Risk Parity: Convex, Non-convex, and Hierarchical Models

a talk by

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(in collab. with Prof. Daniel Palomar)

ELEC/IEDA3180 Data-driven Portfolio Optimization

May 15th, 2020

Contents

Modern Portfolio Theory

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- assume the log-returns $\boldsymbol{r}$ are i.i.d. Gaussian, then for a portfolio $\boldsymbol{w}$:
  - expected return: $\mathbb{E}\left[\boldsymbol{w}^\top \boldsymbol{r}\right] = \boldsymbol{w}^\top \boldsymbol{\mu}$
  - variance: $\mathbb{V}\left[\boldsymbol{w}^\top \boldsymbol{r}\right] = \boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w}$

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- criticisms:
  1. sensitivity to estimation errors in $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$
  2. does not consider risk diversification

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Risk Parity

From Capital to Risk Allocation

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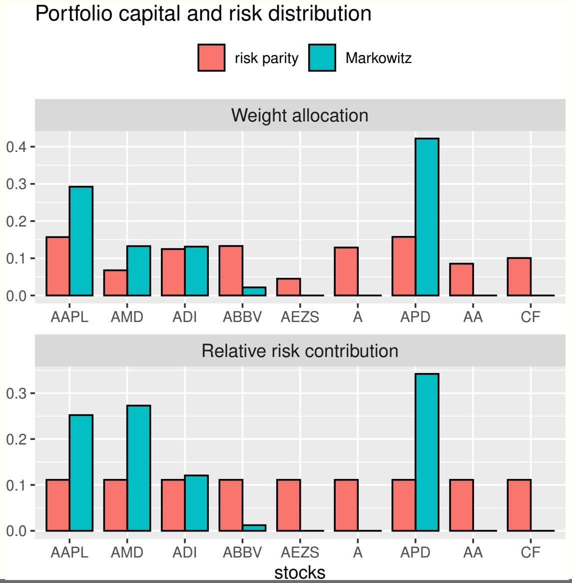
- allocate **risk** rather than **capital**
- first risk parity fund (1996): Bridgewater's All Weather
- Bridgewater publishes "Engineering Targeted Returns and Risks" (2004)
- 2011-onwards: risk parity gains broad adoption
- **basic idea**: design a portfolio such that the risk is equally distributed among the asset classes (stocks, bonds, real state, etc.)

From Capital to Risk Allocation



Problem Formulation

- volatility: $\sigma(\boldsymbol{w}) = \sqrt{\boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w}}$
- from Euler's theorem:

$$\sigma(\boldsymbol{w}) = \sum_i w_i \frac{\partial \sigma(\boldsymbol{w})}{\partial w_i} = \sum_i \frac{w_i \left(\boldsymbol{\Sigma}\boldsymbol{w}\right)_i}{\sqrt{\boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w}}}$$

- $\dfrac{\partial \sigma(\boldsymbol{w})}{\partial w_i}$: **marginal risk contribution**
- measures the sensitivity of the portfolio volatility to the i-th asset
- it can be defined for other risk measures like VaR and CVaR

Problem Formulation

- **risk contribution** of the i-th asset to the total risk $\sigma(\boldsymbol{w})$:

$$\mathsf{RC}_i = w_i \frac{\partial \sigma(\boldsymbol{w})}{\partial w_i}$$

- from Euler's theorem:

$$\sum_i \mathsf{RC}_i = \sigma(\boldsymbol{w})$$

- **relative risk contribution** of the i-th asset to the total risk $\sigma(\boldsymbol{w})$:

$$\mathsf{RCC}_i = \frac{\mathsf{RC}_i}{\sigma(\boldsymbol{w})} = \frac{w_i \left(\boldsymbol{\Sigma}\boldsymbol{w}\right)_i}{\boldsymbol{w}^\top \boldsymbol{\Sigma}\boldsymbol{w}},$$

so that

$$\sum_i \mathsf{RCC}_i = 1$$

Risk Parity: Problem Formulation

- **risk parity portfolio**: $\text{RCC}_i = b_i,\ i = 1, 2, \ldots, N$
- **feasibility** problem:

$$\begin{array}{ll} \underset{\boldsymbol{w} \succ \mathbf{0}}{\text{find}} & \boldsymbol{w} \\ \text{subject to} & \dfrac{w_i \left(\boldsymbol{\Sigma} \boldsymbol{w}\right)_i}{\boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w}} = b_i,\ i = 1, 2, \ldots, N \end{array}$$

  doesn't look trivial
- **approximation**: Σ is diagonal

$$w_i \propto \frac{\sqrt{b_i}}{\sqrt{\boldsymbol{\Sigma}_{ii}}}$$

  i.e. **inverse volatility portfolio**

Solution to Risk Parity

- Spinu (2013):
  - change of variables $\boldsymbol{x} = \dfrac{\boldsymbol{w}}{\sqrt{\boldsymbol{w}^\top \boldsymbol{\Sigma} \boldsymbol{w}}}$, then $\boldsymbol{w} = \dfrac{\boldsymbol{x}}{\mathbf{1}^\top \boldsymbol{x}}$
  - problem becomes: $\boldsymbol{\Sigma}\boldsymbol{x} = \dfrac{\boldsymbol{b}}{\boldsymbol{x}}$
  - $\underset{\boldsymbol{x} \succ \mathbf{0}}{\text{minimize}}\ f(\boldsymbol{x}) = \dfrac{1}{2}\boldsymbol{x}^\top \boldsymbol{\Sigma}\boldsymbol{x} - \boldsymbol{b}^\top \log(\boldsymbol{x})$
  - **optimality condition:** $\nabla f(\boldsymbol{x}) = \boldsymbol{\Sigma}\boldsymbol{x} - \dfrac{\boldsymbol{b}}{\boldsymbol{x}} = \mathbf{0}$
  - then done? it's not a QP, LP, etc.

Cyclical Coordinate Descent (CCD)

Griveau-Billion (2013):

Algorithm 1: CCD to solve risk parity

$k \leftarrow 0$, initial $\boldsymbol{x}^{(0)}$
while *not converged* **do**
  for $i = 1$ *to* N **do**
    $x_i^{k+1} \leftarrow \arg\min_{x_i} f\left(x_1^{k+1}, \ldots, x_i^k, \ldots, x_N^k\right)$
  end
end

- closed-form update:

$$x_i^\star = \frac{-\left(\boldsymbol{x}_{-i}^\top \boldsymbol{\Sigma}_{:,i}\right) + \sqrt{\left(\boldsymbol{x}_{-i}^\top \boldsymbol{\Sigma}_{:,i}\right)^2 + 4\boldsymbol{\Sigma}_{ii} b_i}}{2\boldsymbol{\Sigma}_{ii}},$$

where $\boldsymbol{x}_{-i} = \left[x_1, \ldots, x_{i-1}, 0, x_{i+1}, \ldots, x_N\right]^\top$

Non-convex Formulations

- limitations of the previous formulation: $\boldsymbol{w} \succ \mathbf{0}, \mathbf{1}^\top \boldsymbol{w} = 1$

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- how to "solve" the non-convex formulation?
- general purpose solvers: slow ☹

Non-convex Formulations

- Feng & Palomar (2015):

$$\begin{array}{ll} \underset{\boldsymbol{w}}{\text{minimize}} & \sum_{i=1}^{N} \left[g_i(\boldsymbol{w})\right]^2 + \lambda F(\boldsymbol{w}) \\ \text{subject to} & \mathbf{1}^\top \boldsymbol{w} = 1,\ \boldsymbol{w} \in \mathcal{W} \end{array}$$

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- "convexify" the objective function: fast 😀
- **basic idea**: find a "nice" approximation to the non-convex term, solve it, repeat (Scutari et al. 2014)
- "nice" approximation for $g_i(\boldsymbol{w})$: first order Taylor expansion (Feng & Palomar 2015)

Open Source Software Packages

- dppalomar/riskParityPortfolio (R version)
- dppalomar/riskparity.py (Python version)

Basic Usage

- R version:

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```
> library(riskParityPortfolio)
> my_portfolio <- riskParityPortfolio(Sigma = Sigma, b = b)
> names(my_portfolio)
[1] "w"                  "risk_contributions"
```

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- Python version:

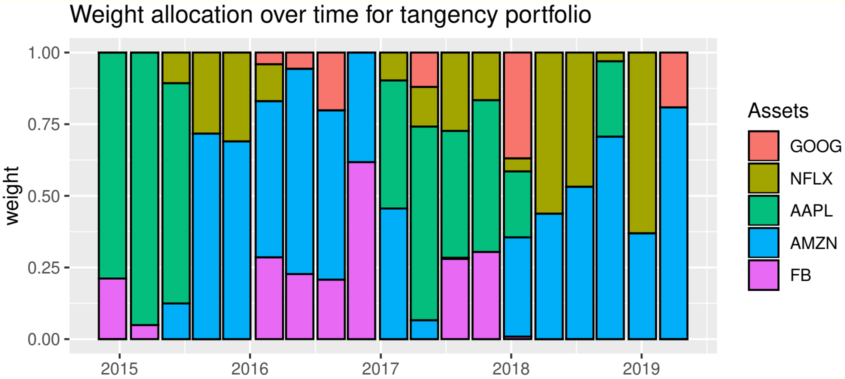
```
>>> import riskparityportfolio as rp
>>> my_portfolio = rp.RiskParityPortfolio(covariance=Sigma,
                                          budget=b)
>>> my_portfolio.design()
>>> my_portfolio.weights.numpy()
>>> my_portfolio.risk_contributions.numpy()
```

Practical Example

```r
library(portfolioBacktest)
library(riskParityPortfolio)
# download price data
faang_data <- stockDataDownload(
                   c("GOOG", "NFLX", "AAPL", "AMZN", "FB"),
                   from = "2014-01-01", to = "2019-06-25")
risk_parity <- function(dataset) {
  prices <- dataset$adjusted
  log_returns <- diff(log(prices))[-1]
  return(riskParityPortfolio(cov(log_returns))$w)
}
bt <- portfolioBacktest(
        portfolio_funs = list("risk parity" = risk_parity,
                              "tangency"    = max_sharpe_ratio),
        dataset_list = list(faang_data), T_rolling_window = 12*20,
        optimize_every = 3*20, rebalance_every = 3*20)
```

Practical Example

```r
backtestChartStackedBar(bt, portfolio = "tangency",
                        legend = TRUE)
```



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backtestChartStackedBar(bt, portfolio = "risk parity",
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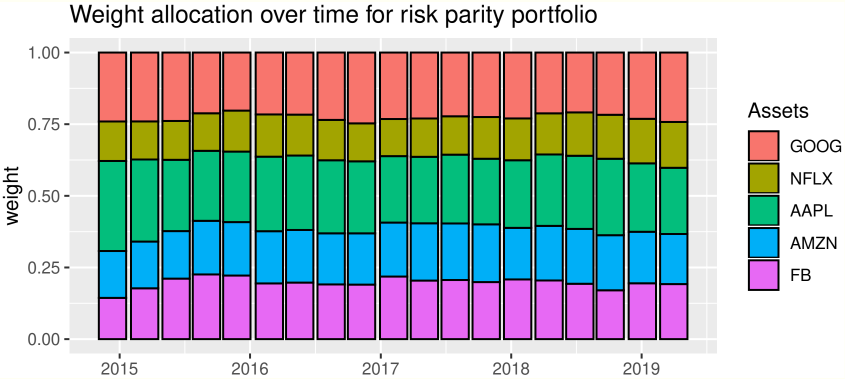
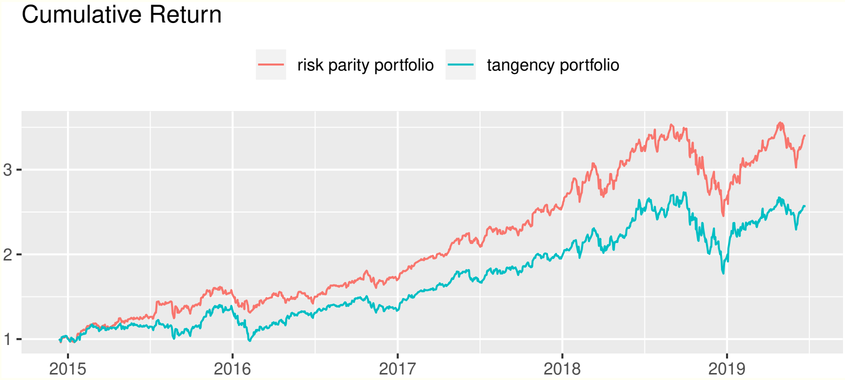


Practical Example

```
backtestChartCumReturns(bt) + theme(legend.position="top")
```



Testimonials

"I can easily run optimizations across new covariance matrices in seconds, which has helped streamline portfolio allocation testing"

Jonathan Dane, CFA. Director of Portfolio Strategy & Market Research, The Coury Firm.

"**riskParityPortfolio** provides a state-of-the-art implementation of rpps, otherwise only available at the top quantitative hedge funds"

Tharsis Souza, PhD. VP at TwoSigma. Founder, OpenQuants.com.

Disclaimer: views, thoughts, and opinions expressed in the text belong solely to the author, and not necessarily to the author’s employer, organization, committee or other group or individual.

Hierachical Risk Parity

Graph Theory 101

- A graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \boldsymbol{W})$

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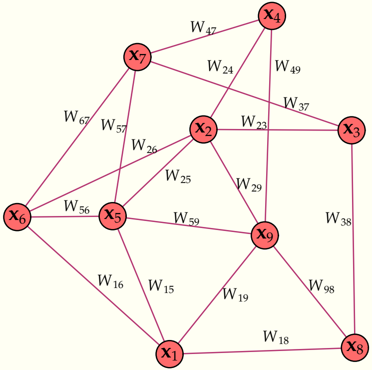

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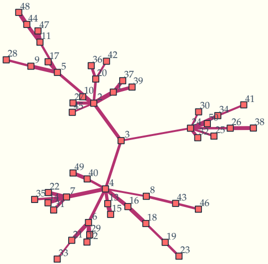
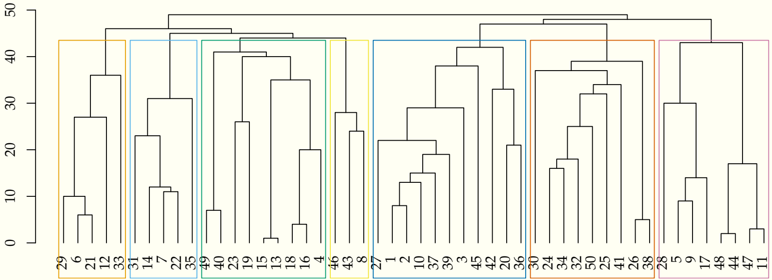
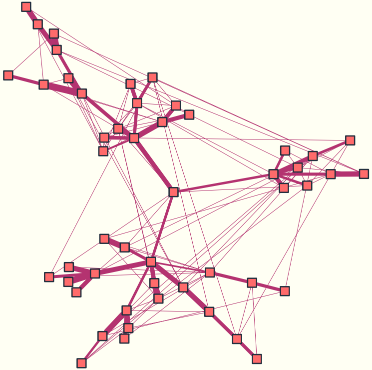




Graph Theory 101: Minimum Spanning Tree

- **def**: an MST of a graph $\mathcal{G}$ is a tree graph spans $\mathcal{G}$ with the minimum possible sum of edge weights
- An MST is **unique** as long as the edge weights are distinct



Graph Theory 101: Minimum Spanning Tree

Algorithm 2: BuildMST (Kruskal's algorithm)

Data: Distances between nodes $\{d_{ij}\}_{1 \leq i,j \leq p}$

Result: $\mathcal{G}_{\text{MST}}$

$\triangleright$ start with an empty graph $\mathcal{G}_{\text{MST}} \leftarrow (\mathcal{V}_{\text{MST}}, \mathcal{E}_{\text{MST}})$

$\mathcal{E}_{\text{MST}} \leftarrow \emptyset$

$\mathcal{V}_{\text{MST}} \leftarrow \{i\}_{1 \leq i \leq p}$

for $\{i, j\} \in \mathcal{V}_{\text{MST}}^2$ *ordered by increasing* d_{ij} **do**

  $\triangleright$ Verify that i and j are not already connected by a path

  if *not* connected(i, j) **then**

    $\mathcal{E}_{\text{MST}} \leftarrow \mathcal{E}_{\text{MST}} \cup \{\{i, j\}\}$

  end

end

Hierarchical Risk Parity

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- Alternatives:
  - Robustness
  - Bayesian priors, e.g., Black and Litterman
  - Improving inverse covariance matrix estimation, e.g., Ledoit and Wolf
- HRP idea: exploit the intrinsic hierarchy of the correlation matrix
- weights are allocated in a top-down fashion, just like how asset managers build their portfolios, e.g., from asset class, to sectors, to individual securities

Hierarchical Risk Parity

The HRP algorithm proposed by Lopez de Prado, 2016, has basically two steps:

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Hierarchical Risk Parity

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1. Build an MST with input matrix: $d_{ij} = \sqrt{\frac{1}{2}(1 - \rho_{ij})}$, where ρ_{ij} is the correlation between stock i and j
2. Use **Recursive Bisection** to allocate the portfolio weights

Hierachical Risk Parity

Algorithm 3: Recursive Bisection

Data: Distances between nodes $\{d_{ij}\}_{1 \leq i,j \leq p}$, estimated covariance matrix $\boldsymbol{\Sigma}$

Result: Portfolio allocation $\boldsymbol{w}$

1 ▷ Initialization

2 • set the list of items $L = \{L_0\}$, with $L_0 = \{i\}_{i=1,\dots,p}$ according to the order obtained by the dendogram

3 • assign a unit weight to all assets: $w_i = 1, \; \forall\, i = 1, \dots, p$

4 ▷ if $|L_i| = 1, \; \forall\, L_i \in L$ then stop.

5 ▷ for each $L_i \in L$ such that $|L_i|$:

6 • bisect L_i into two subsets, $L_i^{(1)} \cup L_i^{(2)} = L_i$, where $|L_i^{(1)}| = \text{round}(\frac{1}{2}|L_i|)$, and the order is preserved

7 • define the variance of $L_i^{(j)}$ as $\tilde{\sigma}_i^{(j)} = (\theta_i^{(j)})^\top \boldsymbol{\Sigma}_i^{(j)} \theta_i^{(j)}$, where $V_i^{(j)}$ is the covariance matrix between the constituents of the $L_i^{(j)}$ bisection and $\theta_i^{(j)}$ are the inverse variance weights

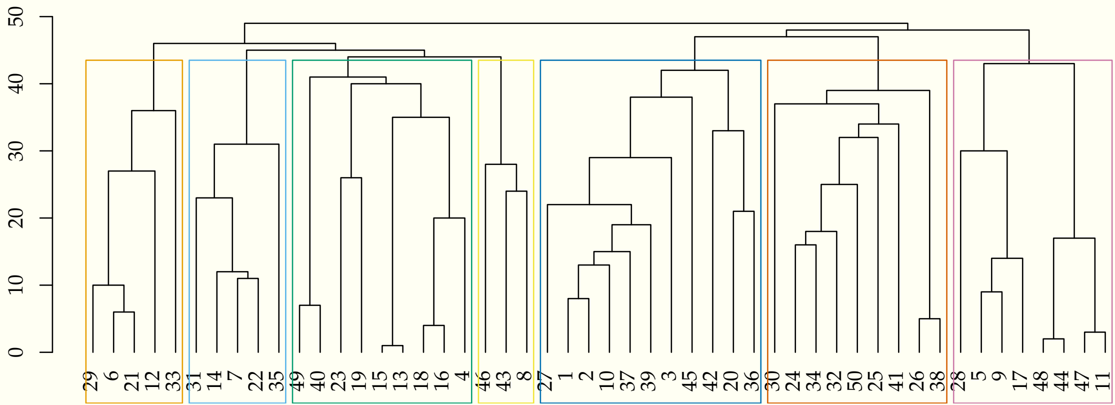
8 • compute the split factor $\alpha_i = 1 - \dfrac{\tilde{\sigma}_i^{(1)}}{\tilde{\sigma}_i^{(1)} + \tilde{\sigma}_i^{(2)}}$

9 • re-scale allocations $\boldsymbol{w}_l$ by a factor of $\alpha_i, \; \forall\, l \in L_i^{(1)}$

10 • re-scale allocations $\boldsymbol{w}_l$ by a factor of $1 - \alpha_i, \; \forall\, l \in L_i^{(2)}$

11 go to **4**

Hierachical Risk Parity



HRP in Practice

- pip install **mfinlab**

```
from mlfinlab.portfolio_optimization.hrp import HierarchicalRiskParity

prices = pdr.get_data_yahoo(stocks_, start="2018-12-31",
                            end="2019-12-31")[['Adj Close']]
hrp = HierarchicalRiskParity()
hrp.allocate(asset_prices=prices)
hrp_weights = hrp.weights
```

HRP in Practice

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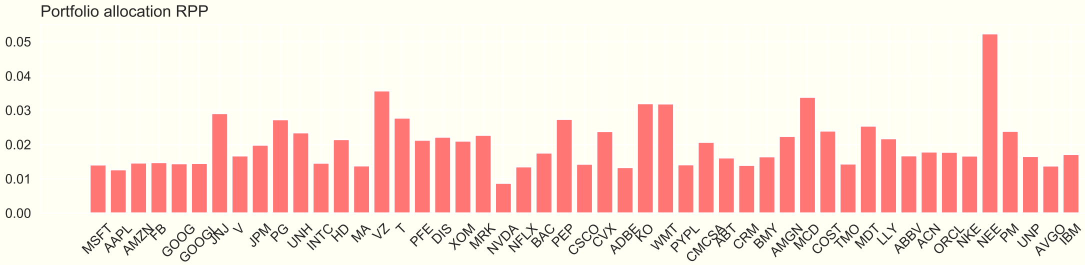

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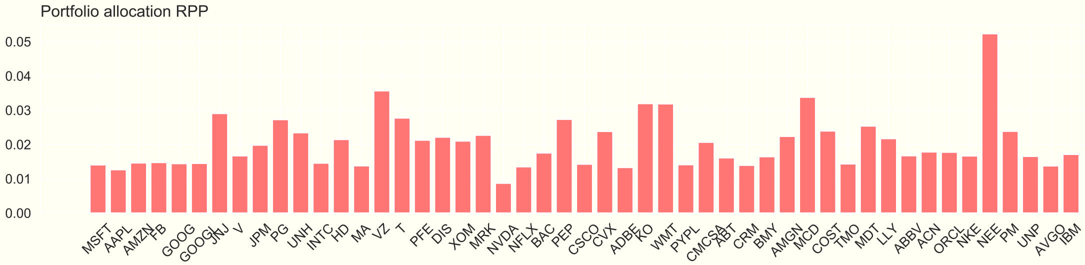

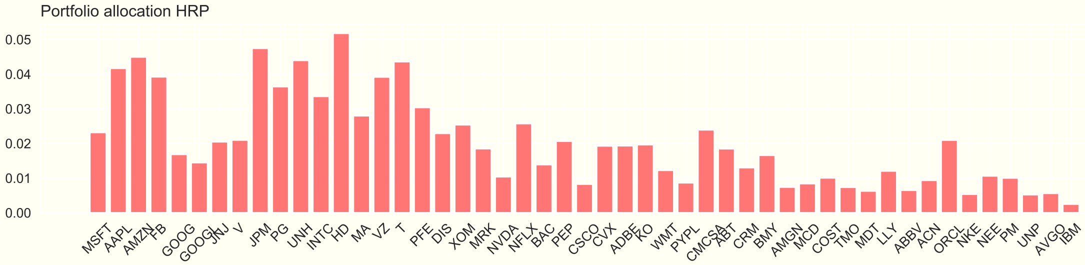

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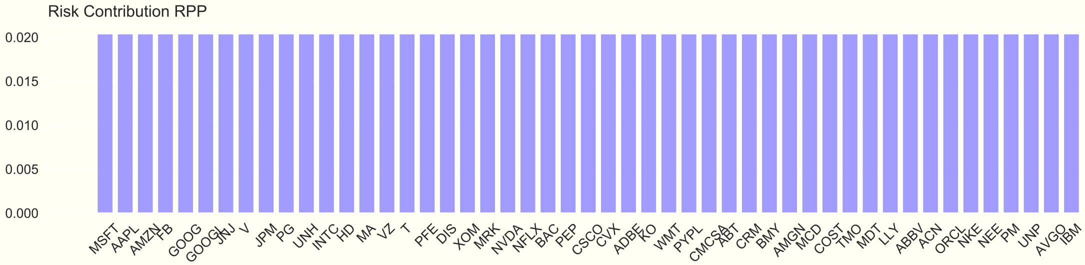
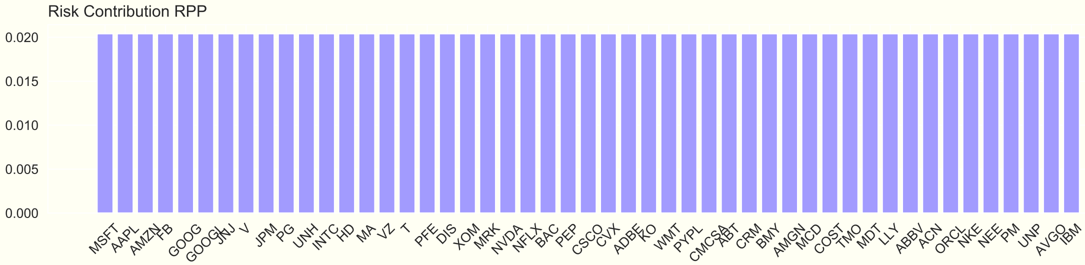
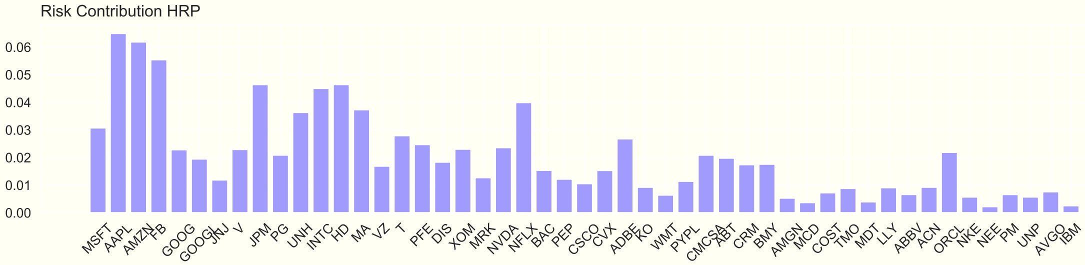


HRP in Practice





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Now, let’s evaluate those portfolios during the COVID-19 2020 crisis:

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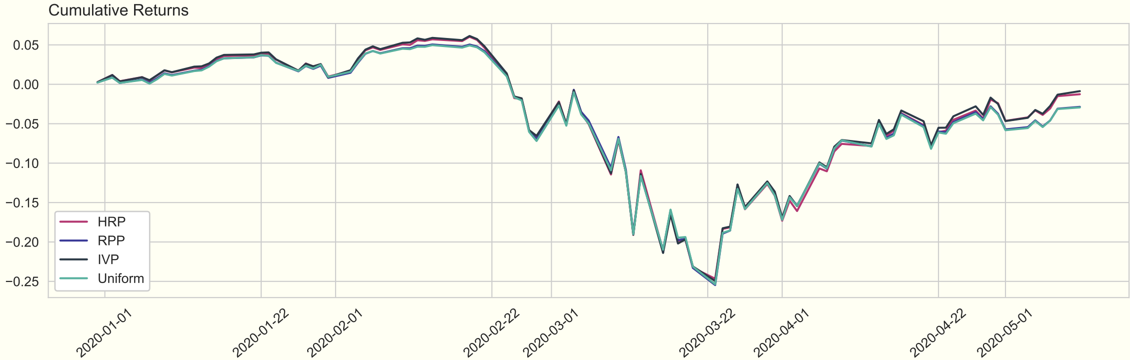

Takeaways

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2. non-convex? no problem!
3. hierarchical risk parity provides a natural way to apply graph theory in portfolio allocation
4. GitHub repositories to stay tuned:
   - dppalomar/riskParityPortfolio
   - dppalomar/riskparity.py
   - dppalomar/portfolioBacktest

Thank you! Questions?

References

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