# Simulating Generalized Fading Channels in Python

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***Abstract*— We present a novel and well tested Python-based library for simulating and computing generalized fading channels, considering $\alpha$-$\mu$, $\kappa$-$\mu$, and $\eta$-$\mu$ distributions. We describe the applicability of this library using a few communications channels scenarios impaired by generalized fading, namely, spectrum sensing and bit error rate computation. The development of the library is open source and its code along with examples are available at `http://github.com/mirca/maoud`.**

***Keywords*— Generalized Fading, Simulation, Python**

## I. Introduction

The study of modern wireless communications systems heavily relies on fading channel simulation. By fading channel simulation, we refer to the generation of samples from a probability distribution that resembles the effects and impairments caused by real communications fading channels on the transmitted signal.

Although accurate and precise distributions for generalized fading have been estabilished in the literature, such as $\alpha$-$\mu$ [1], $\kappa$-$\mu$ [2] and $\eta$-$\mu$ [2], the generation of samples following these distributions is usually a time-consuming task.

In [3], the authors built an efficient algorithm, based on the rejection method, for generation of samples from those distributions. However, there are neither open nor closed source implementations available to the scientific community.

In this paper, we present an open source Python package for generation of samples following the $\alpha$-$\mu$, $\kappa$-$\mu$, and $\eta$-$\mu$ distributions. The usefullness of ths package is illustrated through well known examples involving spectrum sensing and bit error rate (BER) computation.

Section II presents the well known rejection method which is used as the basis for the proposed implementation. Section III and IV demonstrates the features of the proposed package in spectrum sensing over complex generalized fading channels and bit error rate computation on $\alpha$-$\mu$ fading. Section V states the final remarks.

## II. Rejection Sampling

The algorithm implemented in this work, originally proposed in [3], is based on the rejection method, which is one of the most used for the generation of samples of a probability distribution.

We briefly describe the algorithm as follows. Consider that we would like to simulate a random variable $x$ with a probability density $f$. The basic idea is to find an alternative probability density $g$, from which there exists an efficient procedure to generate samples, e.g., inverse methods. Additionally, it is required that $g$ majorizes $f$, i.e., we assume that there exists a real number $c$ such that $f(x)/g(x) < c$.

Basically, the steps of the rejection algorithm are[4]:

1) generate a sample $y$ from the probability density $g$;
2) generate a sample $u$ from a uniform density with support $(0, 1)$;
3) check $u \leq \frac{f(y)}{cg(y)}$; if true then accept $y$; otherwise, return to step 1.

The implementation of this algorithm along with the code to efficiently generate samples from the $\alpha$-$\mu$, $\eta$-$\mu$, and $\kappa$-$\mu$ distributions is available at `http://github.com/mirca/maoud`.

In order to validate the implementation, we apply the routines to two use cases in wireless communications: spectrum sensing and bit error rate computation.

## III. Spectrum Sensing in Complex Generalized Fading Channels

The spectrum sensing problem consists in deciding whether or not a given channel frequency band is being occupied by a licensed (primary) user and, in case that such frequency band is available, how to opportuniscally allocate secondary users such that the interference on the primary user is negligible.

From a probabilistic point of view, the spectrum sensing problem may be framed as a decision theory problem, as follows

$$H_0 : \boldsymbol{y} = \boldsymbol{w}, \tag{1}$$

$$H_1 : \boldsymbol{y} = h\boldsymbol{s} + \boldsymbol{w}, \tag{2}$$

in which $\boldsymbol{y} \in \mathbb{C}^{n\times 1}$ is the received vector signal, $\boldsymbol{w} \in \mathbb{C}^{n\times 1}$ is complex Gaussian noise process with zero mean vector and covariance matrix given as $\sigma^2 \boldsymbol{I}_n$, and $h$ is the channel gain.

In [5], the authors have shown that the cumulative distribution function (cdf) of the energy statistic $\tilde{y} \triangleq \boldsymbol{y}^\dagger \boldsymbol{y}$ conditioned on the knowledge of $h$, in case that $\boldsymbol{s}$ is an $M$-PSK signal such that every symbol has the same probability of occurrence, $\mathbb{P}(s_n = s) = \frac{1}{M}$, is given as

$$P(\tilde{y}|h, H_1) = 1 - \mathrm{Q}_n\left(\sqrt{\frac{2n|h|^2 E_s}{\sigma^2}}, \sqrt{\frac{2\tilde{y}}{\sigma^2}}\right), \tag{3}$$

in which $\mathrm{Q}_n$ is the Marcum-Q function and $E_s$ is the energy per symbol.

Recall that the energy detection rule can be expressed as

$$d_\delta(\tilde{y}) = \mathbb{I}(\tilde{y} > \delta) \tag{4}$$

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in which $\mathbb{I}$ is the indicator function, $\delta$ is a strictly positive real number known as energy threshold, and $d_\delta(\tilde{y}) = j$, $j \in \{0, 1\}$, means that the detector has decided in favor of the hypothesis $H_j$.

As a result, the probabilities of false alarm and miss detection can be written as

$$\mathbb{P}\left(d_\delta(\tilde{y}) = 1 | H_0\right) = 1 - P(\delta|H_0) = 1 - \gamma\left(n, \frac{\delta}{\sigma^2}\right), \quad (5)$$

$$\begin{aligned}\mathbb{P}\left(d_\delta(\tilde{y}) = 0 | H_1\right) &= \mathbb{E}_h\left(P(\delta|h, H_1)\right) \\ &= \int_{-\infty}^{+\infty} P(\delta|h, H_1) p(h)\, \mathrm{d}h,\end{aligned} \quad (6)$$

in which $\gamma$ is the regularized lower incomplete Gamma function, $p(h)$ is the pdf of the fading, and $\delta$ is the energy detection threshold.

The performance of detection schemes can be measured by computing the Receiver Operating Characteristic (ROC), which consists in varying $\delta$ and computing the pairs of probability of false alarm and miss detection, as illustrated in Fig. 1, in which the solid curve represents the theorectical probabilities as stated on (5) and (6) for different values of $\delta$, whereas bullets represent Monte Carlo simulations acquired with $10^6$ realizations. The input signal $\boldsymbol{s}$ consists in a vector of length $n = 25$ in which each entry represents a symbol of a 64-PSK constellation. Symbols are assumed to be equiprobable and the signal-to-noise ratio is set to 5 dB.

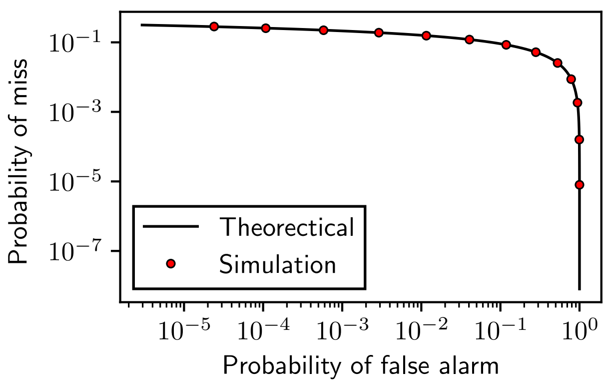


Fig. 1

RECEIVER OPERATING CHARACTERISTIC FOR THE ENERGY DETECTOR IN $\alpha$-$\mu$ FADING CHANNEL WITH $\alpha = 2$ AND $\mu = 1$.

## IV. BIT ERROR RATE IN $\alpha$-$\mu$ FADING

Consider the system

$$y = hs + w \quad (7)$$

in which $s \in \{0, a\}$, $a \in \mathbb{R}_+$, is a transmitted signal, $h$ is an $\alpha$-$\mu$ random variable and $\boldsymbol{w}$ is a Gaussian random variable with zero mean vector and variance $\sigma^2$, and $y$ is the received signal.

Assume that the binary symbols are equiprobable, then the probability of one bit error is given as

$$p_e = \frac{1}{2}\left(\mathbb{P}\left(\hat{y} = 0 | s = a\right) + \mathbb{P}\left(\hat{y} = 1 | s = 0\right)\right). \quad (8)$$

Further, assume that the decoded bit $\hat{y}$ is estimated using the minimum distance decoding rule, i.e.,

$$\hat{y} = \mathbb{I}(|y - a|^2 < |y|^2) \quad (9)$$

therefore

$$\begin{aligned}\mathbb{P}\left(\hat{y} = 1 | s = 0\right) &= \mathbb{P}\left(|w - a|^2 - |w|^2 < 0\right) \\ &= \mathbb{P}\left(w > \frac{a}{2}\right) = 1 - \Phi\left(\frac{a}{2\sigma}\right)\end{aligned} \quad (10)$$

and likewise

$$\begin{aligned}\mathbb{P}\left(\hat{y} = 0 | s = a\right) &= \mathbb{P}\left(ha + w < \frac{a}{2}\right) \\ &= \mathbb{E}_h\left(\Phi\left(\frac{a(1 - 2h)}{2\sigma}\right)\right).\end{aligned} \quad (11)$$

Fig. 2 shows the expression of the BER as a function of the signal-to-noise ratio. As we can see, the simulation presents an excellent agreement against theorectical results.

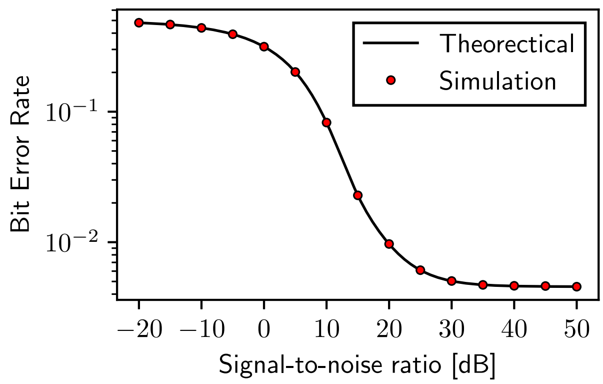


Fig. 2

BER *versus* SNR.

## V. CONCLUSIONS

This paper presents a novel Python-based library for generalized fading simulation. The implementation is tested and validated in two use cases, namely, spectrum sensing and BER computation.

The computational results show that the implementation presents a satisfactory agreement when compared with the theorectical formulation.

We expect that the proposed library, which is open source, will be usefull to researchers, professors, and students of the field of wireless communications.

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