that the interference on the primary user is negligible.

From a probabilistic point of view, the spectrum sensing problem may be framed as a decision theory problem, as follows

$$H_0 : \ \boldsymbol{y} = \boldsymbol{w}, \tag{1}$$

$$H_1 : \ \boldsymbol{y} = h\boldsymbol{s} + \boldsymbol{w}, \tag{2}$$

in which $\boldsymbol{y} \in \mathbb{C}^{n\times 1}$ is the decoded received vector signal, $\boldsymbol{w} \in \mathbb{C}^{n\times 1}$ is complex Gaussian noise process with zero mean vector and covariance matrix given as $\sigma^2\boldsymbol{I}_n$, and h is the channel gain.

In [**?**], the authors have shown that the probability distribution of the energy statistic $\tilde{y} \triangleq \boldsymbol{y}^\dagger\boldsymbol{y}$ conditioned on the knowledge of h, in case that $\boldsymbol{s}$ is an M-PSK signal such that every symbol has the same probability of occurrence, $\mathbb{P}(s_n = s) = \frac{1}{M}$, is given as

$$p(\tilde{y}|h, H_1) = 1 - Q_n\left(\sqrt{\frac{2n|h|^2E_s}{\sigma^2}}, \sqrt{\frac{2\tilde{y}}{\sigma^2}}\right), \tag{3}$$

in which Q_n is the Marcum-Q function and E_s is the energy per symbol.

The pdf of $\tilde{y}$ can be written using the Law of Total Expectation

$$p(\tilde{y}|H_1) = \mathbb{E}_h\left[p(\tilde{y}|h, H_1)\right] = \int_{-\infty}^{+\infty} p(\tilde{y}|h, H_1)p(h)\,\mathrm{d}h. \tag{4}$$

Recall that the energy detection rule can be expressed as

$$d_\delta(\tilde{y}) = \mathbb{I}(\tilde{y} > \delta) \tag{5}$$

in which δ is a strictly positive real number known as energy threshold, and $d_\delta(\tilde{y}) = j, \ j \in \{0, 1\}$, means that the detector has decided in favor of the hypothesis H_j.

As a result, the probabilities of false alarm and miss detection can be written as

$$p_f \triangleq \mathbb{P}\left(d_\delta(\tilde{y}) = 1|H_0\right) = 1 - p(\delta|H_0), \tag{6}$$

$$p_d \triangleq \mathbb{P}\left(d_\delta(\tilde{y}) = 0|H_1\right) = \mathbb{E}_h\left[p(\delta, h|H_1)\right], \tag{7}$$

B. Parameter Estimation in Nakagami-m fading

$$p(\boldsymbol{h}) = \prod_{i=1}^{n} \frac{2m^m}{\Gamma(m)\Omega^m} h_i^{2m-1} \exp\left(-\frac{mh_i^2}{\Omega}\right) \tag{8}$$

$$= \left(\frac{2m^m}{\Gamma(m)\Omega^m}\right)^n \exp\left(-\frac{m\sum_{i=1}^n h_i^2}{\Omega}\right) \prod_{i=1}^{n} h_i^{2m-1} \tag{9}$$

$$\log p(\boldsymbol{h}) \cong n\left(m\left(\log m - \log \Omega\right) - \log \Gamma(m)\right) \\ - m\left(\frac{\sum_{i=1}^{n} h_i^2}{\Omega} - 2\sum_{i=1}^{n} \log h_i\right) \quad (10)$$

A direct maximum likelihood estimator for (10) has been investigated to be infeasible [**?**].

Therefore, we use a Majorization-Minimization algorithm to find smooth and easy to optimize upper bounds for $\log p(\boldsymbol{h})$.

A parte problemática aqui é encontrar um limitante inferior para a função Gamma. Talvez serie de Taylor resolva. Isto é, precisamos encontrar inequacoes da forma

$$m \log m \geq \quad (11)$$

$$-\log \Gamma(m) \geq \quad (12)$$

C. BER in Complex $\alpha - \mu$ Fading

Consider the system

$$\boldsymbol{y} = h\boldsymbol{s} + \boldsymbol{w} \quad (13)$$

in which $\boldsymbol{s} \in \mathbb{C}^{n \times 1}$ is a complex On-Off Keying (OOK) signal, h is a complex $\alpha - \mu$ random variable and $\boldsymbol{w}$ is a complex Gaussian process with zero mean vector and covariance matrix equals $\sigma^2 \boldsymbol{I}_n$, and $\boldsymbol{y}$ is the received complex vector signal.

Assume that the OOK symbols are equiprobable and that there exist no interference between the in-phase and quadrature components, then the probability of one bit error is given as

$$p_e = \frac{1}{2}\left(\mathbb{P}\left(\hat{y}_i = 0 | s_i = 1\right) + \mathbb{P}\left(\hat{y}_i = 1 | s_i = 0\right)\right). \quad (14)$$

Assume that the decoded vector $\hat{\boldsymbol{y}}$ is estimated using the minimum distance decoding rule, i.e.,

V. Conclusions

Acknowledgement

The authors would like to thank the Federal University of Campina Grande (UFCG) and the Institute for Advanced Studies in Communications (Iecom) for supporting this research.