yacoub: a Python package for Simulating Generalized Fading Channels

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Abstract—We present a well tested Python-based library for simulating and computing generalized fading channels, named yacoub. We describe the applicability of yacoub using examples in recent communications systems challenges, namely: spectrum sensing, bit error rate computation, and parameter estimation in generalized fading channels. For the latter, we develop an iterative algorithm based on the Majorization-Minimization framework. The development of yacoub is open source and its code is avaliable at http://github.com/mirca/yacoub.

I. Introduction

Note on notation

Scalars and random variables are denoted as italic smallcase letters e.g. x; vectors and random vectors are denoted as *italic*, boldface, small-case letters e.g. x. The n-th component of a vector x is denoted as x_n . A complex vector of length n is defined as $x \in \mathbb{C}^{n \times 1}$. All vectors are column vectors. Matrices are denoted as italic, boldface, capital letters as in X; the identity matrix of order n is denoted as I_n . We define a discrete-time circularly symmetric Gaussian process z as any collection of random varibles z = x + jy, $j \triangleq \sqrt{-1}$, such that x and y are iid jointly Gaussian with zero mean vector and covariance matrix given by $\mathbb{E} |zz^{\dagger}|$, in which z^{\dagger} means the conjugate transpose of z. The expectation value wrt a random variable x is denoted as \mathbb{E}_x . The probability of an event A is denoted as $\mathbb{P}(A)$. The indicator function is denoted as $\mathbb{I}(\cdot)$ and it evaluates to one if its argument is true and zero otherwise.

II. THE ACCEPTANCE-REJECTION SAMPLER IN LOG-SPACE

III. MAJORIZATION-MINIMIZATION ALGORITHMS

IV. EXAMPLES

A. Spectrum Sensing in Complex Generalized Fading Channels

The spectrum sensing problem consists in deciding whether or not a given channel frequency band is being occupied by a licensed (primary) user and, in case that such frequency band is available, how to opportuniscally allocate secondary users such that the interference on the primary user is negligible.

From a probabilistic point of view, the spectrum sensing problem may be framed as a decision theory problem, as follows

$$H_0: \ \boldsymbol{y} = \boldsymbol{w},\tag{1}$$

$$H_1: \ \boldsymbol{y} = h\boldsymbol{s} + \boldsymbol{w},\tag{2}$$

in which $\boldsymbol{y} \in \mathbb{C}^{n \times 1}$ is the decoded received vector signal, $\boldsymbol{w} \in \mathbb{C}^{n \times 1}$ is complex Gaussian noise process with zero mean vector and covariance matrix given as $\sigma^2 \boldsymbol{I}_n$, and h is the channel gain.

In [?], the authors have shown that the probability distribution of the energy statistic $\tilde{y} \triangleq y^{\dagger}y$ conditioned on the knowledge of h, in case that s is an M-PSK signal such that every symbol has the same probability of occurrence, $\mathbb{P}(s_n=s)=\frac{1}{M}$, is given as

$$p(\tilde{y}|h, H_1) = 1 - Q_n \left(\sqrt{\frac{2n|h|^2 E_s}{\sigma^2}}, \sqrt{\frac{2\tilde{y}}{\sigma^2}} \right), \quad (3)$$

in which Q_n is the Marcum-Q function and E_s is the energy per symbol.

The pdf of \tilde{y} can be written using the Law of Total Expectation

$$p(\tilde{y}|H_1) = \mathbb{E}_h \left[p(\tilde{y}|h, H_1) \right] = \int_{-\infty}^{+\infty} p(\tilde{y}|h, H_1) p(h) \, dh.$$
 (4)

Recall that the energy detection rule can be expressed as

$$d_{\delta}(\tilde{y}) = \mathbb{I}(\tilde{y} > \delta) \tag{5}$$

in which δ is a strictly positive real number known as energy threshold, and $d_{\delta}(\tilde{y}) = j, \ j \in \{0,1\}$, means that the detector has decided in favor of the hypothesis H_{j} .

As a result, the probabilities of false alarm and miss detection can be written as

$$p_f \triangleq \mathbb{P}\left(d_\delta(\tilde{y}) = 1|H_0\right) = 1 - p(\delta|H_0),\tag{6}$$

$$p_d \triangleq \mathbb{P}\left(d_{\delta}(\tilde{y}) = 0 | H_1\right) = \mathbb{E}_h\left[p(\delta, h | H_1)\right],\tag{7}$$

B. Parameter Estimation in Nakagami-m fading

$$p(\mathbf{h}) = \prod_{i=1}^{n} \frac{2m^m}{\Gamma(m)\Omega^m} h_i^{2m-1} \exp\left(-\frac{mh_i^2}{\Omega}\right)$$
 (8)

$$= \left(\frac{2m^m}{\Gamma(m)\Omega^m}\right)^n \exp\left(-\frac{m\sum_{i=1}^n h_i^2}{\Omega}\right) \prod_{i=1}^n h_i^{2m-1} \quad (9)$$

$$\log p(\mathbf{h}) \cong n \left(m \left(\log m - \log \Omega \right) - \log \Gamma(m) \right)$$
$$- m \left(\frac{\sum_{i=1}^{n} h_i^2}{\Omega} - 2 \sum_{i=1}^{n} \log h_i \right)$$
(10)

A direct maximum likelihood estimator for (10) has been investigated to be infeasible [?].

Therefore, we use a Majorization-Minimization algorithm to find smooth and easy to optimize upper bounds for $\log p(h)$.

A parte problemática aqui é encontrar um limitante inferior para a função Gamma. Talvez serie de Taylor resolva. Isto é, precisamos encontrar inequacoes da forma

Inequacao (24) de Sun and Palomar 2016 resolve!!!

$$m \log m \ge$$
 (11)

$$-\log\Gamma(m) \ge \tag{12}$$

C. BER in Complex $\alpha - \mu$ Fading

Consider the system

$$y = hs + w \tag{13}$$

in which $s \in \mathbb{C}^{n \times 1}$ is a complex On-Off Keying (OOK) signal, h is a complex $\alpha - \mu$ random variable and w is a complex Gaussian process with zero mean vector and covariance matrix equals $\sigma^2 \mathbf{I}_n$, and y is the received complex vector signal.

Assume that the OOK symbols are equiprobable and that there exist no interference between the in-phase and quadrature components, then the probability of one bit error is given as

$$p_e = \frac{1}{2} \left(\mathbb{P} \left(\hat{y}_i = 0 | s_i = 1 \right) + \mathbb{P} \left(\hat{y}_i = 1 | s_i = 0 \right) \right).$$
 (14)

Assume that the decoded vector \hat{y} is estimated using the minimum distance decoding rule, i.e.,

V. CONCLUSIONS

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